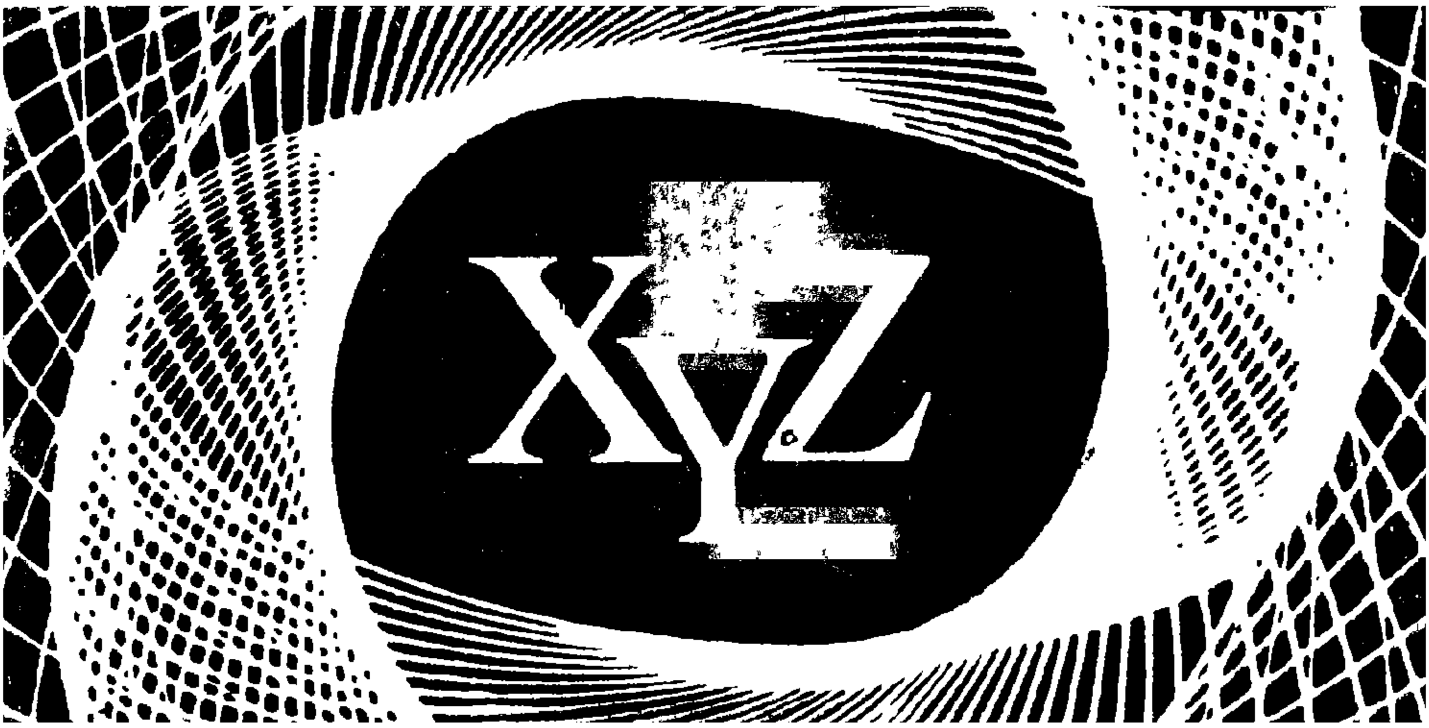
В·П·ДОРОЖКИНА

АНГЛИЙСКИЙ ЯЗЫК для математиков



ENGLISH for mathematicians

V. P. DOROZHKINA

EXTENSIVE ENGLISH COURSE FOR MATHEMATICIANS

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COMPLEMENTED

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В. П. ДОРОЖКИНА

АНГЛИЙСКИЙ ЯЗЫК ДЛЯ МАТЕМАТИКОВ

ИЗДАНИЕ ВТОРОЕ,
ДОПОЛНЕННОЕ И ПЕРЕРАБОТАННОЕ

Под редакцией В. А. Скворцова

Допущено Министерством высшего и среднего специального образования СССР в качестве учебника для студентов математических специальностей университетов

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Настоящий учебник состоит из 9 уроков. Каждый урок включает тексты, представляющие введение в определенную область математики, механики и кибернетики, лабораторные работы и разнообразные упражнения. Содержание и характер учебника направлены на формирование у студентов навыков дифференцированного чтения (т. е. чтения с определенными заданиями, уточняющими вид чтения), учебного перевода и письма, а также развития речевых умений. Речевые упражнения составлены согласно методике проблемного обучения — проблемное высказывание, проблемный вопрос, проблемная ситуация, дискуссия.

Для студентов-математиков и широкого круга специалистов.

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LESSON ONE

BASIC SCIENTIFIC CONCEPTS

Grammar:

1. Present Indefinite Tense.
2. Modal Verbs *can, may, must, should, ought* (*to*).
3. Word-Building Structure.
4. Sentence Structure.
5. Paragraph Structure. Topic Sentence. Generalizing Sentence.
6. The Text.

Word-Building Structure

In this lesson we study some basic concepts of the two sciences — **Textlinguistics** and **Mathematics**. The smallest unit of the text is a WORD, therefore we begin our study with the Word-Building Structure in English. The student can find the necessary unknown or unfamiliar word of the text in the dictionary and must learn its basic meanings and its related words, e. g, **to think** (about, of, out, over) — **thinkable, thinking, thinker, thought, thoughtful, thoughtless, thoughtfulness, thoughtlessly.**

Most long English words are not one word but, as a rule, two or more words put together. We see parts of words over and over again in other words. There are family relationships and resemblances. The learners of English must be architects and wreckers, they must learn to build words up and break them down. We build words up with three kinds of parts. The central part of the word is the ROOT. The part to the left of the root (or before it) is the PREFIX. The part to the right of the root (or after it) is the SUFFIX. So, first, we must observe relationships of words. For example, **imagination, imaginative, unimaginatively** are descendants of **image,** their parent word. Definitely, **finish, finite** and **infinity** are not strangers, they are family-related words.

VOCABULARY EXERCISES

I. *Find the following words in the English-Russian dictionary, learn to master the procedure to spell long English words. They are easy to build up and break down.*

	to de-**ter**-mine		to **sig**-ni-fy (**sign**)		to or-**der**
	de-**ter**-min-ant		**sig**-ni-fi-ca-tion	dis-	or-**der**-ed
in-	de-**ter**-min-ant-ly	in-	**sig**-nifi-cance		or-**der**-li-ness
	to dif-**fer**		to dif-**fer**-en-ti-ate		to de-**rive**
	dif-**fer**-ence		dif-**fer**-en-tial		de-**riv**-a-tive
in-	dif-**fer**-ent-ly		dif-**fer**-en-ti-a-tion		de-**riv**-a-tion

II. *Collect some more family-related words with the same root in the dictionary.*

Un-**lim**-it-ed, **use**-ful-ness, non-**math**-e-mat-i-cal-ly, re-**trans**-la-tor, in-ef-**fi**-cient-ly, **lead**-er-ship, im-**pos**-si-bil-i-ty, un-**busi**-ness-like, pre-**par**-ation, con-**sci**-ence, **sci**-ence, con-**sid**-er-a-ble, mis-un-der-**stand**-ing.

III. *Consult the dictionary and write down the given verbs with their basic meanings into your vocabulary copybook. Collect the family-related words with the same root (their derivatives) i. e., nouns, adjectives, adverbs and set phrases (expr.) in the dictionary.*

Models. 1. **to relate v. t.**, related adj., relative n.; adj., relativity n., relation n., (inter)relationship n.; relatively adv., expr. **in (with) relation to.**

2. **to occupy v. t.**, occupational adj., occupied adj., occupation n., occupant n., occupancy n.

3. **to except v. t.**, exceptional adj., exceptionable adj. exception n., exceptant n., except prep., excepting prep., exceptionally adv., expr. **except that.**

1. to abstract
2. to accomodate
3. to acquire
4. to admit
5. to advance
6. to appear
7. to attend
8. to avail
9. to believe
10. to belong (to)
11. to bound
12. to calculate
13. to change
14. to check (up)
15. to compete
16. to compose
17. to compress
18. to comprise
19. to compute
20. to connect
21. to contain
22. to confound
23. to constitute
24. to contribute (to)
25. to construct
26. to count
27. to define
28. to deliver
29. to depend (on, upon)
30. to descend
31. to describe
32. to determine
33. to develop
34. to direct
35. to discover
36. to discourse
37. to draw
38. to employ
39. to equal
40. to equate
41. to evolve
42. to falcify
43. to familiarize
44. to figure
45. to follow
46. to force
47. to form
48. to found
49. to generalize
50. to graduate (from)
51. to hold
52. to increase
53. to initiate
54. to interpret
55. to invent
56. to mean
57. to measure
58. to mention
59. to miss
60. to ob'ject (to)
61. to observe
62. to originate (in, with)
63. to owe
64. to permit
65. to pertain
66. to produce
67. to provide
68. to recognize
69. to record
70. to regard
71. to remove
72. to render
73. to reproduce
74. to require
75. to research (into)
76. to resemble
77. to reserve
78. to sense
79. to separate
80. to serve
81. to signify
82. to solve
83. to state
84. to sub'ject (to)
85. to suggest
86. to terminate
87. to transmit
88. to value
89. to view
90. to visualize

IV. *Explain the difference (distinction) in the meanings of the words.*

to ob'ject (to) — to dislike | to sub'ject (to) — to be 'subject (to) | to determine — to be determined | to define — to determine | to differ — to differentiate — to discriminate | to draw — to paint | to bound — to limit | to study — to research | to translate — to interpret | to regard — to view | to permit — to let | to deliver — to transmit | to uncover — to discover | to unclose — to disclose | to speak — to converse | to end — to terminate | to find — to found | to understand — to figure out | to use — to employ | to angle — to corner | to break — to wreck — to crash | to visualize — to imagine — to fancy | view —

sight | vision — envision — revision | visible — visionary | to call — to name — to label — to term | to number — to reckon — to count — to figure — to compute — to calculate | to **scribe** — to a**scribe** — to circum**scribe** — to de**scribe** — to in**scribe** — to pre**scribe** — to sub**scribe** — to tran**scribe** | to **course** — to dis**course** — to have inter**course** — to re**course** — to con**course** | to de**scend** — to a**scend** — to tran**scend** | to **form** — to per**form** — to trans**form** — to re**form** — to con**form** | to **quire** — to en**quire** — to in**quire** — to re**quire** — to ac**quire** | to master — to muster | to accept — to except | can — to can | may — to may | must — to must | to cut — to cross — to slice — to sect — to section — to segment.

Sentence Structure

The words of every language fall into definite **grammatical classes** or **parts**, which differ from each other in form, in meaning and in function. The student must learn to differentiate: **parts of speech** in English with their grammatical categories — nouns, verbs, pronouns, adjectives, numerals, adverbs, prepositions, conjunctions, particles (Morphology); **parts of the sentence** — the subject, the predicate, the object, the attribute, the adverbial modifier and their respective clauses (Syntax); **parts of the text** — the sentence and the paragraph (Syntactics). There are close connections and interrelationships of these parts in the text as one and the same word is both a definite part of speech and functions as a definite part of the sentence. We begin our study with the syntax of the sentence — **Sentence Structure**, its fixed word order and **the Predicate** which is the core of the sentence. The student must analyze the predicate from the point of view of its structure (simple and compound) and its content (process and non-process). Syntactic analysis and segmentation of the sentence helps determine the boundaries of the predicate and thereby locate the other principal sentence-parts — **the subject** that precedes the predicate and **the object** that follows it. Recent syntactic research shows that the qualitative type of the compound predicate may be long and expanded: He **is here**. She **likes her friends**. The University **has many Departments** or the University **consists of many Departments**. I **am old enough to understand it**.

Study the models of the English Sentence — the smallest structure which can express a complete thought.

Present Indefinite Tense

The Predicate

Repeat the sentences after the English and Russian instructors. Characterize the predicate.

1. We **are in Moscow** at present.
2. These young men **are first-year students of Moscow State University.**
3. Moscow University **is one of the world's best centres of learning.**
4. It **has many different Departments for humanities and for sciences.**
5. The University Course for students **runs for five-six years.**
6. The University campus **occupies** a large territory.
7. The real wealth of a University **is its faculty (teaching staff) and students.**
8. Many prominent scientists **work** in Moscow University.
9. They **train** specialists in all fields of modern science.
10. The main 33-storied building of the University **houses** three Departments: Geology, Geography and our Department of Mathematics and Mechanics.
11. The Department's Faculty **comprises outstanding mathematicians and scientists of mechanics.**

12. Almost every member of the teaching staff **is both a teacher and a researcher.**

13. They **combine teaching, lecturing and scientific research.**

14. The Heads of the Departments **are academicians, corresponding members of the Academy, eminent professors.**

15. Apart from their lectures they **hold regular seminars and discourses on scientific topics.**

16. There **is** a post-graduate course at the Department as well.

17. Post graduates **carry on research** in all branches of mathematics and mechanics.

18. These students **study** at the Department of Mathematics and Mechanics.

19. Some of them **do computing mathematics** at the Department of Cybernetics.

20. The curriculum for the first term **covers** the fundamental topics in the calculus, higher algebra, analytic geometry, and differential equations together with social sciences, foreign languages and athletics.

21. Moscow University **provides** sport facilities for every level.

22. Most students **are members of different sport teams such as gymnastics, swimming, tennis, football, ice hockey, skiing, etc.**

23. The University Library **has thousands of volumes, rare books, periodicals and scientific journals for study and research in special fields.**

24. Because of the University world-wide reputation it **specializes** students from many foreign countries.

25. Tuition **is free of charge** in all Soviet educational institutions.

26. Students with good progress in their studies **get** state grants.

27. Moscow University hostels **accomodate** non-moscovites and foreign students.

28. Admission to the University **is by competitive examinations.**

29. Candidates **compete** for enrolment.

30. The attendance of lectures and seminars **is obligatory.**

EXERCISES

Declarative Sentence Structure
Parts of the Sentence. Direct Word Order

IV
The Adverbial Modifier

I **The Subject +** The Attribute	II ***The Predicate***	III **The Object +** The Attribute

Models: On [IV] week days / the **[I] first-year students** / [IV] usually / **[II] attend** / [IV] in academic groups / their regular [III] lectures, classes and seminars / in our [IV] Department's lecture rooms.

[IV] There **[II] is a [I] refreshment room** on [IV] the fifteenth floor of the Department.

I. *Arrange the following words into sentences.*

1. Are, academic, two, year, there, terms, every, in, autumn, spring, and. 2. Each, the, end, at, students, term, of, take, and, tests, exami-

nations, credit. 3. Our, trains, mathematics, department, students, of, mechanics, and. 4. Undergraduates, classes, the time-table, attend, set out, in. 5. First, term, and, subjects, carriculum, the, includes, of, mathematics, mechanics, modern. 6. After, classes, go, students, sports, to, ground, the, many, play, to, tennis. 7. Of, hundreds, every, students, year, from, graduate, the, University. 8. Offers, department, residential, its, to, students, the, accomodation. 10. Difficult, is, it, to, study, at, mathematics, department, of, mechanics, and. 11. The, universities, in, most, work, of, fundamental, in mathematics, carry out, mathematicians, the. 12. Science, the, mathematics, a, is, cornerstone, of, modern. 13. Human, the, intellect, noble, the, mathematics, is, of, rigorous, and, activity. 14. Abstractions, nature, the, scientists, of, employ, mathematics, understand, the, to, patterns, of.

Agreement between the Subject and the Predicate

II. *Remove the parentheses (round brackets).*

1. These students (to study) at the Department of Computing Mathematics and Cybernetics. 2. She (to do) well in mathematics. 3. We (to attend) lectures of professor X. He (to be) a great specialist in the theory of elasticity. 4. They (to solve) mathematical problems. 5. The Head of the Department of functional analysis (to deliver) lectures and (to hold) seminars. 6. Dean's office (to be) on the fourteenth floor. 7. At the end of each term undergraduates (to sit) for examinations. 8. She (to specialize) in the theory of 'Probability. 9. Differential equations and equations in partial derivatives (to be) subjects of Analysis. 10. Mathematical Statistics (to play) an important role in many problems of science and engineering. 11. Mathematicians (to base) the Calculus on the concept of the limit of a sequence and the concept of the limit of a function. 2. The basic elements of mechanics — Bodies, Motions, Forces (to constitute) abstractions from physical objects and interactions.

III. *Listen to the tape-recording. Write down each sentence in English. Try to say the sentences without the key on the tape.*

1. Московский государственный университет (с 1775 г.) — старейший и крупнейший центр образования, науки и культуры. 2. Университет готовит молодых специалистов на 15 естественных и гуманитарных факультетах. 3. В университете 280 кафедр, 360 лабораторий, 11 учебно-научных станций, 4 научно-исследовательских института, вычислительный центр, Ботанический сад, 4 астрономические обсерватории, 3 музея, Научная библиотека им. А. М. Горького, издательство и типография. 4. В МГУ в настоящее время обучаются более 27 тысяч студентов и около 5 тысяч аспирантов. 5. Обучение в МГУ бесплатное, и большинство студентов получают государственную стипендию. 6. Срок обучения в МГУ — 5-6 лет. 7. В составе университетского студенчества — представители всех национальностей Советского Союза и большое число студентов, приехавших более чем из 100 стран мира. 8. Преподавательский коллектив университета насчитывает 8,1 тысячи профессоров, преподавателей и научных сотрудников. 9. Механико-математический факультет (с 1933 г.) — ведущий центр математической науки в нашей стране. 10. Московская математическая школа имеет мировую известность. 11. Мехмат готовит специалистов математиков и механиков высокой квалификации. 12. На мехмате работают 10 академиков, 12 членов-корреспондентов АН СССР, 81 профессор, 7 Героев Социалистического Труда, 26 сотрудников факультета — лауреаты Ле-

нинских и Государственных премий. 13. Математика — основа точного естествознания и ее значение в общей системе человеческих знаний все возрастает. 14. Механика — одна из главных научных основ техники.

Interrogative Sentence Structure

General and Special Questions. Answers

Interrogative sentences (questions) have the following characteristics: a) peculiar interrogative intonation (1); b) indirect word order (3, 4, 5, 6); c) interrogative pronouns (question words) *who, whom, whose, what, which, where, why, how much (many)*, etc.

Study the models and generalize the grammar rules.

Models. *Questions*	*Answers*
1. They /know/ the history of mathematics? (coll.)	Yes. (Certainly). No. (Unlikely).
2. Who knows the history of mathematics?	Mathematicians do.
3. Do they know the history of mathematics?	Yes, they do. No, they don't.
4. Don't they know the history of mathematics?	Yes, they do. No, they don't.
5. What do (don't) they know?	They know (don't know) the history of mathematics.
6. What history do (don't) they know?	The history of mathematics.

1. The University course in the history of mathematics and mechanics / is helpful to mathematics and mechanics classes.

What is helpful to classes on speciality? What course is helpful? It is the University course in the history of sciences. Is (isn't) the University course in the history of sciences helpful to classes on speciality? Yes, it is. What is it helpful to? It is helpful to classes on speciality.

2. **The history of the sciences / shows / how they originate, develop, grow and change.**

What (what history) shows how the sciences originate, develop, grow and change? Does (doesn't) the history of the sciences show how they originate...? What does the history of the sciences do? **What** does the history of the sciences show?

3. **Historical topics / increase / the students' interest in their speciality subjects.**

What increases the interest of the students in their speciality subjects? Do (don't) historical topics increase the interest of the students in their speciality subjects? What do historical topics do? What do historical topics increase? Whose interest do historical topics increase? In what subjects do historical topics increase the students' interest?

Disjunctive Questions

1. The use of the history of mathematics and mechanics gives a good understanding of the foundations of these sciences, **doesn't it?** It does, surely. 2. Students are interested in the "men of mathematics" as people, **aren't they?** They are, indeed. 3. A historical topic can be an important tool to create insight(s), **can't it?** It can, in fact.

Alternative Questions

1. Can a story about the discovery and invention in mathematics increase **or** decrease students' interest in mathematics? It can increase it. 2. Do modern mathematicians create abstract **or** empirical mathematics? Abstract mathematics, sure enough. 3. Do(es) mathematics advance through one technique **or** the interplay of many techniques? Of course, through many.

The sentence and sometimes the whole text do not give any information to answer **"problem"** questions. The student ought to produce an answer of his own.

Problem Questions

Model. **A positional numeration system first appears in history at different times and in different countries.**

Problem questions	*Possible answers*
What is numeration?	Numeration is a system of symbols that we use to express numbers.
What is the first positional numeration system known in history?	It is the ancient Babylonians' base-sixty-positional system without a special numeral for zero.
What is the system we use today?	It is the digital Hindu-Arabic base-ten-and-place-value system: 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.
What does the zero symbol signify?	In numeration systems 0 signifies the absence of any units of various number orders.

Negative Sentence Structure

1. Mathematicians **do not know** the early beginnings of the language to express form and quantity (=the early beginnings... are **not known,... are unknown**). 2. The primitive isolated civilizations **do not have** (=**have no**) numbers or number symbols but only words for various human body parts (hands, fingers, toes) to count with. 3. Historians of mathematics **cannot (do not) answer** exactly such questions as: "When (where) does the cardinal number first appear?" or "Who personally invented a positional numeration system?" etc.

Paragraph Structure. Topic Sentence

A **paragraph** is a separate or a distinct section of the text. A paragraph usually contains a number of sentences with one (or more) **main idea**(s). Each paragraph of the text has some extra information unnecessary for the reproduction or writing an abstract. When the student must reproduce the paragraph he can compress it and generalize the main idea(s) with the help of the **topic sentences(s)**. It may happen that the paragraph states the topic sentence (T. S.) that renders the main idea(s) quite obviously, but as a rule the student must construct it himself.

EXERCISES

I. *Reproduce the given paragraph as fully as you can.*

Any positional or place-value numeration system cannot possibly function adequately without a symbol for an empty place or position

(T. S.). In pre-Babylonian Sumerian system as early as 3500 B. C. an empty place in the numeral is actual "blank" or "empty place". The idea of zero (=cipher, nought) as a placeholder first appears in Babylonians' mathematics. Zero as a number has a rather different history. Zero has no place and plays no part in classical Greek mathematics (600—300 B. C.) Hindu and Arab mathematicians of the period 500—1100 A. D. first recognize zero as a number through their attempts to solve quadratic equations of the type $ax^2 - bx = 0$, where one root is zero and through their systematic study of the properties of operations on numbers. We can say that **zero in modern elementary mathematics is both a placeholder and a cardinal number** (T. S.). Its role depends on the context — when we speak about numeration zero is a placeholder, while zero is a cardinal number in any positional number system.

The working rules for compression and generalization of the paragraph

To single out or construct the generalizing sentence(s) the student should a) analyze **all the subjects** of the sentences in the paragraph and choose the one (ones) that repeats itself and constitutes the logical component(s) of the paragraph; b) analyze all **the predicates** and select that one (those ones) that presents some new and important information that characterizes the subject; c) **the object(s)** and **the adverbial modifier(s)** of the generalizing sentence must link its subject with modern times or modern mathematics.

Generalizing Sentence

II. *Choose the generalizing sentence among the given or generalize the paragraph with the sentence of your own.*

1. "Zero as a placeholder" pertains to numeration, while "zero as the cardinal number of the empty set" comes under number with some of its other possible and different meanings in modern mathematics. 2. Zero functions in a different sense in numeration, number theory, cybernetics, physics and engineering. 3. The origin of zero concept dates back to prehistory of computation and results in its numerous different functions in modern mathematics. 4. Zero runs throughout long history of science in different roles and senses and its functions in modern mathematics are numerous. 5. Modern mathematicians operate with zero concept in many different ways. 6. Zero concept originates in history as a placeholder and acquires many new connotations in modern science and engineering.

Read the text. a) Segment it into paragraphs. Generalize the main idea(s) of each paragraph. Write an outline of the main ideas of the text; b) Draw all the geometric figures mentioned in the text and discuss their properties.

THE TEXT
BASIC GEOMETRIC CONCEPTS

The practical value of Geometry lies in the fact that we can abstract and illustrate physical objects by drawings and models. For example, **a drawing of a circle is not a circle, it suggests the idea of a circle.** In our study of Geometry we separate all geometric figures into two groups: **plane figures** whose points lie in one plane and **space figures** or **solids.** A **point** is a primary and starting concept in Geometry. **Line segments, rays, triangles** and **circles** are definite sets of points. A simple closed curve with line segments as its boundaries is a **polygon.** The line segments are **sides** of the polygon and the end points of the segments are **vertices** of the polygon. A polygon with four sides is a **quadrilateral.** We can name some important quadrilaterals. Remember, that in each case we name a specific set of points. A **trapezoid** is a quadrilateral with one pair of parallel sides. A **rectangle** is a **parallelogram** with four right angles. A **square** is a rectangle with all sides of the same length. The **regular polyhedra** are a part of geometric study chiefly in antiquity. They have a symmetrical beauty that fascinate men of all ages. The first question in connection with regular polyhedra is: How many different types are there? Thanks to the ancient Greeks we know that there are exactly five types of polyhedra. All objects in their view are composed of four **basic elements: earth, air, fire** and **water.** They believe that the fundamental particles of fire have the shape of **tetrahedron**, the air particles have the shape of **octahedron**, of water — the **icosahedron**, and the earth — the **cube.** The fifth shape, the **dodecahedron**, they reserve for the shape of the universe itself. Plane geometry is the science of the fundamental properties of the **sizes** and **shapes** of objects and treats geometric properties of figures. The first question is: Under what conditions two objects are **equal** (or **congruent**) in size and shape? Next, if figures are not equal, what significant relationship may they possess to each other and what geometric properties can they have in common? The basic relationship is shape. Figures of unequal size but of the same shape, that is, **similar figures** have many geometric properties in common. If figures have neither shape nor size in common, they may have the same area, or, in geometric terms, they may be **equivalent**, or may have endless other possible relationships. Geometry is the science of the properties, measurement and construction of **lines, planes, surfaces** and different geometric figures. What do we call "**constructions**" in our study of Geometry? Ruler-compass constructions are simply the drawings which we can make when we use only a straightedge and a compass. A compass is a misleading word. It is not only «компас» in mathematics it is usually «циркуль». We call such misleading words «ложные друзья переводчика». For a ruler you ought to use an unmarked straightedge because measurement has no role in ruler-compass constructions. Of course, you can use a marked straightedge if you don't permit yourself to use these marks for measurement. Later you ought to do some measurement to "check" your constructions. We measure segments in terms of other segments and angles in terms of other angles. It seems only natural that we find areas **indirectly** as well. How does a person find the area of a floor? Does he take little squares one foot on a side, lay them out over the entire floor and thus decide that the area of a floor is 100 square feet, for this is indeed the meaning of area? Of course, he does not. He measures the **length** and **width**, quantities usually quite simple and then multiplies the two numbers to obtain the area. This is **indirect measurement,** for we find the area when we measure lengths. The dimensions we take in the case of **volume** are the area and the length or the **height.** Greek mathematicians are the founders of indirect measurement methods. Their contribution to this subject are formulae(=las) for areas and volumes of particular geometric shapes, that we use nowadays. Thus thanks to the Greeks we can find the area of any one single triangle when we take **the product** of its **base** and half its height. We also know due to them, that **the "areas of two similar triangles are to each other as the squares of corresponding sides"**. In other words, even the very common formulae of Geometry which we owe to the Greeks permit us to measure areas and volumes indirectly, when we express these quantities as lengths. We ought not to undervalue this contribution of the ancient Greek mathematicians. Their formulae for areas and volumes represent a great practical and important result. But this type of indirect measurement is not the only one of interest to the Greeks. They measure indirectly **the radius** of the Earth, **the diameter** of the sun and moon, the distances to the moon, the sun, some planets and stars.

GRAMMAR EXERCISES

I. *Write 12 sentences with the active Vocabilary words and ask your pair-mate to translate them. Change the roles over!*

Models. Such verbs as **to think; to meditate, to ponder, to speculate signify** mental activities. When people **count (calculate, compute)** they work with numbers. We can **picture or figure** geometric objects quite easily **by means of** their physical counterparts. We may **imagine** e. g., two sun rays that meet in a point, and thus we **visualize** a geometric point. We may next **idealize, generalize** and **summarize** common properties of geometric points. Hence, geometric concepts are **abstractions from** physical objects.

II. *Ask and answer all possible questions.*

1. The exact period in ancient history of the epochal invention of cardinal number is unknown. 2. History teaches that civilizations develop at various rates in different places. 3. In primitive civilizations "number words" appear with the creation of a language, both spoken and written. 4. The oldest written documents available show the simultaneous appearance of the cardinal number concept in ancient China, India, Mesopotamia and Egypt. 5. These documents contain the question "How many?" 6. People can answer this question best in terms of a cardinal number. 7. The ancients use both their fingers and toes as the natural counting sequence. 8. The modern fundamental concept of a "set" is the man's first abstraction. 9. We name the present so-called Hindu-Arabic numeration system after the Hindus — its first inventors, and after the Arabs, its later transmitters to Europe. 10. The number of ancient numeration systems is about the same as the number of ancient written languages. 11. The Hindu-Arabic base-ten-and-place-value system is universal nowadays because it is unambiguous and easy to count with, but people still use binary, four-, twelve-, twenty-, sixty-, etc., base numeration systems as well.

III. *Make the false statements negative. Paraphrase, if possible, the negative sentences in more than one way.*

Model. Mathematicians define this basic term.
Mathematicians **do not define** this basic term.
No mathematician defines this basic term. (Ни один ... не)
Don't mathematicians **define** this basic term? (Разве ... не)

1. The classical Greeks' (600—300 B. C.) numeration system is positional. 2. Zero plays an important role in nonpositional numeration sytems, e. g., Romans'. 3. Nonpositional numeration follows positional numeration in most civilized regions of the ancient world. 4. Letters of the ancient Greeks' alphabet serve as letters only. 5. There is a special sign in Greek and Hebrew numeration systems which helps interpret the composition of letters as a number. 6. There is a great need for a zero symbol in ancient Greeks' system. 7. People can recognize the Greek letter-

numeral without difficulty. 8. Modern mathematicans use the classical Greeks' numeration system. 9. There can be only one way to represent numbers nowadays. 10. It is possible to describe precisely the development of the Hindu-Arabic numeration system.

IV. *Ask questions to which the following statements may serve as the answers.*

Model. **We are already familiar with basic concepts of Geometry through our highschool studies of mathematics.**
Why are you already familiar with basic concepts of Geometry?

1. In Geometry we study drawings and models that represent geometric concepts. 2. Ruler and compass are the simplest instruments to make a drawing. 3. With a ruler (straightedge) we may draw (that is, construct) a line. With a compass we may construct a circle. 4. Measurements have no role in ruler-compass constructions. 5. The necessity for geometric drawings and models is as old as Geometry itself. 6. Visual method is especially important in Geometry. 7. The power to picture mentally a geometric object is a great talent. 8. The figure is to the geometer what the numerical example is to the algebraist. 9. Plane geometry requires drawings, but solid geometry — models. 10. Models of Geometry are idealizations abstracted from physical objects. 11. All geometric models are inaccurate and misleading. 12. The points of Geometry have no size and no dimensions. 13. Geometric planes have no boundaries, they are endless in both directions. 14. No, there is nothing in the physical world that illustrates these geometric concepts with complete accuracy. 15. A point is a primary and starting concept in Geometry. 16. We can define all other geometric figures in terms of sets of points. 17. Plane geometry is the science of the metric properties and constructions of geometric figures. 18. Solid geometry studies the properties of the figures in space and the measurement of areas, surfaces and volumes of solids.

V. *Write some problem questions that pertain to numeration systems and basic geometric concepts. Ask your groupmates to answer them. Work in pairs in class.*

CONVERSATIONAL EXERCISES

I. *Read the given sentences, find some more information and dispute the (dis)advantages of a certain numeration system.*

1. The concept of number does not appear all of a sudden. 2. Scientits do not have enough evidence to fix the period in history of the invention or discovery of cardinal numbers. 3. The earliest documents available show that the number concept is equally present in many ancient civilizations. 4. The first requirement in computation is a system of numerals, i. e., a way to write numbers. 5. Numeration first evolves through the use of spoken and later on, written languages. Some ancient tribes use a base of 2 and 3 to count by (1-2, 2-1, 2-2) (1-2-3, 3-1, 3-3). 6. Sometime before 2000 B. C. **the Babylonians** develop a base-sixty or sexagesimal system of numeration with the positional principle which is still useful in astronomical calculations. The Babylonians of 2000 B. C. were well-trained and skillful calculators. 7. **The early Egyptian numeration system** uses a base of ten with no more than three symbols to express any number less than 1000 — one for units, one for tens, and one for hundreds. The zero

symbol is unnecessary. 8. Very little is known about the origin of **the Roman notation for numbers**, which is still important. 9. **The ancient Greeks' nonpositional numeration** system employs twenty-four letters of their alphabet to produce letter-numerals and special symbols (M=myriad) for large numbers. To tell a number from a word the ancient Greeks use an accent (=stress) at the end of a number sign or a stroke over it. 10. The traditional **Chinese-Japanese numeration system** is a base-ten system with nine numerals and symbols for the place-value. Numbers go from the top downward or from left to right. 11. **Mayan numeration system** (400 A. D.) uses the base twenty with positional notation and a special symbol for zero. 12. The details of the exact formation of **the Hindu-Arabic symbolic system** are missing. 13. **Binary system** is of recent origin and extremely important in Cybernetics. It needs only a sequence of two digits, 0 and 1, to represent numbers of any size. 14. The advantages of our **present positional numeration systems**, based on place value, with the choice of a certain number as a base are numerous and well-known. 15. **The British monetary system** with its farthings, pennies, threepence, sixpence, shillings, half crown, crowns, pounds and guineas was a 1/4 — 1/2 — 1 —3 —6 —12 —30 —60 —240 —252 system — a mixture of several archaic systems that confounds foreigners so much. It is different nowadays.

II. *Make the right choice and complete the sentences.*

1. The set of all points in Geometry is a ... (volume, plane, line, space, model, surface). 2. Sets of points which all lie in one plane are... (circles, rays, angles, solids, ellipses, plane figures, squares). 3. The regular polyhedra are a part of geometric study in antiquity. How many different types are there? (rhombus, trapezoid, square, cube, tetrahedron, octahedron, dodecahedron, icosahedron). 4. A solid with opposite faces equal and parallel is a... (cube, cylinder, prism, pyramid, sphere). 5. The idea of "betweenness" in mathematics means that... (one point is between two other points, all the given points lie on one line). 6. The set of 3 points not all on one line and all the points between them on the segments is a... (parallelepiped, triangle, rhombus, rectangle, cone). 7. When we cut a cone at different angles we obtain a set of curves such as... (a circle, cycloid, catenary, ellipse, parabola, hyperbola, conchoid, quadratrix, spiral, circumference). 8. It is convenient to have labels for angles and we usually classify them according to... (the measures of their angles, the measures of their sides). 9. The angle of 90° is ... (straight, right, acute, obtuse, adjacent, complementary). 10. A triangle with its sides equal is... (right, acute, isosceles, equilateral). 11. The distance around the circle is a (an)... (perimeter, parabola, hyperbola, circumference, ellipse). 12. We describe the concept of congruence in Geometry by the phrase "..." (is the same size as, has the same length as, is equal to, has the same measure as). 13. The area of a rectangle is the product of two dimensions: (the side and the base; the side and the altitude; the base and the height). 14. If we know the three sides of any triangle, we can find its area by Heron's formula... ($S=6a^2$; $V=a^3$; $S=\sqrt{p(p-a)(p-b)(p-c)}$). 15. The area of a circle contains the number π ... ($\pi d^2/4$; $V=\pi r^2 h$; $2\pi rh$).

III. *Agree or disagree with the following statements.*

Models. **In Geometry we study drawings that illustrate geometric figures.**

It's true. Geometric figures are graphic formulas and no geometer can do without them.

Ancient mathematicians' indirect measurements formulas are out-of-date and useless nowadays.
Not at all. When we calculate the area, surface or volume, we make use of them. They are very valuable though they seem trivial today.

1. Indirect measurement of areas and volumes is an old-fashioned method. 2. The ancient Egyptians initiate the science of measurement. 3. The ancient Greek mathematicians base their mathematics on the results of their predecessors (Egyptians and Babylonians). 4. Archimedes's way to find the area of a circle is well-known. 5. In scientific work we usually measure in units of the metre and the kilogram of the metric or decimal system. 6. The decimal system is not the international system of measures and weights nowadays. 7. The only big countries that still use the imperial system of feet and pounds are America and Canada.

IV. *Try to define all geometric objects mentioned in the text.*

A definition is a phrase that signifies a thing's essence. **A scientific definition** is both a **description** of a scientific concept such as "force", "distance", "energy", "velocity", "acceleration", "momentum", etc., and **the way to measure it.** The formula $d=16t^2$ tells us how a rock falls. We define "acceleration" as $\Delta V/\Delta t=32$ or Gain of Velocity/Time Taken (Rate of-Gain-of Velocity) equals 32.

Model. ... — это (значит, означает, обозначает...)
A drawing is (stands for) a visual picture of a geometric object.

The **predicate** of a **definition**	defines, signifies, means, implies, symbolizes, assigns, marks, notifies, represents, illustrates, pictures, classifies, designates, denotes, fixes, points, describes, manifests, figures, formulates, displays, produces, establishes, is a sign of, gives a name of, models, refers to, functions as, equals, suggests.

An angle, a ray, a square, a circle, a cone, a triangle, a quadrilateral, a prism, a polygon, a tetrahedron.

V. *Disagree with the false statements. Begin your answer with the opening phrases.*

It's not correct,		**Not at all.**
It's not right,	**I am afraid.**	**On the contrary.**
It's wrong,		**Quite the reverse.**

1. A ray is a subset of a point. 2. A straight line extends indefinitely only in one direction. 3. A polygon with exactly four sides is a polyhedron. 4. We call decimals in which one digit or one group of digits repeats over and over nonterminating decimals. 5. We find the area directly by laying out little squares over the entire floor of the room. 6. There is no formula to calculate the volume of a cube. 7. We undervalue the contribution of the ancient Greek mathematicians.

VI. *Translate the following sentences into English.*

1. Геометрия занимается построениями и изучением свойств и отношений геометрических фигур и тел. 2. Геометрические построения должны выполняться только с помощью циркуля и линейки. 3. Геометрические фигуры определяются через понятие множества точек. 4. Чертежи — это модели, изображающие геометрические объекты. 5. На-

чальное понятие геометрии — точка. Представление о ней дают многие модели, но все они неточны, так как точка в геометрии не имеет измерения. Точка — это определенное положение в пространстве. 6. Отрезки прямой, лучи, углы, круги и треугольники — геометрические фигуры на плоскости, т. е. это множество точек, лежащих в одной плоскости. 7. Каждая замкнутая ломаная линия делит плоскость на 3 различные множества точек: фигура, ее внутренняя часть, ее внешняя часть. 8. Существуют различные типы углов, треугольников и многоугольников. 9. Чтобы получить множество важных кривых, мы должны разрезать конус под разными углами. Получаемые кривые известны как конические сечения. 10. Многогранники — это геометрические тела, каждая грань которых многоугольник. 11. Понятие «конгруэнтность» помогает в геометрии объяснить значение длины отрезка и меры угла. 12. Мы измеряем отрезки с помощью других отрезков и углы через другие углы. 13. Мы находим площади геометрических фигур, перемножая длину на ширину. Это — косвенное измерение, так как мы находим площадь перемножая длины отрезков. 14. Для того чтобы найти объем тела, мы должны использовать ту же идею и метод и перемножить площадь на высоту. 15. Существуют формулы для измерения площадей и объемов, которыми мы обязаны греческим математикам. 16. Мы высоко ценим вклад греческих математиков — основателей косвенных измерений.

COMPREHENSION EXERCISES

I. *Speak English aloud as you draw all geometric figures, illustrating each step of the problem solution.*

1. For each case below try to determine if it is a plane geometric figure: a) Two lines which are not parallel and have no common points. b) Two triangles with a common side. c) Two intersecting lines. 2. Draw illustrations for each of the plane geometric figures and define them all. 3. Find illustrations of space geometric figures in the physical world. 4. We can define a sphere in much the same way as a circle. Give the definition of a sphere. 5. Draw a triangle with sides 3,4 and 5 inches. Use a protractor and measure the angles of this triangle. What kind of a triangle do you have? What is the area of this triangle? 6. a) Two triangles are congruent. Do they have the same area? b) Two triangles have the same area. Are they congruent?

II. *Determine whether the following statements are true or false. Draw figures to help with your decision.*

1. Every square is a rhombus. 2. Every trapezoid is a parallelogram. 3. The "opposite sides" of a parallelogram are congruent to each other. 4. A rectangle that is inscribed in a circle is a square. 5. No parallelogram is a trapezoid. 6. Some quadrilaterals are triangles. 7. Every rhombus with one right angle is a square. 8. No trapezoid has two right angles. 9. If a rectangle has a pair of congruent sides then it is a square 10. If a trapezoid has one right angle then it has two right angles. 11. If a quadrilateral has two pairs of congruent sides then it is a parallelogram. 12. If two diameters of a circle are perpendicular to each other then their end points determine the vertices of a square. 13. There is a square that is not a parallelogram. 14. No rhombus is a trapezoid 15. No trapezoid has a pair of congruent sides.

III. *Determine the fallacy in the given statements. Make a drawing to help you find the errors.*

1. A right angle is equal to an angle which is greater than a right angle. 2. A part of a line is equal to the whole line. 3. The sum of the lengths of two sides of any triangle is equal to the length of the third side. 4. Every triangle is isosceles. 5. $\pi/4$ is equal to $\pi/3$. 6. If two opposite sides of a quadrilateral are equal, the other two sides must be parallel. 7. Every ellipse is a circle.

LAB. PRACTICE

Intensive Listening and Text Recording

I. *Listen to the text (15-20 sentences) from tape.*

II. *Listen to it again (and, if necessary, again). Stop the tape-recorder after each sentence and write it down.*

III. *When the whole text is written down listen to it again to check it up.*

IV. *Hand in your written text to the teacher for correction.*

V. *Be ready to reproduce the text orally in class.*

The students must do similar exercise in every lesson of this textbook.

LESSON TWO

WHAT IS MATHEMATICS?

Grammar:

1. Countable and Uncountable Nouns.
2. Degrees of Comparison.
3. Indefinite Tense-Aspect Forms. Active and Passive Voice.

LAB. PRACTICE

Repeat the sentences after the instructors. Mind the logical division of the sentence.

Model. **Among all the sciences / mathematics is distinguished / for its universality.**
Математика выделяется / среди всех наук / своей универсальностью.

1. It is impossible to give a concise and readily acceptable definition of mathematics as it is a multifield subject. 2. Mathematics in the broad sense of the word is a peculiar form of the general process of human knowledge of the real world. 3. Mathematics deals with the space forms and quantity relations abstracted from the physical world. 4. Mathematical abstractions are idealizations that have material or physical origin. 5. Numbers are abstracted ideas or mental notions only, for numbers do not exist in nature. 6. In mathematics the abstracted notions and laws become divorced from the real world. 7. In a formal mathematical system the content is put aside as irrelevant. 8. Mathematics enjoys an unparallel world-wide reputation of objectivity. 9. Contemporary mathematics is a mixture of much that is very old and still important (e. g., counting, the Pythagorean theorem) with new concepts such as sets, axiomatics, structure. 10. The totality of all abstract mathematical sciences is called **Pure Mathematics**. 11. Pure mathematics is borrowed from the physical world; it represents only one part of its forms of interconnection. 12. The totality of all concrete interpretations is called **Applied Mathematics**. Together they constitute **Mathematics** as a science. 13. Mathematics is the science dealing primarily with what can be obtained by reasoning alone. 14. Human thought moves from the concrete to the abstract, from specific individual cases to general principles. 15. Mathematical thought involves special kind of thinking and reasoning. 16. Despite the usefulness of **analogy** and **induction**, mathematics does not rely upon these methods to establish its conclusions. **All mathematical proofs must be deductive.** 17. The need for careful and rigorous reasoning in proofs is not at once intuitively apparent to a non-mathematician. 18. Mastery of mathematics does not demand a "mathematical mind", peculiar talents

or genius. The subject is within anybody's grasp. 19. The common phrase "There is no royal road to mathematics" can be paraphrased by saying that "There is no royal road to learning". 20. "Language is as old as the mind" (Karl Marx). 21. Human knowledge and notions about the universe are expressed, represented and stored in **Language**. 22. There are two main forms of Language. They are distinguished in the concepts of Language as a specific written Code and Speech. 23. Speech is the realization and representation of this written code. 24. Language is a foremost means of both human communication and human knowledge. 25. Natural spoken language has numerous and limitless applications. 26. The mass media — the press, radio and TV — make for the correctness of the formal language spoken in the country. 27. Colloquial language is vague, ambiguous and unreliable for science. 28. Spoken words may have different meanings determined by the context. 29. Scientists set up formalized languages to avoid confusion. 30. The essential and peculiar feature of modern mathematics is its symbolic language. 31. Mathematical language is designed and ingeniously devised by the prominent mathematicians. 32. Much of the mathematical language has the form of signs, symbols, equations and formulas. 33. The development of a meaningful, adequate and consistent system of notations in various branches of mathematics is part of the history of mathematics. 34. Modern terminology and symbolism are a relatively new development. 35. **Mathematical notation** involves signs and symbols that represent objects, concepts, statements, operations, relations, functions, etc. 36. Symbols permit clear, concise, unambiguous representation of ideas which are sometimes very complex. 37. Mathematical writing is remarkable because it encompasses much information in few words. 38. Most mathematical texts involve the basic symbols used in algerba, analytic geometry, calculus, set theory and mathematical logic with the meanings usually ascribed to them. 39. The precise signification of the symbol is fixed by usage, i. e., by the context. 40. A formal mathematical system bears some analogy to a natural language, for it has its own vocabulary and rules. 41. Symbols of a formalized language are combined in strict accordnace with the rules of semantics and syntax. 42. The creations of calculus, non-Euclidean geometries, set theory and cybernetics may be considered as revolutions in mathematics. 43. Modern methods of carrying out arithmetic operations (addition, subtraction, multiplication and division) and their applications become sophisticated through modern computers. 44. Nowadays mathematicians frequently liken mathematics to art or game rather than to science. 45. Most mathematicians claim there is great beauty in mathematics. 46. Mathematical and scientific problems demand solution. Mathematicians seek to solve problems in the most beautiful, elegant and simple manner. 47. The solution of difficult mathematical problems evokes aesthetic emotions. 48. There is an agreement on the fact that a "beautiful" mathematical result must be nontrivial. 49. An essential element in the "beauty" of a theorem lies in its simplicity and generality. 50. The search for simplicity is a leitmotive of scientific thought in general. 51. To develop a rigorous and elegant proof the mathematician builds a structure of logic and form which to his eye is as beautiful as the finest poem.

Key Grammar Patterns

To gain a general idea of the Tense-Aspect Forms of the verb in English analyze the table given below. Our starting point is the group of **Indefinite Tense-Aspect forms** which express recurrent (repeated) or permanent actions in the **Present**, **Past** or **Future**. The group of Indefinite Tenses is basic for most scientific texts. **Present Indefinite**

Tense (we deal with in this lesson in detail, though Past Indefinite tense is also found in the texts) represents an action as **a fact**. It is used in conversation, lectures, scientific prose, newspaper and radio reports, etc. Remember, that all Present Tense-Aspect forms include the moment of speaking.

Verb Finite Tense-Aspect Forms

Finite Tense-Aspect Forms of the verb have the function of **the predicate** in the sentence and are characterized by the following categories: Tense-Aspect, Person, Number, Voice and Mood. They show the connection of the subject and the predicate of the sentence.

Aspects or Groups and Voice Infinitives	**Past** Actions preceding the moment of speaking	**Present** From speaker's point of view	**Future** Actions following the moment of speaking
Indefinite **Act.** to solve — решать	He **solved** the problem *yesterday*	He *always* **solves** problems in mathematics.	He **will solve** the problem *tomorrow.*
Pas. to be solved — решаться	The problem **was solved** *yesterday.*	Problems **are** *always* **solved** in mathematics.	The problem **will be solved** *tomorrow.*
Continuous **Act.** to be solving — решать в определенный момент	He **was solving** the problem *yesterday at that time.*	He **is solving** the problem *now.*	He **will be solving** the problem *at this time tomorrow.*
Pas. to be being solved—решаться в определенный момент	The problem **was being solved** *at that time.*	The problem **is being solved** *now.*	—
Perfect **Act.** to have solved — (уже) (раз) решить	He **had solved** the problems before we came.	He **has** *already* **solved** *the problem.*	He **will have solved** the problem by this time tomorrow.
Pas. to have been solved — (уже) было решено	The problems **had been solved** by that time.	The problem **has** *already* **been solved.**	The problem **will have been solved.**
Perfect Continuous **Act.** to have been solving — решать в течение определенного отрезка времени	He **had been solving** the problems *for two hours when we came.*	He **has been solving** the problem for an hour.	He **will have been solving** the problem *for a long time when we come tomorrow.*
Pas. —	—	—	—

Parts of speech:
1) **notional words** = nouns, verbs (both finite and nonfinite forms, i. e., infinitives, participles and gerunds), adjectives, adverbs, pronouns.
2) **structural words** = articles, particles, prepositions and conjunctions; link-verbs, modal verbs.
3) **independent elements** = Yes, no, certainly, please, etc. **Sentence connectives** = yet, nevertheless, that is, in fact, etc.

I. *Translate the following sentences and a) locate and analyze the Predicate Tense-Aspect forms; b) qualify the parts of speech.*

1. Among the many adjectives given to the present century (e. g., electronic, atomic and space) the term "mathematization of science age" is often come across. 2. Some people define the unprecedented development of modern mathematics as the "revolution in mathematics". Others call it "mathematics power". 3. However correct or incorrect these terms may seem, one thing is obvious: mathematics is a key science nowadays. 4. Mathematics has a peculiar and remarkable language. 5. Certainly, it is unlike any human language as, in a sense, it is an unspoken language. 6. Mathematical language may be called the language of science. 7. Scientific language must be precise, concise and universal, i. e., (= that is,) it must be the same throughout the world. 8. Unlike the natural languages, the language of science is man-made or artificial. 9. Some laymen unaccustomed to its forms find it confusing. 10. Mathematical reasoning is of the highest level known to man.

Parts of the sentence: 1) the subject; 2) the predicate (verbal and nominal); 3) the object; 4) the attribute; 5) the adverbial modifier.

II. *Translate the following sentences and single out the parts of the sentences in them. Mind, that both structural words and independent elements do not perform any syntactical function in the sentence. They express relations between the words, specify the meaning of a word or are used as parenthesis.*

1. Obviously, the meaning of the word becomes clear from the context. 2. No mathematician prefers a wordy and lengthy statement of a theorem or law. 3. A mathematical sentence of signs and symbols is formed by means of rules of syntax of a corresponding formalized theory. 4. Scientists determine the meaning of symbols by definitions and use them by common agreement. 5. The attention paid to rigour and precision in mathematics points to the requirements underlying mathematical research. 6. Certainly, mathematics is more than a language or technique, it is, in fact, a body of knowledge that serves all other sciences. 7. The study of mathematics is sometimes discouraging to weak-willed minds, indeed.

The sentence: 1) simple; 2) compound; 3) complex.

III. *Translate the following sentences and analyze their syntax.*

1. The sign $\angle$ means an angle in mathematics. 2. The expression $a^2+2ab+b^2$ has three terms of algebra. 3. The Pythagorean theorem is the theorem everybody is familiar with. 4. Highschool geometry is a subject in which the idea of rigorous definition is meant and given for the first time and where we learn to think in terms of axioms and theorems. 5. The need for careful reasoning in proofs is not at once intuitively apparent to a non-mathematician. 6. These two theorems are distinct and they must be clearly distinguished. 7. There is some opposition to his theory, perhaps because of the complexity of the ideas involved. 8. We call decimals in which one digit or one group of digits is repeated over and over repeating decimals and those which repeat zeroes terminating decimals, e. g., $1/3=0.333\ldots$, $1/4=0.25000\ldots$

Listen to the recording watching the text in class and try to translate each sentence into Russian with the teacher's assistance. The text should be read and reread as many times at home as it is necessary for every student to grasp its message. A written translation is recommendable. The active vocabulary of the text ought to be learnt.

TEXT ONE

WHAT IS MATHEMATICS?

The students of mathematics may wonder where the word "mathematics" comes from. Mathematics is a Greek word, and, by origin or etymologically, it means "something that must be learnt or understood", perhaps "acquired knowledge" or "knowledge acquirable by learning" or "general knowledge". The word "mathematics" is a contraction of all these phrases. The celebrated Pythagorean school in ancient Greece had both regular and incidental members. The incidental members were called "auditors"; the regular members were named "**mathematicians**" as a general class and not because they specialized in mathematics; for them mathematics was a mental discipline of science learning. What is mathematics in the modern sense of the term, its implications and connotations? There is no neat, simple, general and unique answer to this question.

Mathematics as a science, viewed as a whole, is a collection of branches. The largest branch is that which builds on the ordinary whole numbers, fractions, and irrational numbers, or what, collectively, is called **the real number system.** Arithmetic, algebra, the study of functions, the calculus, differential equations, and various other subjects which follow the calculus in logical order are all developments of the real number system. This part of mathematics is termed **the mathematics of number.** A second branch is **geometry** consisting of several geometries. Mathematics contains many more divisions. Each branch has the same logical structure: it begins with certain **concepts**, such as the whole numbers or integers in the mathematics of number, and such as point, line and triangle in geometry. These concepts must verify explicitly stated **axioms.** Some of the axioms of the mathematics of number are the **associative, commutative**, and **distributive properties** and the axioms about **equalities.** Some of the axioms of geometry are that two points determine a line, all right angles are equal, etc. From the concepts and axioms **theorems are deduced.** Hence, from the standpoint of structure, **the concepts, axioms and theorems are the essential components of any compartment of mathematics.** We must break down mathematics into separately taught subjects, but this compartmentalization taken as a necessity, must be compensated for as much as possible. Students must see the interrelationships of the various areas and the importance of mathematics for other domains. Knowledge is not additive but an organic whole and mathematics is an inseparable part of that whole. The full significance of mathematics can be seen and taught only in terms of its intimate relationships to other fields of knowledge. If mathematics is isolated from other provinces, it loses importance.

The basic concepts of the main branches of mathematics are **abstractions from experience**, implied by their obvious physical counterparts. But it is noteworthy, that many more concepts are introduced which are, in essence, creations of the human mind with or without any help of experience. Irrational numbers, negative numbers and so forth are not wholly abstracted from the physical practice, for the man's mind must create the notion of entirely new types of numbers to which operations such as addition, multiplication, and the like can be applied. The notion of **a variable** that represents the quantitative values of some changing physical phenomena, such as temperature and time, is also at least one mental step beyond the mere observation of change. The concept of **a function,** or a relationship between variables, is almost totally a mental creation.

The more we study mathematics the more we see that the ideas and conceptions involved become more divorced and remote from experience, and the role played by the mind of the mathematician becomes larger and larger. The gradual introduction of new concepts which more and more depart from forms of experience finds its parallel in geometry and many of the specific geometrical terms are mental creations.

As mathematicians nowadays working in any given branch discover new concepts which are less and less drawn from experience and more and more from human mind the development of concepts is progressive and later concepts are built on earlier notions. These facts have unpleasant consequences. Because the more advanced ideas are purely mental creations rather than abstractions from physical experience and because they are defined in terms of prior concepts it is more difficult to understand them and illustrate their meanings even for a specialist in some other province of mathematics. Nevertheless, the current introduction of new concepts in any field enables mathematics to grow rapidly. Indeed, the growth of modern mathematics is, in part, due to the introduction of new concepts and new systems of axioms.

Axioms constitute the second major component of any branch of mathematics. Up to the XIX century axioms were considered as basic self-evident truths about the concepts involved. We know now that this view ought to be given up. The objective of mathematical activity consists of the **theorems** deduced from a set of axioms. The amount of information that can be deduced from some sets of axioms is almost incredible. The axioms of number give rise to the results of algebra, properties of functions, the theorems of the calculus, the solutions of various types of differential equations. **Mathematical theorems must be deductively established and proved.** Much of the scientific knowledge is produced by deductive reasoning; new theorems are proved constantly, even in such old subjects as aglebra and geometry and the current developments are as important as the older results.

Growth of mathematics is possible in still another way. Mathematicians are sure now that sets of axioms which have no bearing on the physical world should be explored. Accordingly, mathematicians nowadays investigate algebras and geometries with no immediate applications. There is, however, some disagreement among mathematicians as to the way they answer the question: Do the concepts, axioms, and theorems exist in some objective world and are merely detected by man or are they entirely human creations? In ancient times the axioms and theorems were regarded as necessary truths about the universe already incorporated in the design of the world. Hence each new theorem was a discovery, a disclosure of what already existed. The contrary view holds that mathematics, its concepts, and theorems are created by man. Man distinguishes objects in the physical world and invents numbers and number names to represent one aspect of experience. Axioms are man's generalizations of certain fundamental facts and theorems may very logically follow from the axioms. Mathematics, according to this view-point, is a human creation in every respect. Some mathematicians claim that pure mathematics is the most original creation of the human mind.

Listen to the recording of the text. Analyze each paragraph and its Russian translation. Practise back translation and questions. Work in pairs.

TEXT TWO

THE SUBJECT MATTER OF MATHEMATICS
СОДЕРЖАНИЕ МАТЕМАТИКИ

..."**Pure mathematics deals with the space forms and quantity relations of the real world** — that is, with material which is very real, indeed. The fact that this material appears in an extremely abstract form can only superficially **conceal** its origin from the external world.

But in order to make it possible to investigate these forms and relations in their pure state, it is necessary to separate them entirely from their content, to put the content aside as irrelevant...

But, as in every department of thought, at a certain stage of development the laws, which were abstracted from the real world, **become divorced** from the real world, and are set up against it as something independent, as laws coming from outside, to which the world has **to conform.**

That is how things happened in society and in the state, and in this way and not otherwise, **pure mathematics was** subsequently **applied** to the world, although it is borrowed from this world and represents only one part of its forms of interconnection — and it is only just **because of this** that it can be applied at all".

Engles F. Anti-Dühring. M., 1959, p. 58—59.

...«Чистая математика имеет своим объектом пространственные формы и количественные отношения действительного мира, стало быть — весьма реальный материал. Тот факт, что этот материал принимает чрезвычайно абстрактную форму, может лишь слабо затушевать его происхождение из внешнего мира.

Но чтобы быть в состоянии исследовать эти формы и отношения в чистом виде, необходимо совершенно отделить их от их содержания, оставить это последнее в стороне как нечто безразличное...

Но, как и во всех других областях мышления, законы, абстрагированные из реального мира,... противопоставляются ему как нечто самостоятельное, как явившиеся извне законы, с которыми мир должен сообразоваться.

Так было с обществом и государством, так, а не иначе, *чистая* математика *применяется* впоследствии к миру, хотя она заимствована из этого самого мира и только выражает часть присущих ему форм связей, — и как раз *только поэтому* и может вообще применяться».

Маркс К. и Энгельс Ф. Собр. соч., т. 20, с. 37—38.

Read and translate the text at home. Be ready a) to illustrate different meanings of the words bold-faced in the text with examples of your own; b) *to speak on the topic*: "*The Language of Science*"

TEXT THREE

MATHEMATICS — THE LANGUAGE OF SCIENCE

What distinguishes the language of science from language as we ordinarily understand the word? How is it that scientific language is international? The supernational character of scientific concepts and scientific language is due to the fact that they are set up by the best brains of all countries and all times.

A. Einstein

One of the foremost reasons given for the study of mathematics is, to use a common phrase, that "mathematics is the language of science". This is not meant to imply that mathematics is useful only to those who specialize in science. No, it implies that even a layman must know something about the foundations, the scope and the basic role played by mathematics in our scientific age.

The language of mathematics **consists** mostly of signs and symbols, and, in a **sense**, is an unspoken language. There can be no more universal or more simple language, it is the same throughout the civilized world, though the people of each country translate it into their own particular spoken language. For instance, the symbol 5 means the same to a person in England, Spain, Italy or any other country; but in each country it may be called by a different spoken word. Some of the best known symbols of mathematics are the numerals 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 and the signs of **addition** (+), **subtraction** (—), **multiplication** ($\times$), **division** (:), **equality** (=) and the letters of the alphabets: Greek, Latin, Gothic and Hebrew (rather rarely).

Symbolic language is one of the basic characteristics of modern mathematics for it determines its true aspect. With the aid of symbolism mathematicians can make transitions in **reasoning** almost mechanically by the eye and leave their **mind** free to grasp the fundamental ideas of the **subject matter**. Just as music uses symbolism for the **representation** and **communication** of sounds so mathematics expresses quantitative **relations** and spatial forms symbolically. Unlike the common language, which is the **product** of custom, as well as social and political movements, the language of mathematics is carefully, purposefully and often ingeniously **designed**. By virtue of its compactness, it permits a mathematician to work with ideas which when expressed in terms of common language are **unmanageable**. This compactness **makes for** efficiency of thought.

Mathematical language is precise and concise, so that it is often **confusing** to people unaccustomed to its forms. The symbolism used in mathematical language is essential to distinguish **meanings** often confused in common speech. Mathematical style aims at brevity and formal perfection. Let us suppose we wish to express in general terms the Pythagorean theorem, well-familiar to every student through his high-school studies. We may say: "We have a right triangle. If we construct two squares each having an arm of the triangle as a side and if we construct a square having the hypotenuse of the triangle for its side, then the area of the third square **is equal** to the sum of the areas of the first two". But no mathematician expresses himself that way. He **prefers**: "The sum of the squares on the sides of a right triangle **equals** the square on the hypotenuse". In symbols this may be stated as follows: $c^2=a^2+b^2$. This economy of words makes for conciseness of presentation, and mathematical writing is remarkable because it encompasses much in few words. In the study of mathematics much time must be **devoted** 1) to the expressing of verbally stated facts in mathematical language, that is, in the signs and symbols of mathematics; 2) to the translating of mathematical expressions into common language. We use signs and symbols for convenience. In some cases the symbols are **abbreviations** of words, but often they have no such relation to the thing they **stand for**. We cannot say why they stand for what they do, they **mean** what they do by common agreement or by **definition**.

The student must always remember that the understanding of any **subject** in mathematics presupposes clear and definite knowledge of what precedes. This is the **reason** why "there is no royal road" to mathematics and why the study of mathematics is discouraging to weak **minds**, those who are not able and willing to master the subject.

Translate the text in class. a) Generalize ideas for writing an abstract of the text. Use the bold-faced words in your abstract. Expand the text by speaking about Russian (Soviet) mathematicians involved in the mathematical analysis of the objects of art.

TEXT FOUR

MATHEMATICS AND ART

All science as it grows towards **perfection** and **sophistication** becomes mathematical in its ideas.

A. N. Whitehead

Today mathematicians frequently liken mathematics and its creations to music and art **rather than** to science. It is convenient to keep the old classification of mathematics as one of the sciences, but it is more **just** to call it an art or a game. Unlike the sciences, but like the art of music or a game of chess, mathematics is foremost a free creation of the human mind. Mathematics is the sister, as well as the servant of the arts and is touched with the same genius. In an age when specialization means isolation, a layman may be **surprised** to hear that mathematics and art are intimately related. Yet, they are closely identified from ancient times. To begin with, the visual arts are spatial by **definition.** It is therefore not surprising that geometry is evident in classic architecture or that the ruler and compass are as familiar to the artist as the artisan. Artists search for ideal proportions and mathematical principles of composition. Many trends and traditions in this search are mixed.

Mathematics and art are mutually indebted in the area of **perspective** and **symmetry** which express relations only now fully explained by the mathematical theory of groups, a **development** of the last centuries. But does not art, in breaking away from academic canons nowadays, also break with mathematics? On the contrary. In the last one hundred years mathematics also has its liberation. From the science of number and space, mathematics becomes the science of all relations, of structure in the broadest sense. A mathematician, like a painter or a poet, is a maker of **patterns.** The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas, like the colours or the words must fit together in a harmonious way. Beauty and elegance are the true test for both. The revolutions in art and mathematics only deepen the **relations** between them. It is a common observation that the emotional drive for creation and the satisfaction from success are the same whether one constructs **an object** of art or a mathematical theory.

In ancient Greece mathematics was transformed from a tool for the advancement of other activities to an art. Arithmetic, geometry and astronomy were to the classical Greece music for the soul and the art of the mind; indeed, rational and aesthetic can hardly be separated in Greek thought. Mathematics and art were fused harmoniously in a single individual during the Renaissance. Though the further developments tended to weaken the connection, it was reinforced again in the last century and recent revolutionary changes in both fields open new possibilities for interaction without weakening the potential role of each as **inspiration** to the other. In both areas the creative process **involves** observation and experiment, judgement and **rejection**, intuition and feeling, careful **calcu-**

lation and analysis, sophistication, flashes of insight, and possibly **results** that are thrilling, satisfying and useful to both the artist and his audience. Patterns in either field may illustrate, explain, or inspire work in the other. The new mathematics and the new art are capable of an intimacy that we have not seen since the Renaissance.

Since mathematics and the arts often deal with the same material in different idioms only the most careful study can show which precedes the other, but there is certainly much in modern art to inspire the mathematician, and there are many mathematical ideas whose artistic exploitation may reap a rich harvest. Perhaps modern art expresses intuitively many relations that appear deductively in mathematical theories. The professional mathematician has a strong poetic sense of form in his own language of mathematics and most mathematicians **claim** that there is great **beauty** in their science. Mathematics means problems, and problems **demand** solutions. When every mathematician is confronted with a problem he **does his best** to solve it by whatever **means** he can think of. But he also tries, if possible, to solve it in the most beautiful and simple manner which is the most fruitful **in the long run.** A mathematical problem or theory has a history, which follows the same pattern as in every science. But the fascination of mathematics has a flavour of its own. Mathematical problems not only **appeal to** the scientist's delight in solving riddles but they definitely evoke aesthetic emotions. Contrary to the attitude of the experimental scientist, the result alone is **not what matters** to the mathematician, but the difficulty coped with to obtain it. That is, what is beautiful in mathematics can never be merely skin-deep; it must **penetrate** deep into the bottom of the mathematical organism where all difficult problems **converge.**

In 1933 **George Birkhoff**, one of the most distinguished mathematicians of his generation, attempted to apply mathematics to art in the manner that proves so successful in other areas. He began with a presice formulation of the old idea that beauty depends on the relations of the parts of an art object. He defined aesthetic measure as varying **inversely** with the number of elements present and **directly** with the number of relations between them. Of course, the difficulty of the problem is to determine these two numbers in specific contents to discover the **implications** for **design** and to **test** and **verify** the conclusions against human aesthetic **judgement.** This Birkhoff attempted to do for painting, poetry and music. His work was an integration in the main stream of artistic and mathematical thought and showed great insight, ingenuity and sophistication.

During the many years from the age of Pythagoras to the nineteenth century, mathematicians and musicians alike sought to understand the nature of musical sounds and to find the relationship between mathematics and music. The climax to this long series of investigations, **from a mathematical standpoint**, came with the work of the mathematician **Joseph Fourier**, who showed that all sounds, vocal and instrumental, simple and complex, are completely describable in mathematical terms. Because of Fourier's work not even the elusive beauty of a musical phrase **escapes** mathematical formulation. Whereas Pythagoras was content to pluck the strings of a lyre, Fourier sounded the whole orchestra. Stated as a theorem of pure mathematics Fourier's formula $y = \sin x$ says about **the relations among variables**, which can be represented by means of a graph.

The graph shows that the function is **regular** or **periodic**; or we may say, the cycle of y-values repeats itself after every 360-unit interval of x-values. This function does not quite represent the sound of music but a very simple modification of it does. A little effort produces the proper modification and it can be summarized by the statement that the function $y = a \sin bx$, where a and b are any possitive numbers, has the amplitude a and the frequency b in 360 units of x-values. The formula represents sounds mathematically. But of course very few musical sounds are as simple as those that may be represented by the formula. What can the mathematician say about more complex musical sounds?

Part of the answer to the question is learned by observing the graphs of various sounds. The graphs of all musical sounds show regularity. In "graphic" terms we have, then, the distinguishing feature between pleasing and displeasing sounds, between musical sounds in the broad sense and noise. Unfortunately, such a great variety of musical sounds possesses this feature of regularity that further analysis and characterization is necessary — and yet this seemed impossible until the nineteenth century. Then Fourier entered the scene and **dispelled the confusion.**

What is the significance of Fourier's theory? In mathematical language the theorem tells us that the formula for any musical sound is a sum of terms of the form $a \sin bx$. Since each such term can represent a simple sound, the theorem says that **each musical sound, however complex, is merely a combination of simple sounds.** The mathematical deduction that any complex musical sound can actually be built up from simple sounds is **physically verifiable.** This resolution of complex sounds into partials or harmonics helps us describe mathematically the chief characteristics of all musical sounds. Thus, thanks to Fourier, the nature of musical sounds is now clear to us. But what can mathematics say about harmonic combinations of sounds, about the essence of beautiful musical compositions, about the "soul" of music? The role of mathematics in music stretches even to the composition itself. Masters such as Bach constructed and **advocated** vast mathematical theories for the composition of music. In such theories cold reason rather than feeling and emotions produce the creative pattern.

Of course the mathematical analysis of musical sounds is of great practical importance. The musical sounds of most instruments are considerably improved and perfected by the application of mathematics. The fact cannot be denied that mathematics not only aids in the design of musical instruments but sometimes mathematics rather than the ear is the arbiter of a perfect design. The engineering of practically all the components of complex instruments relies heavily on Fourier's analysis of musical sounds. Even the layman can soon learn to speak Fourier's language. In view of the many shares and bearings of mathematics to the production and reproduction of musical ideas the modern music lover evidently owes as much to Fourier as to Beethoven. There are philosophical overtones to Fourier's work. The essence of beautiful music is obviously more than what mathematical analysis can show. Nevertheless, through Fourier's theorem this major art leads itself perfectly to mathematical description. Hence, the most abstract of the arts can be transcribed into the most abstract of the sciences and the most reasoned of the arts is clearly recognized to be akin to the music of reason.

ACTIVE VOCABULARY

1. to abbreviate
2. to accustom (to)
3. to add
4. to apply
5. to avoid
6. to claim
7. to communicate
8. to condense

9. to confirm
10. to confuse
11. to consider
12. to consist (of, in)
13. to converse
14. to deal (with)
15. to deduce
16. to denote
17. to deny
18. to design
19. to devise
20. to distinguish
21. to divide
22. to establish
23. to introduce
24. to imply
25. to inquire
26. to inspire
27. to involve
28. to justify
29. to manage
30. to mind
31. to multiply
32. to per'fect
33. to prefer
34. to reason
35. to reject
36. to rely (on, upon)
37. to represent
38. to seek
39. to simplify
40. to sophisticate
41. to subtract
42. to suppose
43. to validate
44. to vary
45. to verify
46. to wonder

VOCABULARY EXERCISES

Self-Study

This type of exercises must be done alone by the student at home as one's drilling in Grammar and to master Active Vocabulary.

Common nouns (e. g., *a text — texts*) are **countable.** Abstract and material nouns are **uncountable** and they cannot have plural forms. The uncountable nouns may be combined only with the **zero article, the, their, that, much, little, a little, some, any, a piece of, a lot of.**

I. *Translate the sentences into Russian.*

Model. This information is new. It comes regularly.
Эти сведения новые. Они поступают регулярно.

1. What is the **news**? There is some, but I am not sure if it is good **news** or bad. 2. He makes good **progress** in English. 3. Let me give you some **advice**. 4. It is a fine piece of **music.** 5. Sound **knowledge** can be obtained through study and research work. 6. **Information** that is wrong is not useful. 7. This theory shows much **ingenuity** and **sophistication.**

It is sometimes difficult to draw a line of division between countable and uncountable nouns. There are many **abstract nouns** in English that have more than one meaning and which are countable in one meaning but uncountable in another.

II. *Use the following nouns in sentences of your own combining them with "these — those", "much", "many", "few", "a few", "little", "a little".*

Model. time —	There is still **much time** before the lecture.
a time	**How many times** must I add those numbers?
much time	$2\times2=4$ (Two **times** two equals four).
many times	

Science, people, mind, art, beauty, quality, quantity, development.

III. *Use the following nouns with the zero, definite or indefinite articles in the sentences of your own.*

M o d e l s. **Mathematics:** 1. In a sense it is easier to say what **mathematics** is not than what it is. 2. Do we have **a mathematics** accessible to all today? 3. By "modern" **mathematics** we mean **the mathematics** of the past century.

Art — arts: 1. Model designing is **art** in science. 2. Mathematics is **art** and as such gives the pleasures which all **arts** give.

Aids, news, habits, mechanics, customs, aims, ideas, advances, judgements, contents, calculi, semantics, transitions, physics, means, computations, numerals, assertions, ends, works.

IV. *Don't mix these words up! Illustrate their difference by examples of your own.*

word	what	very	some	than	must	can	poor	part	ever
world	that	vary	same	then	mast	a can	pure	path	every

since	since	quite	area	only	accept	simple	effect
sense	science	quiet	square	the only	except	sample	affect

basic	at last	solve	formally	unable	precede	employ
basis	at least	serve	formerly	enable	proceed	apply

quantity	precise	numerous	convenient	differ	variation
quality	concise	numerical	conventional	distinguish	verification

nature	devote	peculiar	application	number	validate	ingen**ious**
nurture	denote	particular	implication	numeral	violate	ingen**uous**

(al)	though	verse	like	describe
	thorough	inverse (ly)	alike	inscribe
	through	reverse (ly)	the like	prescribe
	thoroughly	converse (ly)	to liken	transcribe
	throughout	vice versa	unlike (ly)	circumscribe
	evoke	cause	formal	spell
	evolve	course	former	spelt
	evolution	recourse	foreign	spelled
	revolution	discourse	foresight	spelling
	evolutionary	intercourse	foremost	dispelled

V. *Say the verbs related to the following words.*

Computation, representative, confused, preference, devisible, undefined, managerial, involvement, objectively, sophistication, insensitive, subtraction, minded, reasoning, introductory, claimant, application, computing, determination, designed, meaningfully.

VI. *Say what part of speech the following words belong to.*

Additive, avoidance, countable, subjectively, inexperienced, divider, objectivity, perfection, determined, misconception, distinguishable, multiple, conceptual, decimal, numerator, unsophisticated, applied, nonsense, perception, search.

VII. *Paraphrase the following verbs or give their synonyms.*

To reason, to subject, to relate, to claim, to sophisticate, to multiply, to divide, to count, to devise, to manage, to avoid, to mind, to subtract, to apply, to communicate.

VIII. *Give all the derivatives of the following verbs.*

To add, to calculate, to divide, to define, to mean, to deal with, to subtract, to object, to mind, to relate, to sense, to sophisticate, to subject, to represent, to involve, to multiply, to communicate, to avoid, to compute.

IX. *Classify the prefixes and suffixes of the following words and give their Russian equivalents.*

Meaningfulness, sophistication, additivity, interrelationships, points (to), consistency, unambiguously, indistinguishable, involved, managerial, reasonlessly, mindedness, subjective, subtractor, variable, development, conclusion, multinational.

X. *Consult the dictionary and give the Russian equivalents of the following phrase verbs.*

to be on	to stand for	to make for	to point at
to be off	to stand up	to make away	to point to
to be over	to stand over	to make out	to point out
to be through	to stand back	to make up	to point up

XI. *Give the Russian equivalents of the following words adding the negative prefixes.*

un: defined, known, wanted, willing, familiar, accustomed, manageable, ambiguous, suitable, like, spoken, common, official, reasonable, avoidable, sophisticated;
in: exact, complete, accessible, dependent, efficient, sensible, sensitive, divisibility, ability, accurate, correct, convenient, definable, attention, adequate, determinate, different, distinct, formal, human, elegant, experienced, escapable, valid;
im: precise, possible, perfect, proper;
ir: rational, regular, relevant;
non: specialist, mathematician, sense, trivial, repeating, essential;
mis: conception, understanding, pronunciation, calculation;
dis: information, courage, proof, agreement, regarding, associate, belief, taste;
ill: logical, legal, limitable.

XII. *Give the Russian equivalents of the following antonyms.*

good — bad
true — false
right — wrong
absolute — relative
natural — artificial
alike — different
ancient — modern
lend — borrow
inner — outer

inside — outside
inward — outward
internal — external
interior — exterior
implicit — explicit
include — exclude
converge — diverge
divide — unite
increase — decrease

encourage — discourage
forget — remember
general — particular
simple — complicated
knowledge — ignorance
positive — negative

XIII. *Choose the proper word and complete the sentences.*

Physicist, a student (specialist) of Mechanics, Lawyer, Philologist, Mathematician, Geographer, Historian, Physician.

1. He studies mathematics. He means to be ... 2. They have nothing in common with medicine. They are not ... 3. These girls study at some Department for Humanities. They are either ... or ... 4. He likes to de-

termine the meaning of geographical names as a true ... 5. He knows history so well. To my mind he is a ... 6. She practises law, she is a ... 7. His field is Theoretical Mechanics. He is ...

XIV. *Translate the sentences into Russian paying attention to: "rather", "rather than", "other than".*

1. Mathematics is the study of relations between certain ideal objects such as numbers, functions, and geometric figures. These objects are not regarded as real but **rather** as abstract models of physical situations. 2. Mathematicians want from mathematical objects not their material or physical existence but **rather** the right to use them in proofs. 3. The mathematical concept is a notion or method **rather than** content. 4. Mathematics is an active **rather than** a passive activity. 5. Mathematics not only aids in the design of musical instruments but sometimes mathematics **rather than** the ear is the arbiter of perfect design. 6. In this century the skill of reading is divided into many types among which intensive, extensive and silent are most commonly used. Extensive reading is aimed at ideas **rather than** grammatical structure and is definitely distinguished from translation. 7. For many physical phenomena no exact concepts exist **other than** mathematical notions. 8. The concepts of number and space figure do not come from any source **other than** the world of reality.

XV. a) *Choose the proper Russian equivalent and translate the sentences into Russian. Mind the interrelation within the complex: the meaning ↔ the context.*

Model. Development(s) *n.* развитие; изложение; раскрытие;, разъяснение; преобразование; построение; становление, разработка; теория; событие; результат; совершенствование; достижение.

1. The mathematical theory of groups is a **development** of the last centuries. 2. The **development** of the rigorous mathematical (as opposed to the dictionary) type of definition is the product of the modern mathematics. 3. Educated people must be familiar with all the important scientific **developments** of their day. 4. In mathematics the basic **development** from concrete individual matter through abstraction and back again to the concrete and individual gives a theory its meaning and significance. 5. The concept of number and the process of counting developed so long before the time of recorded history that the manner of this **development** is largely conjectural. 6. The requirements for quicker aids to computation lay at the root of the **development** of multiplication tables, tables of reciprocals and the like. 7. Mathematics ranks among the highest cultural **developments** of man. 8. It is especially true that in mathematics the creative work is done by individuals mostly; nevertheless the results are the product of centuries of thought and **development.** 9. What we call "mathematics" consists of several discrete individual **developments**, each manifesting its own birth and growth. 10. No subject can be effectively learnt or taught without an adequate understanding of its historical **development.** The **development** of mathematical knowledge is essentially in a continuous evolution. 11. The **development** of the means of mass communication in the modern world makes for greater understanding among nations. 12. Lab. practice and self-training make for the **development** of important language skills: listening, understanding, reproducing and improvising.

b) *Consult the dictionary and give all possible Russian equivalents of the English words "division", "power", "consideration" with illustrative examples.*

XVI. *One and the same Russian word may be expressed by different English equivalents which are not synonyms in the proper and strict sense of the term. The following verbs coincide in one of their possible meanings.*

Model. Обращаться (к, с, за)

1. **Speak to** Comrade Petrov. 2. He **turns to** me **for** help. 3. She **applies** for help. 4. He **treats** the child kindly. 5. She **addresses** the meeting with a long speech. 6. The speaker **appeals to (calls upon)** all those present to sign the paper. 7. You should **consult** a doctor (a lawyer, an expert, etc.). 8. The students must translate the text without **referring** to the dictionary. 9. Don't **invoke** so much the manager.

понятие представление	**idea — term — notion — concept — conception**
утверждение высказывние суждение	**phrase — sentence — statement — expression — wording — utterance — assertion — affirmation — judgement**
значение	**meaning — sense — value — bearing — implication — importance — significance — signification**
цель	**end — aim — intent — goal — target — object — objective — purpose**
средство	**aid — means — tool — device — medium — instrument — apparatus — technique — appliance**
обычный обычно	**usual** (ly) — **common** (ly) — **ordinary** (ly) — **customary** (ly) — **average — familiar — popular** (ly)
очевидный очевидно	**clear** (ly) — **plain** (ly) — **evident** (ly) — **obvious** (ly) — **apparent** (ly)
строгий строго строгость	**strict** (ly) — **severe** (ly) — **rigorous** (ly) **strictness — severity — rigour**
точный (не) точно точность	(in) **exact** (ly) — (in) **accurate** (ly) **exactness** — **accuracy** (im) **precise** (ly) — (not) **neat** (ly) **precision** — **neatness**
изучать исследовать исследование	**to learn — to study — to investigate — to explore —** **learning — study — investigation — exploration** **to analyze — to make research — to inquire (into)** **analysis — to research — to make an inquiry** **research — inquiry**
доказывать доказательство	**to prove — to argue — to demonstrate — to give (to bear) evidence or reasons** **proof — argument — demonstration — evidence, reasons** Q. E. D. = **which was to be demonstrated**

предполагать	to assume —	to (pre) suppose —	to presume —
предположение	assumption	supposition	presumption
допущение	to conjecture —	to hypothesize	
	conjecture	hypothesis	

равняться равно	to be — to make — to equal — to be equal to — to amount to

соединять	to tie —	to bind —	to link —	to couple —	to combine —
объединять	tie	bond	link	couple	combination
	to associate —	to relate —	to connect —		
	association	relation	connection		
	to consolidate —	to amalgamate			
	consolidation	amalgamation			

требовать	to call for —	to claim —	to demand —	to require
требование	call for	claim	demand	requirement

XVII. *Give one Russian equivalent of the following groups of words.*

a) Series — a number of — row — array / time — tense / cause — reason — excuse / content(s) — subject-matter / mind — intellect — reason — brains — senses — intelligence — mentality — wits / language — tongue — lingua / talk — conversation — discourse / track — road — way — path — route / ingenuity — sophistication — deftness / theorem — proposition — conclusion / motion — movement / convention — agreement / test — experiment — experience / decision — solution — determination — settlement / progress — advance / non-specialist — layman / change — alteration / quantity — value — magnitude / help — aid — assistance / viewlook — sight / corner — angle / couple — pair / mistake — fault — flaw — error — lapse — blunder / imagination — fancy — fantasy / worker — craftsman — artisan / thing — matter — object / business — affair / part — area — branch — field — domain — division — department of mathematics.

b) Big — large — great / all — overall — full — whole — total — utter — entire — complete / peculiar — strange / convenient — comfortable / previous — preceding / unmanageable — uncontrollable / unaccustomed — inexperienced — uninitiated / today's — of today — nowadays — up-to-date / present — modern — current — contemporary — sophisticated / famous — distinguished — prominent — outstanding / mean — middle — medium — common — average — ordinary / different — various — alternative / true — geniune — veritable.

c) To shorten — to abbreviate — to abridge — to contract / to make for — to contribute to / to want — to wish — to crave — to covet — to desire — to long / to define — to determine — to notify — to qualify / to test — to check up — to verify / to use — to employ — to apply — to utilize / to discuss — to argue — to debate — to dispute / to choose — to pick up — to single out — to select / to surprise — to amaze — to astonish / to conform — to adapt / to reject — to refuse — to decline / to conceal — to hide / to realize — to put into practice / to understand — to realize — to become aware / to codify — to put into the form of a code — to record — to systematize / to get — to gain — to obtain — to attain — to receive / to divorce — to dissociate — to separate — to detach / to stand for — to symbolize — to replace — to substitute / to clarify — to make clear / is thought — is considered — is regarded — is judged — is viewed — is conjectured / to avoid — to escape — to avert — to evade — to pass by / to grasp — to catch — to seize — to understand — to realize / to inspire — to encourage / to make stronger — to strengthen — to reinforce.

d) With the aid of — with the help of / by virtue of — by means of / due to — owing to — thanks to / in spite of — despite / in terms of — through / as well as — also / in a sense — in a way / both mathematicians and laymen — mathematicians and laymen alike.

e) The problem **involved**, the functions **concerned**, the objects **under study**, the relations **under consideration**, the argument **in question**, the theory **at issue**.

f) On the contrary, quite the contrary, in contrast, just the opposite, quite the reverse, vice versa, conversely, inversely, just the other way round.

Many mathematical terms have numerous non-mathematical meanings and conversely many spoken words are commonly used in mathematics as terms. For example, "At this **point** we wish **to add** a **number** of **arguments** along the same general **line**". And conversely, such concepts as **"set"**, **"function"**, **"relation"** and **"operation"** have mathematical meanings that are almost entirely divorced from their everyday meanings. That is why mathematicians prefer to introduce such terms explicitly by definition or use them as undefined terms.

XVIII. *Use the following active vocabulary words in non-mathematical situations.*

Model.	**to relate** **a relation**	— Is he any **relation** to you? (What **relation** is he to you?)
	a relative	— Yes, he is a **relative** of mine. We **are related** by marriage, he is my brother-in-law, besides I have business **relations** with him.

To reason — reasoning — reason — reasonable / to apply — application — an applicant / to add — addition — in addition to / to count — counter — in the final count / to calculate — calculator — calculation.

LAB. PRACTICE
Grammar Rules Patterns

In **the Present Indefinite Tense** either the **subject** or the **predicate** may have s-ending (The Law of one **"s"**, e. g., mathematicians prove, a mathematician proves.

I. *Say the following sentences with the subject in the singular and transform the predicate accordingly.*

Model. Many young boys **mean** to be mathematicians.
(this young man)
This young man **means** to be a mathematician.

1. Mathematicians **claim** that mathematics is the language of science. (a scientist). 2. They **define** this concept in a precise and formal way. (she) 3. We **determine** the strict meaning of this word from the context available. (a philologist) 4. Different fields of mathematics **involve** specific problems. (this new field) 5. People very often **mix up** the words "some" and "same". (a careless student) 6. We **introduce** a new term to distinguish the notions often confused. (he) 7. They **make up their minds** to study the problem involved. (she) 8. We **mean** that your reasoning is illogical. (the teacher) 9. Scientists **relate** these two facts to one known cause. (a mathematician) 10. Human reasoning **differ** according to the problem under study. (his reasoning) 11. The students **get confused** when they are unable to solve the problem on the blackboard. (she) 12. They **learn** to think in terms of equations. (an algebraist)

II. *Ask a general question altering the subject.*

Model. **Mathematicians** express such relations in terms of a formula. (she)
Does **she** express such relations in terms of a formula?

1. **We** multiply the examples to clarify our viewpoint. **(he)** 2. **Mathematicians** never confuse the words "undefined" and "meaningless". **(a true scientist)** 3. **They relate** undefined terms to meaningful concepts. **(the mathematician)**. 4. **Mathematicians** prefer to introduce new mathematical terms explicitly by definitions. **(the teacher)** 5. **We** add that many mathematical terms have non-mathematical meanings in everyday situations. **(he)** 6. The sophisticated **readers** master mathematical formalized texts quite easily. **(the mathematician)** 7. **They** divide the task and each of them has his share of work. **(he)**. 8. **We** try to communicate the information by all the means available. **(a scientist)**.

III. *Agree with the following statements and develop them further.*

Model. Mathematicians **do not like** a long wordy statement of a theorem.
No, they don't. No mathematician likes it. He prefers a symbolic statement of a theorem.

1. Mathematicians **do not confuse** basic signs and symbols. 2. Scientists **do not devote** much time to transitions in reasoning. 3. A professional mathematician **does not develop** his arguments verbally. 4. Weak-willed minds **cannot master** maths *. 5. An algebraist **does not solve** equations by arguing in words. 6. Mathematicians **do not rely on** intuition in their proofs. 7. Analogy and induction **do not always lead** to true conclusions. 8. A layman **does not claim** that maths is easy to manage. 9. Mathematical language **was not developed** all of a sudden. 10. The mathematician **does not object** to the fact that there is a language barrier between scientists and laymen. 11. The "translation" of problems into the language of mathematics **is**, in fact, **not** an easy task. 12. Common people **do not understand** the formalized language of maths.

IV. *Disagree with the following negative statements and develop them further.*

Model. Symbols **do not have** the same meaning.
But they do. Most symbols have the same meaning throughout all mathematical texts. Certain symbols, in fact, have numerous connotations.

1. Symbols **do not play** the role of words. 2. Mathematical language **has no** design and no rules. 3. **There are no** man-made or artificial languages. 4. Language **is not** a means for communicating human thoughts. 5. Both the vocabulary and syntax of the mathematical language **are not** strictly **devised**. 6. Scientists **do not think** and **reason** in terms of formulas. 7. Mathematics **is not distinguished** for its universality. 8. In mathematics the major method of reasoning **is not deduction**. 9. Analogy and induction **are not employed** in mathematics at all. 10. **There is no difference** between inductive and deductive reasoning.

V. *Give both short and full answers.*
a) **General questions**

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* Maths is an abbreviation of (for) the word mathematics.

Model. Is scientific language peculiar? **Yes, it is.**
Scientific language is peculiar.

1. Do peoples have their particular national language? (Yes, ...) 2. Must there be a universal language for codifying science? (Yes, ...) 3. Do linguists design and devise the language of mathematics? (No, ...) 4. Is there any ambiguity in the meaning of symbols and signs? (No, ...) 5. Does a professional mathematician confuse mathematical concepts? (No, ...) 6. Can a layman understand formalized scientific language? (No, ...)

b) **"There is", "there exists" construction.**

Model. **There holds** a common agreement among mathematicians about this method of proof.
Does there hold such an agreement? Yes, there does. No, there does not.

1. There (is) circulates another variant of this common phrase. (Yes, ...) 2. There appears a new meaning of this term. (Yes, ...) 3. There passes a line through these two points. (No, ...) 4. There lives a distinguished mathematician in this city. (Yes, ...) 5. There arises a need for a new formalized language. (No, ...)

c) **Special questions**

Model. What distinguishes the language of science from common language? (mathematical symbolism)
Mathematical symbolism distinguishes the language of science from common language.

1. What constitutes the most part of mathematical language? (signs and symbols) 2. What makes for efficiency of thought in mathematical reasoning? (the compactness and precision of mathematical language) 3. Who designs and devises the language of mathematics? (distinguished mathematicians). 4. Who sets up the rules of abstract language? (logicians and semanticists) 5. Who confuses signs and symbols of mathematics? (laymen) 6. What do we call the result of addition, subtraction, multiplication and division? (a sum, a difference, a product, a quotient, respectively) 7. What do scientists use mathematical symbolism for? (for compactness, conciseness and unambiguity of presentation) 8. What theorem is familiar to every student of mathematics? (Pythagorean) 9. How is it that scientific language is universal? (due to the supernational character of scientific concepts) 10. When does a statement of a theorem appeal to mathematician? (concise and precise) 11. Which wording is more concise: verbal or symbolic? (symbolic, sure enough) 12. Who can dissociate, generalize and abstract mathematical concepts? (abstract-minded scientists) 13. Why do scientific laws seem independent? (due to their abstract character) 14. Why are scientific laws abstract? (their content is put aside as irrelevant)

VI. *Disagree with the false statements. Begin your answer with the following opening phrases and develop them further.*

I am afraid, you are mistaken here. It is quite the reverse. On the contrary. Just the other way round.

Model. The language of science is vague and unreliable.
In fact, **it is quite the reverse.** The language of science is precise and quite safe.

1. Mathematical language is a natural spoken language. 2. The language of science is the foremost means of human intercourse. 3. Ordinary people communicate by means of formalized language of maths. 4. Mathematical language is set up by distinguished linguists. 5. The only reason for studying maths is to master basic arithmetical operations. 6. When people try to do without maths they lose a colloquial language. 7. Mathematical language and techniques do not penetrate into fields outside the mathematical sciences. 8. Statements expressed in mathematical terms are vague and ambiguous. 9. Scientists can't use maths as a shorthand script to codify relationships. 10. Razor-sharp calculus operates with wordy statements. 11. Pure science is knowledge obtained from experiments and calculations. 12. Mental reasoning from obvious truths, i. e., axioms and postulates constitutes applied science. 13. Basic undefined terms of maths are meaningless concepts. 14. A sharp dividing line between "pure" and "applied" maths can, in fact, be drawn. 15. The present role of maths is the same as in previous stages of its development. 16. There is a great deal of confusion about symbolism.

Degrees of Comparison

Notice that there are some adjectives which owing to their meaning have neither comparative nor superlative degreees, e. g., **perfect, unique, full, empty, square, round, daily, upper, major, outer, whole, only.**

I. *Translate into Russian the sentences, paying attention to the adjectives.*

1. The solution of the problem must be **perfect** but not always **unique.** Now its **whole** solution is available. 2. A sphere is a geometrical name for a **round** or ball-shaped solid. 3. Counting arose from **daily** needs of calculating objects in **outer** physical world. 4. This is the **only** problem that may attract our attention. 5. The **major** method of proof in mathematics is deduction.

II. *Follow the models and make sentences in which comparison is expressed.*

Models. a) comparison of **equality** (as ... as)
This problem is **as** difficult **as** the first problem.
This definition is **as** rigorous **as** the definition given before.
b) comparison of **inequality** (not so ... as, not as ... as)
This problem is **not as** interesting **as** people may think.
The proof is **not so** valid **as** he **supposed** at first.
c) comparison of **parallel increase or decrease** (the ... the)
The sooner the problem is solved, **the** better (so much the better).
The longer he refuses to recognize the impossibility of the solution, **the** worse for him (so much the worse for him).
d) comparison of **superiority** (...er than, ...est of)
This contribution of the ancient Greeks is much great**er than** the formulas of the Egyptians.
This solution is a great deal **better than** the last suggestion.
Her answer is **the most** convincing **of** all.
e) comparison of **inferiority** (less ... than; still, a little worse than)
His argument is **less** elegant **than** her proof.
Your drawing is **a little worse than** my drawing.

Participles Constructions

Participle I

to go — going	to lie — lying	to study — studying

I. *Transform the following sentences into Participle I constructions.*

Model. 1. The sign **which (that) stands for an angle** is $\angle$.
The sign **standing** for an angle is $\angle$. (Attribute)
2. **When (if) you add fractions in arithmetic**, you must determine the least common denominator of the fractions involved.
Adding fractions in arithmetic ... (Adverbial modifier)

1. The line **which passes through these two points** is a diameter. 2. **If you express these statements in mathematical terms**, you obtain the following equations. 3. A decimal fraction is a fraction **which has a denominator of 10, 100, 1000 or some simple multiple of 10** (e. g., 0.05). 4. **When we amalgamate several relationships**, we express the resulting relation in terms of a formula. 5. **If we try to do without mathematics**, we lose a powerful tool for reshaping information. 6. The mathematical language **which codifies present science so clearly** has a long history of its development. 7. The formula **which describes acceleration** is $\Delta v/\Delta t = 32$. 8. Calculus **which is the main branch of modern mathematics**, operates with the rules of logical arguments. 9. **When we use mathematical language**, we avoid vagueness and unwanted extra meanings of our statements. 10. **When scientists apply mathematics**, they codify their science more clearly and objectively.

Participle II

to give — given	to mean — meant	to mind — minded
to determine — determined	to speak — spoken	to see — seen
to imply — implied	to communicate — communicated

II. *Transform the following sentences into Participle II constructions.*

Models. 1. The reasons **which are given for the study of maths.**
The reasons **given** for the study of maths. (Attribute)
2. **When (if) they are expressed in terms of symbols**, these relations produce a formula.
Expressed in terms of symbols ... (Adverbial modifier)

1. A common phrase **which is used in such cases**. 2. **When they are used as scientific terms**, these concepts have different meanings. 3. The formal language **which is spoken in this country** is Russian. 4. The meanings of words **which are always confused in speech**. 5. **If it is expressed in mathematical terms**, this theorem gives a general method of calculating the area. 6. The sense **which is implied in this assertion** is not clear. 7. **If it is designed and devised in a proper way**, the symbolic language becomes universal. 8. The time **which is devoted to the translating of mathematical expressions into common language** is wasted for a mathematician.

III. *Turn from Active into Passive.*

Models. 1. Scientists **introduce** new concepts by rigorous definitions.
New concepts **are introduced** by rigorous definitions.
2. Mathematicians **cannot define** some notions in a precise and explicit way.
Some notions **cannot be defined** in a precise and explicit way.

1. Students of the Department of Mathematics and Mechanics **can give** the principal reasons for the study of maths. 2. People often **use** this common phrase in such cases. 3. Even laymen **must know** the foundations, the scope and the role of mathematics. 4. In each country people **translate** mathematical symbols into peculiar spoken words. 5. All the specialists **apply** basic symbols of mathematics. 6. Students **may express** this familiar theorem in terms of an equation. 7. Scientists **devote** little time to master symbolism. 8. A student **may use** basic principle to determine the relation. 9. All the specialists **must** thoroughly **remember** the preceding material. 10. By the aid of symbolism mathematicians **can make** transitions in reasoning almost mechanically by the eye. 11. The students **verify** the solution of this equation easily. 12. People **abstract** number concepts and arithmetic operations with them from physical reality. 13. Mathematicians **investigate** space forms and quantitative relations in their pure state. 14. Scientists **divorce** abstract laws from the real world. 15. Mathematicians **apply** abstract laws to study the external world of reality. 16. A mathematical formula **can represent** some form of interconnections and interrelations of physical objects. 17. A mathematical law **involves** abstractions built upon abstractions, i. e., abstractions of higher order. 18. Scientists **can avoid** ambiguity by means of symbolism and mathematical definitions.

IV. *Add the introductory phrase and translate the sentences into English using both active and passive constructions. Check up the resultant sentences from tape.*

Model. **Как это принято среди математиков** **значения знаков и символов точно согласуются.**

Mathematicians are accustomed (to agreeing) **to agree** precisely **upon** the meanings of signs and symbols.

It is customary among mathematicians
It is a custom with mathematicians
A customary practice of mathematicians is
that the meanings of signs and symbols **are** precisely **agreed upon.**

1... символический язык используется для интерпретации содержания текста. 2... математические тексты формализуются согласно определенным правилам математической логики. 3... каждое новое понятие вводится эксплицитно строгим математическим определением. 4... неопределяемые термины используются как имеющие смысл (meaningful) основные понятия, с помощью которых определяются все другие понятия. 5... требования математической строгости разрабатываются. 6... математические теории основываются на допущениях, абстрагированных из практики. 7... универсальность математики определяется широтой ее предмета и приложений. 8... количественные отношения и пространственные формы записываются в виде символических высказываний. 9... связи теории и практики, соотношения мира математического и мира реального изучаются. 10... граница между чистой математикой и прикладной (напр. вычислительной математикой) не проводится. 11... методы математического исследования применяются в других науках. 12... абстрагированные законы рассматриваются как отношения математических объектов или между объектами (сущностями).

To give an exact and explicit definition of maths is not a simple task. Numerous definitions of maths are given by different writers but none of them is utterly satisfactory. The point is, it is impossible to compress into a few words the definition of so vast and multifield subject. Mathematicians determine the essence of maths in different ways according to the following roles of the science: maths as a science, as a language, as a tool, as a method of inquiry.

I. *Analyze the following definitions of maths and say which role of the science is implied in each. Compare them with F. Engels's definition. Which of them may be called scientific and rigorous? Can you give a definition of maths?*

Model. **Maths means problems.**
It's not a definition at all, to my mind.
Rather, it's schoolchildren's view of maths.

1. Maths is what mathematicians do. 2. Maths is what maths does. 3. Maths is the queen of the sciences. 4. Maths is a universal tool for describing the world around. 5. Maths is a device designed to enlarge human powers. 6. Maths is a symbolic representation of human perceptions. 7. Maths is a game, a free creation of the mind divorced from practical problems. 8. Maths is a tool for codifying information, for simple transmission and communication of thought. 9. Maths is a general approach to the whole class of quite dissimilar situations. 10. Maths is a logical study of shape, quantity, arrangements and structures. 11. Maths is the classification and study of all possibe patterns. 12. Maths is the art of giving the same name to different things. 13. Maths is an inspiration to the artist as well as a tool to scientist. 14. Maths is the classification of all possible problems and the means appropriate to their solutions. 15. Maths is the technique of discovering and expressing in the most economical possible way useful rules of reliable reasoning about calculation, shape and measurement. 16. Maths is an activity which has as its goal the formulation and understanding of a complete model of the universe.

II. *The transition from pure to applied mathematics usually proceeds unnoticed for the nonspecialist. Read the statements of the following theorems and say which of them is a theorem of pure mathematics. Justify your choice.*

1. The area of a circle is πr^2. 2. The area of a circular field is π times the square of a certain physical length.

III. *Are the following statements true or false? Justify your choice. Add some sentence(s) to substantiate your claim.*

Models. 1. **Mathematical abstractions are mental conveniences only. In a sense, it is true.** Many mathematical abstractions stand for no real object in practice (e. g., a variable, a function), they are, in fact, mental creations and conveniences. Mathematicians deal with abstractions to gain generality, but something else is involved, of course. Abstractions make for the efficiency and compactness of thought. Through the employment of abstractions there results a considerable economy, both in mathematical thought and in communication of thought.

2. **Current maths is unmanageable for most social scientists. Not quite so.** Mathematical methods of inquiry constitute the essential part of and contribute to practically all the sciences nowadays. Most social scientists manage to master and apply mathematical methods in their work.

1. It is a common misconception to look at mathematicians as the high priests of learning. 2. Mathematical ability is a rare and unusual gift. 3. Mathematicians are by no means as peculiar as they may seem at first. 4. No special gifts or qualities of mind are needed to master maths. 5. Any man of average intelligence can have access to maths. 6. Mathematicians are human beings like "you and me". 7. The mathematician needs only critical intellect, common sense and a little imagination. 8. Maths is a temple and only the initiated are allowed to enter it. 9. Maths is a wide science which is open to anyone who enjoys thinking and precise thinking in particular. 10. Maths requires professionals not amateurs. 11. Like all other sciences maths arose out of the needs of men. 12. The abstract ideas of modern maths are related to real-life situations. 13. It seems that real-life situations have little to do with maths. 14. The intimate connection between maths and the objects and events in the physical world is obvious and reassuring. 15. The subject matter of maths is completely empirical, borrowed from the external world and then divorced from it. 16. To understand the external world scientists must seek its mathematical essence. 17. Pure maths is a fascinating subject.

IV. *Confirm or deny the statements. Use the introductory phrases.*

Right you are (it is).
Quite so. Absolutely correct.
I quite agree to it.
I think, it is right.

I am afraid, it is wrong.
I don't quite agree to it.
Excuse me, but... Not at all.
On the contrary. Far from it.

1. The transition from concrete and individual to the abstract involves some difficulty. 2. The attempts to define verbally the "meaning" of mathematical terms lead to confusion and ambiguity. 3. It is more difficult to think and reason in terms of abstractions than in concrete physical objects. 3. A mathematical formula has a direct real physical counterpart. 5. Mathematicians do not rely on their intuitive judgement — they seek to give a rigorous proof. 6. Proof is a thread connecting the statements in a mathematical theory. 7. If you want to know what a mathematical theorem states, see what its proof proves. 8. The need for careful and exact reasoning in proofs is not at once apparent for laymen. 9. Symbolism often leads to misunderstanding among mathematicians. 10. An idea expressed in symbols is more scientific than the same thought presented in words. 11. Maths is both intelligible and enjoyable. 12. Scientists alone know about the scope of maths in our age. 13. In our age in particular maths attains its wide range, scope and extraordinary applicability. 14. A scientist finds that his own thinking is enriched and enlarged by an insight into mathematical thought. 15. Computing procedures are not all maths of today. 16. There is nothing more practical than a sound fundamental theory. 17. Mathematicians do not deal with applications of maths.

V. *Disagree with the following negative statements and keep the conversation going where possible.*

Model. **Pure maths theory cannot be useful at all.**
Yes, it can. (But it can.) Not rarely mathematicians find new

interpretations and applications for theories formerly considered as "pure science".

1. Scientists do not think and reason in terms of abstractions. 2. The abstract idea of "number" was not the first grand step in maths. 3. Mathematical abstractions are not reflections of material objects and their interrelations in the real world. 4. Maths is not a free creation of the human mind and reasoning. 5. Pure maths does not conceal its origin from the external world. 6. To investigate the space forms and quantity relations of the real world maths does not separate them entirely from their content. 7. In pure maths theory the physical content of the objects involved is not irrelevant. 8. The laws abstracted from the real world are not usually divorced and set up against the real world. 10. The world does not conform to these laws. 11. Pure maths cannot be applied to the physical world. 12. Maths does not present the forms of interconnections of objects in the physical world. 13. The subject matter of maths is not borrowed from reality. 14. Mathematicians do not seek useful applications of their theories. 15. Most pure maths theories do not find any practical applications. 16. Pure maths theories are often not significant and they are entirely forgotten in, say, 50 years.

VI. *Correct the following statements.*

Model. **Modern maths is not sophisticated.**
It is, in fact, sophisticated and claims good training.

1. The word "mathematics" is unambiguous. 2. To the short question "What is maths?" there is a short answer. 3. Mathematicians perfect and refine everyday language and apply it to scientific formalization. 4. Maths is dry, barren and boring. 5. Mathematicians are high priests of mysterious and powerful mind of magic cult. 6. Mathematical activity takes place in high spheres open only to the few initiated. 7. Maths is a free creation of the mind unconditioned by anything except the nature of the mind itself. 8. There is nothing necessary in any of the fundamental concepts, definitions and axioms of any branch of mathematics. 9. There is no interconnection and interrelation between maths and the real world. 10. Mathematicians never care for the useful application of their science. 11. Lengthy and boring calculations give satisfaction and aesthetic emotions. 12. Most mathematicians are men of genius with extraordinary mental abilities. 13. Maths deals with empty equations signifying nothing.

VII. *Discuss the given alternatives, choose the right one to your mind or complete the sentences with an ending of your own.*

To bridge the gap between the mathematical and non-mathematical communities the mathematicians ought...

1) to popularize maths for the laymen; 2) to make maths more popular; 3) not to use precisely accurate mathematical terms; 4) to define every mathematical concept; 5) to speak straightforward, unpretentious language in the explanations; 6) to apply techniques familiar to non-mathematical audiences; 7) to enlighten the readers on different subjects of today's maths; 8) to co-author their mathematical textbooks (manuals) with foremost scientific fiction (sci-fi) writers; 9) to teach non-specialists to reason in mathematical terms and reach all the conclusions by the deduction only; 10) to explicate the history of every great discovery in maths.

To establish a rigorous deductive proof of a theorem the mathematician should...

1) get familiar with models of mathematical proofs; 2) master deductive reasoning procedures; 3) develop a set of definitions of basic terms, axioms and postulates; 4) collect new results available; 5) be systematic and use perfect logic; 6) apply the rule of inference; 7) avoid vagueness, ambiguity, implicitness and circularity; 8) prove its converse; 9) use his inspired intuition and insight in a proper way; 10) reason deductively and rigorously.

To develop a new fundamental and consistent mathematical theory the mathematician must ...

1) review the preceding material on the topic; 2) revise basic mathematical concepts; 3) introduce new sings and symbols; 4) invent novel mathematical notions; 5) deduce new theorems; 6) create a new hypothesis (conjecture); 7) find the applications of the theory; 8) verify the results of other mathematicians; 9) reconstruct the old theory; 10) vary and modify old techniques and devices; 11) invent new sophisticated maths; 12) carry out complicated arguments and demonstrations.

VIII. *What is meant to imply by the following assertions?*

Models. 1. **Maths is an applied logic.**

This assertion **means** that logic is an integral component of maths both pure and applied. It **implies** as well that mathematicians must reason logically while developing their theories and establishing their rigorous proofs. But maths is not logic only, it has the subject matter of its own. The assertion is an example of unscientific definition of maths.

2. **Living maths involves new results and the work of predecessors.**

This assertion **means to imply** that maths is always in the making. The work of predecessors should be mastered, developed further and corrected, if necessary. Not infrequently new results in maths appear when the mathematician does not take the classical contribution for granted.

1. There is no **national** prejudice in maths. 2. The assumptions from which maths starts are simple; the rest is not. 3. We say that while the creative work in maths is done by individuals, the results are the fruition of centuries of thought and development. 4. Maths is something over and above mere development of axioms, theorems and proofs. 5. Maths is abstract and it is hard. It is distinctive and effective and thus important. 6. Maths is a living plant which flourishes and fades with the rise and fall of civilizations, respectively. 7. We are all laymen outside the field of our speciality. 8. An educated mind is composed of all the minds of preceding ages. 9. Where there is pattern, there is significance. 10. The intuitive side of the basic concepts is mathematically irrelevant. 11. Without maths it is impossible to gain any deep insight into the essence of space, time and matter. 12. All things in the whole wide universe happen mathematically. 13. Mathematical universe is an ideal universe. 14. Pure maths does not arise of pure thought. 15. While mathematicians produce formulas, no formula produces mathematicians. 16. Maths is a body of knowledge, but it contains no truths. 17. Wherever we have deductive reasoning we have maths. 18. Much of the scientific knowledge is produced by deductive reasoning. 19. There is no absolute rigour. Rigorous proof is a slow and torturous method. 20. Rigour in the proof is the enemy of simplicity.

IX. *Agree or disagree with the statements given below. Use the introductory phrases. In case of agreement repeat the statement, and add some sentence(s) to justify your choice.*

That's right. It's O.K.	**Quite the contrary (the reverse).**
Exactly. Quite so.	**You're wrong there, I am afraid.**
It's correct to say.	**It's unlikely.**
I quite agree with you.	**Not at all. Not quite so.**
I share this viewpoint.	**Just the other way round.**

Models. 1. **People communicate by means of language.**
That's right... Language is the foremost means of communication.

2. **Mathematicians speak only the formalized language of maths.**
This is not the case. Nodoby speaks the formalized language of maths. It is an unspoken language. It helps codify scientific knowledge. Symbolism must be avoided wherever it is not of real help or a necessity.

1. People make their thoughts known to one another by means of speech only. 2. Natural human languages exist in speech. 3. Speech is the only means of communication. 4. People use speech in preference to other means of intercourse. 5. National language is the product of manners and customs of a country. 6. Every generation of people develops its own new language. 7. The history of every national language is indistiguishable from the history of the particular nation. 8. Speech is the creation and formulation of human thoughts in terms of colloquial language. 9. Speech is the essential means in human development. 10. The language of maths like common language is natural. 11. Symbolism often leads to misunderstanding. 12. The language of maths is set up by distinguished linguists. 13. Scientific language is designed and devised with the definite purpose and ingenuity. 14. Most mathematical texts are written in everyday language. 15. Today's mathematical language is the same as the language set up by Newton and Leibnitz in the XVIIth century. 16. There is nothing more horrible that the current misapplication of symbolism in maths. 17. Basic terms of present maths no longer have their usual intuitive meaning, they become abstract concepts.

X. *Suppose that the statement seems to you too concise and you want to add. Repeat the statement and add your own reasoning, thus developing the idea further. Use the following phrases.*

There is one more point...	**One more remark seems reasonable, namely...**
I may as well add that ...	
Moreover... More than that...	

Models. 1. **Writing is a powerful means in mastering a foreign language.**
Certainly, ... I may as well add that writing involves: graphics, spelling and composition. So, writing is a very difficult language skill, to my mind.

2. **Mathematics is often termed the language of science.**
Exactly. Mathematics is, indeed, the language of science. But **I like to emphasize** that maths is not only a language but a science in its own right.

1. Written and spoken languages are different forms of human communication. 2. Speech is spontaneous while writing allows time for think-

ing, reasoning and choosing the right word. 3. Speaking is many times faster than writing. 4. Early verbal thinking was rough and confused. 5. Reasoning was unfinished and words mistaken for the things they represent. 6. With speech and writing came the beginnings of science. 7. Numbers or the written figures called "numerals" have a long history. 8. Number names were evidently the first words used when people began to talk. 9. Numbers may have different representations. 10. Symbolism distiguishes the language of science from spoken everyday languages. 11. When maths is used in any science it brings precision, rigour and objectivity about. 12. Rigour in science depends upon the language used. 13. Maths is the source for precise language for all the sciences. 14. The function of the formalized language is to codify and present mathematical theories. 15. Bare mathematical formulas explain nothing; they simply describe in precise language. 16. Yet, such formulas are the most valuable knowledge man can acquire about nature.

XI. *Practise problem Questions and Answers. Work in pairs. Change over!*

Models. Q. How many signs and symbols are there in the language of maths?

A. As far as I know, there are 50, not less, and of various categories: symbols of mathematical objects, relations, and operations.

Q. Which symbol or sign is the most important in the language of maths?

A. Certainly, it's the little sign "=" which is translatable as "is another name for". Such basic mathematical concepts as an **equality** e. g., ($a \cdot a \cdot a = a \cdot 3$), an **identity** e. g., ($ab = ba$), an **equation** e. g., ($2x+5=11$) all involve this sign. Most scientific laws are expressed in terms of equations.

Q. The equality sign is basic in maths, sure enough. However, among numerous symbols and signs one is remarkable, with the big meaning: "∞". How should we word it?

A. It is pronounced "**infinity**". The mathematical notation "$\to \infty$" must be worded "tends to infinity".

Q. Is there any need to go out of this world to locate mathematical infinity? Where is it exactly situated?

A. In the scientists' mind. It's an abstract concept. In the calculus "∞" means limit and nothing more.

Q. In algebra we must use the symbol for a **variable.** How can you complete the definition, "A variable is ..."?

A. Men use variables readily and never try to define the term precisely. Very roughly speaking, **a variable is a placeholder symbol for the unspecified numbers.** Generally, literal symbols x, y, z, w are variables regardless of what happens to them.

Q. Is it enough to look at a symbol and say, "Ah, this is a variable!"

A. By no means. Variables must be understood in their context. Suppose, we have a formula $x+y=3$, without additional remarks, very little meaning is conveyed.

1. What do you feel looking at a book page sprinkled with x's and y's, $=$'s and other mathematical symbols and signs? 2. Symbolism burdens the memory and is a bar to understanding. Isn't it the case? 3. Does the mathematician write in the language of maths to hide his

knowledge from the world at large? 4. Is it possible to make Mathematical Language-to-Russian dictionary or a "translation key"? 5. Does every branch of maths have its own "language"? 6. Do new symbols often appear in maths? 7. When do mathematicians introduce new symbols and signs into the language of maths? 8. How old is the language of maths? 9. To whom is the language of maths "foreign"? Can it be mastered overnight? What is the reason for this? 10. The purpose of mathematical symbolism is to facilitate concise communication among scientists, isn't it? How does it work? 11. Does the language of maths change from generation to generation as any common language does? 12. Are mathematicians always consistent in the way they use their notations? Is the language of maths perfect? 13. The words "variable" and "constant" sound like antonyms. Are they used in this sense in maths? 14. The symbol x may have more than **ten** different interpretations. What does its signification depend on? 15. Are arithmetic operations more convenient to perform in Arabic numeral notation or in Roman numerals?

XII. *Add the opening phrase, repeat the statements and keep the conversation going.*

It is not meant that ...
I don't mean to say ...
Mathematicians object and say...
The statement does not imply ...
Scientists do not claim ...
It is too much to say that ...

Model. **The calculus is a brain-twisting subject.**
Mathematicians object and claim that the calculus is the most rational subject in maths. It is a razor-sharp algebra.

1. Ordinary languages are absolutely unreliable for science. 2. The language of maths is the language of the brain. 3. Formalized text needs no explanatory footnotes. 4. Unsophisticated reader can grasp the fundamental ideas of a formalized text quite easily. 5. Mathematical formulas are easy to understand. 6. The formulas avoid vagueness and unwanted extra meanings. 7. Routine services of maths for sciences include only computations. 8. Maths customarily performs marvels in science and develops entirely new viewpoints.

XIII. *Agree or disagree with the following negative statements and develop them further.*

Models. 1. **Scientists do not codify science in terms of colloquial phrases.**
No, they don't. They use formalized languages for that purpose.
2. **Maths is not a tool for reshaping information.**
But it is. It is like an automaton that operates with the rules of logical arguments instead of wheels and pistons.

1. The scientist building knowledge needs not express himself in clear language. 2. Ordinary languages are not vague and they are reliable for science. 3. The language of maths does not say what it means. 4. Mathematical symbols do not have any unwanted extra meanings. 5. The formula $y=16\Delta t^2$ does not tell anything about mass or gravity. 6. There is no formula for uniformly accelerated motion. 7. Scientists do not amalgamate several relationships in their arguments. 8. The compact shorthand of algebra is not the main part of the language of maths. 9. Maths is not a clever servant for science. 10. The knowledge of the mathematical formulas does not represent knowledge about all the situations encompassed by the formulas. 11. The person who looks at a mathe-

matical formula and complains of its abstractness, dryness and uselessness fails to grasp its true value.

XIV. *Express surprise, consent or disagreement with the statements given below. Try to prove your viewpoint or advocate the opinion of others. Summarize the discussion.*

Model. **Maths is an art, with a beauty of its own.**

1. Is it really? It is too much to say that maths is an art, to my mind. It runs counter to common sense, indeed. I prefer a conventional definition of maths as one of the oldest sciences.
2. Exactly. Maths has nothing to do (=nothing in common) with art. There is no poetry and no beauty in maths. From a mathematical standpoint, it's nonsense to claim it.
3. I share this viewpoint. Science is not the object for art and beauty. It's, in fact, a meaningless statement.
4. You're all a bit wrong, I am afraid. To say that maths is an art is not to say that it is a mere amusement. The highest compliment to a mathematical work is to call it elegant, though it is not easy at all to define elegant maths.
5. Surely. The inclusion of maths among arts is apparently not illogical. Mathematical creations have design, symmetry, harmony and inner beauty, i. e., characteristics of art, in the long run.
6. One more remark seems reasonable, namely, in the search for a method of proof the mathematician must use not only his creative ability and insight but **inspiration** that we usually associate with the creation of a piece of art or music.
7. Certainly. I quite agree to it. I may as well add that the role of maths as an art is especially emphasized when conjectures (hypotheses) are proved.
8. That's right. It's common knowledge that **a rigorous and elegant proof** is beautiful to mathematical eye. It's a poem and a delight for the mathematician.
9. There is one more point that I think is relevant. The analogy between maths and art makes sense only to a person who loves both.
10. **Summing up the discussion**, it seems correct to say that there may be different viewpoints, but maths is more than only a language or technique. It's an art in the broad sense of the word.

1. Most mathematicians are not insensitive to art and beauty. 2. Mathematicians see beauty where others find only confusion of signs and symbols. 3. Maths and art are intimately related. 4. Art is beyond the scope of a scientist's interests. 5. Poetry is read only by artistic-minded people — not by mathematicians. 6. Matematicians pay no attention to the elegance of presentation. 7. Mathematicians' search for beauty and elegance likens them to artists. 8. The beauty of a theorem lies in its simplicity and generality. 9. Mathematical ability is often classed with artistic ability. 10. A beautiful mathematical result is always non-trivial. 11. Elegant and beautiful ideas enrich maths. 12. The mathematician like an artist is a maker of patterns. The mathematician not rarely chooses his patterns for beauty's sake. 13. In mathematicians' view the formula $c^2=a^2+b^2$ is elegant and beautiful.

XV. *Confirm or deny the following statements.*

1. From the time of Pythagoras the study of music is regarded as mathematical in nature. 2. The relationship between maths and music is obvious. 3. Music — the most abstract of the arts — apparently appeals to mathematicians. 4. Not few mathematicians are excellent musicians. 5. Masters such as Bach constructed and advocated mathematical theories for the composition of music. 6. In such theories cold reason rather than spiritual feeling gives the creative pattern. 7. Music lovers can enjoy beautiful music thanks as much to a mathematician Fourier as to Beethoven. 8. Unlike the sciences but like the art of music maths is a free creation of the mind.

XVI. *Translate the text into English.*

Что такое математика? Что она изучает? Существует ли математика единая как система органически связанных между собой знаний или она скопление научных дисциплин, изолированных друг от друга по своим методам, целям и даже по языку выражения своих результатов? Ответить на эти вопросы — совсем не легкое дело. Определение предмета и сущности математики, высказанное Ф. Энгельсом сто лет назад, сохраняет свою справедливость и актуальность. Чистая математика имеет своим предметом пространственные формы и количественные отношения действительного мира. Все объекты и процессы, реально существующие в мире, обладают такими свойствами, которые выражаются в категориях количества и формы, т. е. они присущи всей действительности. Математики абстрагируют упомянутые количественные отношения и пространственные формы, устанавливают связи в реально протекающих процессах, формулируя их в виде логических высказываний, записанных символами и формальными определениями. Дальнейшее развитие этих абстракций включает в себя доказательство теорем, образование новых понятий, построение новых теорий. Эти понятия, теоремы, теории применяются впоследствии к изучению действительности. По мере восхождения к более высоким абстракциям связь теоретической математики с практикой, с действительностью становится все менее непосредственной и осуществляется во многом через другие науки. По отношению к этим наукам математика выступает как метод и язык формулировки количественных закономерностей, как средство решения задач, как аппарат для построения и разработки теорий. В них она также черпает новые понятия, задачи и импульсы для своего развития.

Математика есть лишь **специфическая форма процесса человеческого познания.** Математическое мышление — одна из форм этого общего процесса познания. Математики мыслят абстракциями. Математические абстракции имеют материальное происхождение, они представляют определенные свойства реальных вещей. Математические теории являются не произвольными построениями ума, а отражением сущности вещей. Математика выделяется среди других наук своей универсальностью. Методы математического исследования составляют неотъемлемую часть всех наук. Применение математических методов исследования повышает объективную ценность научных теорий. Математика должна рассматриваться в развитии. Развитие математики не означает добавление новых теорем; оно включает в себя качественные изменения содержания математики. Универсальность математики объясняется широтой ее предмета. Трудно, если вообще возможно, провести границу между математикой чистой и прикладной. Обособление этих частей математики не соответствует объективным закономерностям развития математической науки, противоречит им. Быстрый рост состава математики и ее приложений привели к ряду революционных преобразований содержания математики, к осознанию ее значимости. Борьба противоположных взглядов на природу и методы математики или отдельных ее частей обостряются в те периоды ее истории, **когда происходит становление новой теории, ведущей к существенному пересмотру сложившихся представлений, смысла основных понятий, операций логического анализа, систем исходных высказываний (аксиом), средств вывода новых теорем и т. п.** Подобные революционные преобразования были, например, в период создания математического анализа, формирования неевклидовых геометрий, введения в математику теории множеств и создания кибернетики.

COMPOSITION

Reproduction Writing

Reproduction is a composition in which the ideas are stated explicitly and the task is to remember correct English ways expressing them. The student must listen attentively to the text, read by the teacher or recorded on the tape and then write the gist (the essence, the main points) using some words and expressions from the text either given by the teacher or memorized.

I. *Listen several times to tha recording and reproduce* (*in writing*) *any part of Text One "What Is Mathematics?"*

Dialogue Writing

A dialogue is a conversation or talk.

II. *Reconstruct Text Three "Mathematics — the Language of Science" into a dialogue. Write questions and correct answers.*

Paragraph Writing

A paragraph consists of a number of sentences which are closely related and deal with the same topic.

Model. Scientific methods of reasoning seem so different from the reasoning used in our ordinary life, because they are more refined, more elaborate and sophisticated. Yet, essentially, they are the same. The object of our reasoning in life and in science is the same: to order events; to choose the most essential points of the events; to extract their relations and interconnections; to understand and explain the world of our sense perceptions.

III. *Write paragraphs showing how you reason* a) *while crossing a busy crossroad with heavy traffic*; b) *while trying to prove your viewpoint in a scientific dispute*; c) *while developing a formal mathematical theory. Your paragraph must not exceed 5-8 sentences.*

Abstract* (prècis)** in Scientific Abstract Journals

An abstract (реферат) is the expression (reproduction, representation) in a condensed form of the content of any piece of scientific writing in a limited number of

* The adjective „abstract", e. g., abstract concepts, must not be confused (mixed up) with the noun „abstract".

** **A prècis** (конспект, реферат) is any abbreviated and condensed restatement (reproduction) of essential facts, the main ideas, points, details of a text given for collateral readings made by the student as an exercise in composition. In scientific literature the term „abstract" is more preferable.

sentences. The student must briefly formulate the main ideas in his own words excluding and omitting the unnecessary details. Facts must become plain statements. An abstract consists of a) an introduction, i. e., data (the printed source, the author's name, the title); b) principal part; c) conclusion.

Abstract Reading Practice

I. *Read, translate and analyze the given below abstract.*

Model. a) Mathematical Reviews, June 1983, p. 2259. 83 (f) Bambah, R. P. **Mathematics and Society.**

b) The author points out some discreet and indirect contributions of mathematics to the behefit and welfare of society. Firstly, he mentions some of the great achievements of mathematics during the last two decades, arguing that whenever difficult and important problems are solved, "the whole human race shares in the glory". As for the "indirect" contributions, the relations between mathematics, physics and chemistry are considered. The study of functions of several variables is assumed to be one of the most favoured research areas with respect to these relations. Symmetry and qualitative analysis, leading to group theory, resp., topology are general principles used in handling the corresponding problems. More "direct" contributions of mathematics to society can be seen in the applications in economics, planning, industry, management and the author gives some examples, e. g., linear programming and input-output analysis. He mentions some special relations of mathematics to other areas, such as mathematics and biology, mathematics and communications, etc. Finally the author confesses his belief in the benefit of mathematical education for gifted young people.

c) The book shows the author's view of the beauty and usefulness of mathematics. It should be useful supplementary reading for students who seek an introductory overview to mathematics, its utility and beauty. The book encourages the mathematics students to be involved more deeply into the history of current mathematics.

ASSIGNMENTS

II. *Look through abstracts in scientific abstract journals, e. g., Mathematical Reviews of the current year, choose one and study it carefully. Be ready: 1) to read and translate it in class; 2) to display its components structure*; 3) *to assess its subject matter*; 4) *to appreciate its grammatical and stylistic peculiarities*; 5) *to justify your choice.*

Abstract Reading Practice is compulsory for students in every of this textbook's lessons.

Abstract Writing Practice

III. *Read Text Four "Mathematics and Art" carefully several times. When it is completely understood start writing an* a b s t r a c t, *i. e., a brief condensation of the whole text.* a) *Give your abstract a suitable title*; b) *Begin the abstract with the introductory general phrase*: The text deals with ... (speaks about, says that, presents, shows, relates, points out, discusses, describes, reviews, sketches, surveys, consists of, is devoted to, throws light on, gives some comments of, traces the history of, outlines the development of, offers an overview, etc.); c) *The principle part must not exceed 5-6 plain statements generalizing the main ideas of the text in a logical sequence*; d) *Conclude* the abstract with your personal viewpoint (opinion, judgement, critical comments, etc.) of both the content and the language of the text, using the given phrases*: The text is (non) informative ... The information is up-to-date (out-of-date) ... The language is quite (un)manageable ... The style is formal (academic) ... There are practically no (many) unknown words ... The meaning of the unfamiliar words can (not) be grasped from the context ... The text is dry, dull and boring ... It is (not) worth reading and abstracting ... The reading of the text (does not) gives some satisfaction and pleasure ... The reading of the text (de)increases language skills.

Composition Writing

A **composition** is a creative literary work dealing with one problem or topic in detail.

IV. *Write out from all the texts and exercises of the lesson the definitions of mathematics which, to your mind, are correct. Arrange them up to your liking and add some more relevant and important information. Make a plan for the composition. Write a composition, using the active vocabulary, on the topic: "What Is Mathematics?"*

COMPREHENSION EXERCISES

Questions

Choose and answer some problem question(s) in writing and be ready to take part in the discussion.

1. Where does the word "mathematics" come from, I wonder. 2. Does mathematical knowledge come as a consequence (result) of studying and learning alone? 3. How many subject-fields (branches, domains, divisions, compartments) of maths do there exist nowadays? 4. What are the fundamental components of any branch of maths? 5. Can you name some new branches of modern maths? What field of maths is the most interesting (important, essential, significant), to your mind? 7. Why are axioms necessary in a deductive system? 8. Why ought the mathematician to reason deductively? 9. Can we distinguish between whole numbers and irrational numbers from the viewpoint of their origin? 10. What are the factors that make possible the growth of maths? 11. What can research in maths mean? 12. Is the use of abstractions peculiar to maths alone? 13. Are the concepts of force, mass, energy, wealth, liberty, justice, democracy, etc., mental creations? 14. What is meant by the phrases "pure maths", "applied maths"? What else can be "pure" and "applied"? 15. What is more important: a mathematical theory or practical applications? 16. Can a single person be a specialist in many if not all the branches of present-day maths? 17. Where is progress more rapid in pure or applied maths? 18. Where do mathematical concepts come from?

* No personal viewpoint is wanted in a **formal abstract** meant for publication in a scientific journal, e. g. **Mathematical Reviews**, but for teaching purposes such a conclusion is desirable.

19. Most abstract mathematical concepts have their physical counterparts, haven't they? 20. Are mathematical concepts discovered or invented? 21. Do mathematicians mean the objective existence of the objects they study? 22. Although maths is a science, it is usually distinguished from science by its relative independence from empirical considerations. How does pure maths manage to conceal its origin from the real world? 23. Does maths deal only with numbers and geometrical forms and the concepts built upon these basic ideas? 24. What does the degree of abstraction of a mathematical notion depend on? 25. Why do mathematicians not deal with abstractions of other sciences? 26. Do mathematicians make an agreement with physicists, economists, chemists, sociologists and others and divide abstract concepts among themselves? 27. What is a mathematical postulate (axiom, theorem, proof, theory)? 28. How is maths created and developed? 29. What mathematicians may be called distinguished (famous, prominent, outstanding, of genius, etc.)? Who are the greatest Russian and Soviet mathematicians according to your personal viewpoint and criterion? 30. Do modern young mathematicians advance more rapidly than the mathematicians of the previous (preceding) ages? 31. Do the phrases "mathematical language", "mathematical notation", "mathematical symbolism" mean the same thing? 32. What were the first mathematical signs and symbols, to your mind? 33. What is the distinction between natural language and the language of maths? 34. What do they have in common? 35. Is there any difference between the language of algebra and the language of the calculus? 36. What language(s) do mathematicians speak? 37. How do mathematicians prefer to express themselves in their mathematical writing? 38. What language (Latin, French, English, native, formalized) do mathematicians use for their scientific publications? 39. What is the distinction between a strict scientific presentation and a popular scientific presentation of a mathematical theory? 40. Can every mathematician introduce any symbols he likes or prefers into the language of maths? 41. Can a common language render the subtle art of scientific reasoning, designing hypotheses and developing mathematical theories? 42. If a speaker uses a word we are not familiar with, the context usually gives the clue (key) to its meaning. Can a scientist understand the meaning of the unfamiliar symbol in a formalized text? 43. How do mathematicians make transitions in reasoning? 44. What do we call reasoning from particular and individual to general, and vice versa? 45. If we wish to avoid "circular reasoning" what must we do? 46. What method of reasoning is the most reliable in science? 47. What is the difference between a dictionary (encyclopaedic) and a rigorous mathematical definition? 48. Implicit (explicit) definition. The range of definition. What do all these phrases mean in maths? 49. While proving a theorem what can the mathematician rely on: intuition, instinct, imagination, a flash of insight, logic, the power of deduction, inspiration, common sense, experience? 50. If there is no ready-made solution for the problem involved what must the mathematician do? 51. Is the inclusion of maths among arts senseless? 52. What does the creative process in both maths and art involve? 53. Are aesthetic requirements (beauty, elegance of proof, etc.) subject to formal and logical analyses? 54. What is meant by the phrase "revolutions in art and maths"? 55. Do mathematicians seek to understand the nature of musical sounds? Why? 56. What are some of the cultural bearings and humanistic implications of maths?

Discussion

1. What do **the given definitions** emphasize? What do they have in common?

a) Pure "mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true" (B. Russell).

b) "Mathematics is a meaningless game played with meaningless marks on paper and shifting these marks about in accordance with certain tacitly assumed or explicitly formulated rules of play" (D. Hilbert).

2. ***Mathematicians*** — what are they? When (why) does a person make up his mind to become a mathematician? What motivates and directs the activities of mathematicians? What mathematician(s) to your mind, is the most distinguished and why?

3. What are the mathematicians and scientists who made for the development of **mathematical language** and introduced the basic signs and symbols. Consult the articles: Знаки математические. БСЭ. Изд. III. Том 9. М., 1972, с. 548—550. Is mathematical language no more than a system of signs and symbols? Has it any content or is it above reality? Has it any roots in some actuality or are the truths of mathematical language independent of all experience? If language is a means of communication between human beings, what is communicated by mathematical language? How is this communication effected?

4. How is an **abstract mathematical science** constructed? Does the abstract science consist of abstract concepts alone? Is modern maths abstract or emperical? What are the (dis)advantages of maths being abstract? **Symbolic language determines the true aspect of modern maths.** What does this statement imply? Is an abstract mathematical theory of use to any one but mathematicians?

5. Strictly speaking, the careful mathematician cannot say: "It is true". He must instead make statements like this' "If *A* is true, then *B* is true". If you wish to prove a statement in any subject of maths why do you prove it by deducing it from other statements? **The truth of mathematical** statements is relative and relevant only to maths and has no direct bearing on the physical world. Why?

6. Most mathematicians object to the separation of pure and applied aspects of mathematics. Why? Do pure and applied maths have common language, methods, applications?

7. **Reasoning** may be: **true, false, (in)valid, rigorous, vague, (un)scientific.** What do all these phrases mean? What's the distinction between everyday reasoning in life and scientific reasoning?

8. The requirement of rigour in reasoning is proverbial in mathematics. **Mathematical rigour** — who needs it?

9. What is the role of a) experience, b) common sense, c) intuition, d) talent, e) genius, f) imagination, g) flashes of insight in maths?

10. **Language** that is reliable for science. What characteristics must it have? Its vocabulary and syntax. "English cannot be taught, it must be learnt" (M. West). Do scientists learn formalized languages or do they master them without learning? Your own experience. What other formalized languages besides maths do you know? How did you manage to master them?

11. There exist several types of **translation** viz., **literal, word-for-word, verbal, adequate, literary.** Explain what all these terms mean. Which type of these translations is the best? Does the translation of the verbal or

wordy statement of a mathematical problem into a symbolic language involve similar difficulties as the translation from foreign languages?

12. **Grammar** is the study and analysis of how the language is spoken and written by most educated people. As language habits and customs change with years, the "rules" of grammar change, too. What about the "rules" of formalized languages? Do they change with time?

13. What is meant by the phrase: "**Mathematization of sciense**"? What are its (dis)advantages and implications? In the present age specialization means isolation. Give the reasons and possible arguments. The use of mathematical methods enlarges **the objective value** of scientific theories. Why? Illustrate it by some examples.

14. "The science of **Pure Mathematics** in its modern development may claim to be the foremost original creation of the human mind. Another claimant for this position is music" (A. Whitehead). Is maths the creation of human mind alone? What about applied maths? Can we liken it to music? Why?

15. The professional mathematician has a strong poetic form in his own way of **scientific presentation.** Illustrate it by examples to show the compactness and elegance of mathematical style. What's the layman's viewpoint on this question?

16. "Music is the pleasure the human soul experiences from counting without being aware that it is counting" (G. Leibnitz). Explain in your own words the meaning (sense) of this statement. Was G. Leibnitz a mathematician or a musician?

17. **J. Fourier's theorem** says that all sounds, vocal or instrumental, simple or complex, are completely describable in mathematical terms, i. e., by a formula. Every musical sound, however complex, is merely a combination of simple sounds. Is this theorem of pure maths or of physics? What is its significance in maths? How can it help distinguish between musical sounds and noise? Can the theorem explain the sweetness of some sounds and harshness of others? Why does the same note given off by both violin and piano sound different to the ear? **The role of maths in music.**

18. "In the interest of clarity, I did not hesitate to repeat myself and did not pay the slighest attention to the **elegance of presentation.** I sincerely stuck to the prescription of the great theoretician L. Boltzmann that the business of elegance should be left to the tailors and shoemakers" (A. Einstein). Are **Einstein's creations** elegant, to your mind? Do scientists like Einstein's form of scientific presentation? How much information does Mass-Energy-Relation ($E=mc^2$) encompass? Why did Einstein have to repeat himself? Are Einstein's creations easy to understand? Why do you think Einstein made such a statement?

19. Wherein is **the beauty of maths**? Beautiful mathematics is the greatest contribution of the man's mind to all the civilizations. Prove it.

20. It is convenient to keep the old classification of maths as one of the sciences, but it is more just to call it an art. If maths is an art with cultural bearings it must be a part of the liberal education of a doctor, lawyer or average educated person. Agree or disagree.

LESSON THREE

UNSOLVED PROBLEMS

Grammar:

1. Past Indefinite Tense-Aspect Forms.
2. "ing"-ending forms.
3. Indefinite Pronouns, Adverbs and their Derivatives.
4. Modal Verbs and their Equivalents.

LAB. PRACTICE

Repeat the sentences after the instructors.

1. In this lesson we are to get familiar with geometric constructions under the conditions specified and the famous unsolved problems in mathematics. 2. Students are taught mathematics at school and college with the idea that all mathematical problems can be solved. Unfortunately, this is not the case. 3. In mathematics there exist problems that can be readily solved as well as the problems that are impossible and the ones demanding the right often ingenious technique for their solution. 4. However, the conviction of the solvability of every mathematical problem is a powerful challenge and stimulus to the researcher. 5. The Pythagoreans' discovery that $\sqrt{2}$ is irrational was the first example of **a proof of impossibility** in mathematics. 6. Every mathematical problem must be settled either in the form of a direct answer to the question posed, or by the proof of the impossibility of its solution. 7. Numerical evidence counts for very little, the only luxury a reputable mathematician allows himself is proof. 8. Mathematical rigour in reasoning demans that **the solution of the problem must be established by means of a finite number of steps based upon a finite number of hypotheses precisely formulated**. 9. This high standard of mathematical rigour was formulated by the ancient Greek mathematicians and philosophers in order to make mathematics finite, rigorous and coherent. 10. The search for the construction problem solution is a favourite subject in Geometry. 11. The ancient Greeks are given credit for posing famous unsolved construction problems that challenge mathematicians and amateurs alike even today. 12. The Greeks imposed severe restrictions upon the instruments used for the construction. Ruler-compass constructions are the drawings made by using only a straightedge (=an unmarked ruler) and a compass. The constructions must be performed with the highest degree of accuracy and precision. 13. The Greeks gave special attention to geometric constructions, as each construction served as a sort of **existence theorem** for the figure or concept involved.. 14. To prove that a certain object exists meant for the Greeks to construct it. With a straightedge we may draw (that is, construct) a line determined by any two points. With a compass we may

construct a circle. 15. The classical Greeks were able to carry out many constructions with these two permissible tools. Nevertheless, despite the persistent efforts, the Greeks failed to solve the three famous construction problems, viz., "squaring the circle", "doubling the cube" and "trisecting the angle". 16. The Greek geometers realized that the allowable instruments were inadequate for the solution sought. 17. Though the construction was the main part of the solution it was not the whole task. The problems were of both practical and theoretical interest. 18. The Greeks sought to prove that the constructions could be performed in principle, that the solution could be found **theoretically.** 19. They tried ot devise a theory in terms of which they could rely on the construction in place of the existence theorem, but they did not succeed in creating it, however. 20. The theory in question was developed successively by a Danish geometer **G. Mohr** (1672), then by an Italian engineer **L. Mascheroni** (1797) and by a Swiss scientist **J. Steiner** (1833). 21. In the 19th century it was finally proved that the famous unsolved problems defy solution under the restrictions specified. 22. It should be emphasized that though the Greeks failed to find the solution satisfying their criterion, they made great mathematical discoveries on the way, for in mathematics there is no futile search. 23. The failure with the classical unsolved problems was the stimulus for many novel developments in mathematics. 24. Every generation of mathematicians ever since the Greek times on has to seek a proof that certain problems are solvable or insoluble in principle. 25. The number of problems in mathematics is inexhaustible and as soon as one problem is solved others come forth in its place. Mathematics offers an abundance of unsolved problems.

Key Grammar Patterns

"ing"-ending forms

Numerous "ing"-ending forms in English may have different meanings and different functions in the sentence The translation of a particular "ing"-ending form depends on its part of speech and its function in the sentence.

Analyze the following parts of speech with "ing"-ending.

Noun	**Adjective**	**Participle I**	**Gerund**
a morning	interesting	expressing	in solving
a building	following	(выражающий;	by measuring
a meaning	misleading	выражая)	without finding

Verbal Noun	**Preposition**	**Conjunction**	**Adverb**
The measuring of	concerning	providing	according
areas and volumes.	regarding	supposing	notwithstanding
The understanding of	owing to	seeing	running
the article.			

As an example let us take the word "**meeting**" and analyze its meaning, functions and possible translation. As a **noun** it may have the following meanings: *собрание, заседание, митинг, встреча, пересечение, схождение, слияние, соединение, разъезд, стык* и т. д. It may have all the functions of a noun in the sentence (the subject, the predicative, the object, the attribute). As a **Participle I** or **Gerund** it may be translated as: *удовлетворять (-ющий, -ая), отвечать, соответствовать требованиям* и т. д. So, be careful while translating "ing"-ending forms!

Read and translate the following text, identifying all the "ing"-ending forms in it.

There is much **thinking** and **reasoning** in mathematics. Students master the subject matter not only by **reading** and **learning**, but by **proving** theorems and **solving** problems. The problems therefore are an important part of **teaching**, because they require students to discuss and reason and polish up their own knowledge. To understand how experimental knowledge is matched with theory and new results extracted, the students need to do their own **reasoning** and **thinking.** Of course, it is quicker and easier, for both teacher and student, if the text states all the results and outlines all the **reasoning**; but it is hard to remember such **teaching** for long, and harder still to get a good **understanding** of science from it. So, in this textbook many of the problems ask you to do your own **thinking**; and for this reason they form a very important part of the **teaching.**

Some problems raise general questions whose discussion can do much to advance your **understanding concerning** particular points of the theory. Such general questions ask for opinions as well as **reasoning**; they obviously do not have a single, unique or completely right answer. More than that, the answers available are sometimes **misleading**, **demanding** more **reasoning** and further **proving.** Yet, **thinking** your way through them and **making** your own choice of opinion and **discussing** other choices is part of a good education in science and a good method of **teaching.**

Indefinite Pronouns, Adverbs and their Derivatives

some	somebody	someone	something	somewhere	whatever
every	everybody	everyone	everything	everywhere	whoever
any	anybody	anyone	anything	anywhere	whichever
no	nobody	none	nothing	nowhere	wherever

Read and translate the following sentences.

1. We know **something** about his work. 2. You can't find this book **anywhere**, it is practically unavailable. 3. **No** mathematician confuses these basic terms. 4. **Whatever** book you choose it is good to begin with. 5. **Everything** is ready for the experiment. 6. **Whoever** says it he is wrong. 7. **Whenever** she comes she asks about you. 8. **Wherever** we see him, he is always in a hurry. 9. **Everybody** knows this familiar theorem. 10. **Whichever** of these problems you try to solve you must use this method of reasoning.

Modal Verbs and their Equivalents

can	**may**	**must**
to be able	to be allowed	need, shall, should, ought to, to be (to), to be due (to), to be bound (to), to have (to), to have got (to), to be obliged (to)

1. **Are** you **able to draw** a straight line without a ruler? 2. We **are allowed to use** only a straightedge and a compass in ruler-compass constructions. 3. **Are** we **to define** all the plane geometric figures? No, you **need not.** You **should perform** the construction first. 4. I **have to make** another drawing as this one is inaccurate. 5. He **shall use** this method of

proof for the problem. 6. **I have got to check** my results. 7. The solution **is due to be found.** 8. The plan **is bound to succeed.** 9. He **is obliged to fulfil** his task. 10. She describes how the constructions **are to be performed.**

Read and translate the text. Practise "question — answer" type of reproduction of the main ideas of the text. Work in pairs.

TEXT ONE

UNSOLVED PROBLEMS OF ANTIQUITY

Greek mathematics is significant for the questions it raised and did not answer. Among such questions are three famous construction problems known to every amateur in mathematics. They are referred to as **"squaring the circle"**, **"doubling the cube"** and **"trisecting the angle".** To square the circle means to construct a square, the area of which is equal to the area of a given circle. To double a cube means to construct the side of a cube whose volume shall be double that of a given cube. To trisect an angle means to divide any angle into three equal parts. These constructions are to be performed only with an unmarked ruler and a compass. No other instruments are to be used.

The reason for this restriction sheds light on the classic attitude towards mathematics. A ruler and a compass are the physical counterparts suggesting the concepts of a straight line and a circle. This restriction, self-imposed and arbitrary, was motivated by the desire to keep geometry simple and harmonious. The three construction problems were very popular in Greece. The first historical reference to them states that the philosopher Anaxagoras passed his time in prison trying to square the circle. Despite the repeated efforts of the best Greek mathematicians the problems were not solved. Nor were they to be solved for the next two thousand years. It was finally proved that the constructions cannot be performed under the conditions specified.

Duplication of the Cube

One of the "three famous problems of antiquity" was to find a geometrical construction for the edge of a cube having twice the volume of a given cube. It probably dates back to the time of the Pythagoreans (c. 540 B. C.). The Pythagorean theorem suggests a simple means for finding a square with twice the area of a given square — it is the "square" on the diagonal. If the side of the square is of unit length, we can thus solve the problem of finding a line segment of lenght $\sqrt{2}$. The corresponding problem of finding a segment of lenght $\sqrt[3]{2}$ was stated in a much more interesting form by the Greeks.

The Greek commentator of the period tells us of a letter supposedly written to Ptolemy I (not to be confused with the mathematician of the same name) concerning King Minos, who had a cubical tomb constructed for his son. The king was displeased with the size of the monument, however, and so ordered it doubled in size — by doubling the side. The commentator points out that this was an error as the tomb would thereby be increased fourfold in area and eightfold in volume; but he says, the geometers then tried to solve the problem.

A second and better known story is also told of the source of the problem. It is said that the gods sent a plague to the people of Athens.

The people sent a delegation to the oracle at Delos to ask what could be done to appease the gods. They were told to double the size of the cubical altar to Apollo, and the plague would cease. They built a new altar, each edge of which was twice as long as each edge of the old altar. But since the gods' demand was not fulfilled, the plague continued. The story fails to relate what was finally done to appease the gods, but evidently the plague eventually left the city.

The search for solutions to this problem, to be carried out if possible with the restriction of using only straightedge and compass, was to lead the Greeks to many mathematical discoveries during the next several centuries. A compass-and-straightedge construction for this problem was not one of their discoveries, however, it can be proven that this cannot be done under these restrictions. **Menaechmus** (c. 350 B. C.) is given credit for discovering **the conic sections** in the process of trying to find a solution to this problem. He gave two solutions, one involving the intersection of two parabolas, and the other the intersection of a hyperbola and parabola. (It can easily be seen by analytic geometry, that when the equations $y=x^2$ and $xy=2$ are solved simultaneously, then $x=\sqrt[3]{2}$). It should be emphasized that these were perfectly legitimate solutions, but they did not satisfy the Greek criterion of restricting the tools used, to straightedge and compass. **Plato** (340 B. C.) discovered a mechanical solution, and during the third century B. C. **Nicomedes** used the curve called **the conchoid. Diocles** (c. 180 B. C.) used **the cissoid** to effect duplication.

Vietè in 1593 proved that every cubic equation not otherwise solvable leads to either a duplication or a trisection problem. It remained for **Descartes** in 1637 to prove the impossibility of a solution by means of lines and circles. He showed that a parabola and a circle can be used to find the roots of a cubic equation, if the second-degree term is missing. Since every cubic may be reduced to one with no second-degree term, every cubic may be solved by means of a circle and a parabola. But the parabola may not be constructed with straightedge and compass, hence, neither the duplication of the cube nor the trisection of the angle may be so performed.

But people still try and often claim success. They are either wrong or they misunderstand the problems. The problems are insoluble for the same sort of reason, viz., that the solution involves a kind of irrational number which cannot be constructed by Euclidean methods. A good approximation to the solution is not what is wanted.

While it is customary to emphasize the futile search of the Greeks for the solutions (perhaps because amateur mathematicians at all periods of time eagerly exercised their ingenuity on these problems) a more accurate appraisal must be made that even the early Greek geometers realized that the allowable means were inadequate. They set to work to find other means to solve these problems and here they did not fail. By making use of certain curves, not circles, supposedly already completely drawn, they were able to solve many of the construction problems.

The discovery of the conic sections and the use of such curves as the **conchoid** and the **quadratrix** to effect solutions is an obvious evidence of the ingenuity of the Greek geometers. The fact that they lacked the necessary mathematical tools of analytic geometry and algebraic theory to describe the possibilities of various geometrical instruments (and thereby also to show what is impossible) cannot be held against the Greeks. A valid and rigorous proof that "squaring the circle" problem cannot be solved by compass and straightedge alone was not given until 1882.

Nowadays it is well known that problems in construction can be solved by various uses of the basic geometrical tools, and in most cases in more than one way with each (by straightedge and compass, by compass only, by compass with the same opening throughout the construction, i. e., "fixed compass", and other limited means). A natural question is: "Which way is best?" Possible criterion was established in 1907: the simplicity of the construction is the sum of the numbers of the simple operations (steps) used in the construction.

The early Greeks had to give special attention to geometric construction because each served as a sort of **existence theorem** for the figure or concept involved. The establishing of the various equivalence theorems (e. g., that the compass alone is equivalent to straightedge and modern compass) **reverses** the approach — now a geometer is interested in showing that **theoretically**, at least, the results are attainable even without carrying out the actual construction, i. e., that the construction can be performed in principle.

Read and translate the text. Write out all "ing"-ending forms and arrange them into groups according to their part of speech.

TEXT TWO

UNSOLVED MATHEMATICAL PROBLEMS

by D. Hilbert

Extracts from the lecture delivered before the International Congress of Mathematicians in Paris, 1900.

Who of us cannot be glad to lift the veil behind which the future lies hidden; to cast a glance at the next advances of our science and at the secrets of its development during future centuries? What particular goals can there be which the leading mathematical minds of coming generations will strive? What new methods and new facts in the wide and rich field of mathematical thought can the new centuries disclose?

History teaches the continuity of the development of science. We know that every age has its own problems, which the following either solves or casts aside as worthless and replaces by new ones. If we could obtain an idea of the probable development of mathematical knowledge in the immediate future, we must let the unsettled questions pass in our minds and consider the problems which the science of today sets and whose solution we expect from the future. To such a review of present-day problems, raised at the meeting of the centuries, I wish to turn your attention. For the close of a great epoch of the 19th century not only invites us to look back into the past but also directs our thought to the unknown future.

The deep significance of certain problems for the advance of mathematical science, in general and the important role which they play in the work of the individual investigator are not to be denied. As long as a branch of science offers an abundance of problems, so long it is alive, a lack of problems foreshadows extinction or the cessation of independent development. Just as every human undertaking seeks after certain objects, so also mathematical research requires its problems. It is by the solution of problems that the researcher tests the temper of his steel; he finds new methods and new outlooks, and gains a wider and freer horizon.

It is difficult often impossible to judge the value of a problem correctly in advance; for the final award depends upon the gain which science obtains from the problem. Nevertheless, we can ask whether there are general criteria which mark and label a good mathematical problem. An old French mathematician said: "A mathematical theory is not to be considered completed until you made it so clear that you can explain it to the first man whom you meet in the street". This clearness and ease of understanding, here claimed for a mathematical theory, I should still more demand for a mathematical problem that it ought to be perfect; for what is clear and easily understandable attracts, while the complicated repels us. Moreover, a mathematical problem should be difficult in order to appeal to us, yet not completely inaccessible, lest it mock at our efforts. It should be to us a guide post on the mazy paths to hidden truths, and ultimately a reminder of our pleasure in the successful solution.

The mathematicians of past centuries were accustomed to devoting themselves to the solution of difficult particular problems with passionate zeal. They knew the value of difficult problems. I remind you only of the "problem of quickest descent" proposed by J. Bernoulli, of Fermat's assertion $x^n+y^n=z^n$ (x, y, z integers) which is unsolvable except in certain self-evident cases. The calculus of variations owes its origin to this problem of Bernoulli and to similar problems. The attempt to prove the impossibility of Fermat's theorem offers a striking example of the inspiring effect which such a very special and apparently unimportant problem may have upon science. I can remind you as well of the Problem of Three Bodies. The fruitful methods and the far-reaching principles which Poincare brought into celestial mechanics and which are today recognized and applied in practical astronomy are due to the fact that he sought to treat anew that difficult problem and to come nearer to its solution.

But it often happens also that the same special problem finds application in the most diverse and unrelated branches of mathematics. So for example, the problem of the **shortest line** plays a chief and historically important part in the foundations of Geometry, in the theory of curved lines and surfaces, in mechanics and in the calculus of variations. And F. Klein convincingly pictured, in his work on the icosahedron, the significance which is attached to the problem of the regular polyhedra in elementary Geometry, in group theory, in the theory of equations and in the theory of linear differential equations.

After referring to the general importance of problems in mathematics, let us return to the question from what sources this science derives its problems. Surely, the first and oldest problems in every field of mathematics spring from experience and are suggested by the world of external phenomena. Even the rules of calculation with natural numbers were discovered in this fashion in a lower stage of human civilization, just as the child of today learns the application of these laws by empirical methods. The same is true of the first unsolved problems of antiquity, such as the duplication of the cube, the squaring of the circle. Also the oldest problems in the theory of the solution of numerical equations, in the theory of curves and the differential and integral calculus, in the calculus of variations, the theory of Fourier series and the theory of potential to say nothing of the abundance of problems properly belonging to mechanics, astronomy and physics.

But, in the further development of the special domain of mathematics, the human mind, encouraged by the success of its solutions becomes convinced of its independence. It evolves from itself alone, often without appreciable influence from outside by means of logical combination, generalization, specialization, by separating and collecting ideas in elegant ways, by new and fruitful problems and the mind appears then as the real questioner and the source of the new problems. Thus arose the problem of prime numbers and the other unsolved problems of number theory, Galois' theory of equations, the theory of algebraic invariants, the theory of abelian and automorphic functions; indeed, almost all the nicer problems of modern arithmetic and function theory arose in this way.

In the meantime, while the creative power of pure reason is at work, the outer world again comes into play, forces upon us new questions from actual experience, opens up new divisions of mathematics and while we seek to conquer these new fields of knowledge for the realm of pure thought, we often find the answers to old unsolved problems and thus simultaneously advance most successfully the old theories, thanks to this ever-recurring interplay between pure thought and experience.

It remains to discuss briefly what general requirements may be proposed and laid down for the solution of a mathematical problem. I want first of all say this: that it shall be possible to establish the correctness of the solution by means of a finite number of steps based upon a finite number of hypotheses which are implied in the statement of the problem and which must always be exactly formulated. **This demand for logical deduction by means of a finite number of processes is simply the requirement of rigour in reasoning.** Indeed, this requirement of rigour, which became proverbial in mathematics, corresponds to a universal philosophical necessity of our understanding; and on the other hand, only by satisfying this claim do the problems attain their full effect.

Besides it is an error to believe that rigour in the proof is the enemy of simplicity. On the contrary, we find it proved by numerous examples that the rigorous method is at the same time the simpler and worthy in the long run and easier to understand. The very effort for rigour helps us come across a simpler method of proof. It also frequently leads the way to methods which are more capable of development than the old methods of less rigour. Thus, the theory of algebraic curves experienced a considerable simplification and attained greater unity by means of a more rigorous function-theoretical methods and the introduction of transcendental curves.

To the new concepts correspond, necessarily, new signs. These we choose in such a way that they remind us of the phenomena of the external world. Likewise the geometric figures are signs or symbols of space intrusion and are used as such by all mathematicians. Who does not always use along with the double inequality $a>b>c$ the picture or drawing of three points following one another on a straight-line as the geometrical idea of "betweenness"? Who does not make use of drawings of segments and rectangles closed in one another, when it is required to prove with perfect rigour a difficult theorem on the continuity of functions or the existence of points of condensation? Who can do without the figure of the triangle, the circle with its centre, or with the cross of three perpendicular axes? The arithmetical symbols are written diagrams and the geometric figures are graphic formulas and no mathematician can do without them or avoid them.

Some remarks upon the difficulties which mathematical problems may offer and the means of overcoming and coping with them may be worth discussing. If we do not manage and are not able to solve a mathematical problem the reason often consists in our failure to recognize the more general standpoint from which the problem under study appears only as a single link in a chain of related problems. After finding this standpoint, the problem becomes more accessible to our investigations and we possess then a method which is applicable also to related problems. This way for finding general methods is certainly the most fruitful and the most certain; for who seeks for methods without having a definite problem in mind seeks for the most part in vain.

In dealing with mathematical problems, specialization plays, to my mind, a still more important part than generalization. Perhaps in most cases where we seek in vain the answer to a question, the cause of the failure lies in the fact that problems simpler and easier than the one at issue were either not at all or incompletely solved. All depends, then, on finding out these easier problems, and on solving them by means of devices as perfect as possible and of concepts capable of generalization. This rule is one of the most important levers for overcoming mathematical difficulties and I think, that it is used wherever it is possible, though sometimes unconsciously.

Occasionally it happens that we seek the solution under insufficient hypotheses or in an incorrect sense and for that reason do not surmount the difficulty. The problem then arises: to show the impossibility of the solution under the conditions specified. Such proofs of impossibility were effected by the ancients, for instance, when they showed that the ratio of the hypotenuse to the side of an isosceles right triangle is irrational. In later mathematics, the question of the impossibility of certain solutions plays a great part and we realize in this way that old and difficult problems, such as the proof of the axiom of parallels, the squaring the circle, of the solution of equations of the fifth degree by radicals found fully satisfactory and rigorous solutions, although in a different sense than that originally intended. It is probably this important fact along with other philosophical reasons that gives rise to the conviction (which every mathematician shares but which as yet no one supported by a proof or refuted) that **every definite mathematical problem must necessarily be settled, either in the form of a direct answer to the question posed, or by the proof of the impossibility of its solution** and hence the necessary failure of all attempts.

Is this axiom of the solvability of every problem a peculiar characteristic of mathematical thought alone, or is it possibly a general law inherent in the nature of the mind, that all questions which it asks must be answerable? For in other sciences there exist also old problems which were handled in a manner most satisfactory and most useful to science by the proof of their impossibility. For example, the problem of perpetual motion. The efforts to construct a perpetual motion machine were not futile as the investigations led to the discovery of the law of the conservation of energy, which, in turn, explained the impossibility of the perpetual motion in the sense originally presupposed.

This conviction of the solvability of every mathematical problem is a powerful stimulus and impetus to the researcher. We hear within us the perpetual call: There is the problem. Seek its solution. You can find it for in mathematics there is no futile search even if the problem defies solution. The number of problems in mathematics is inexhaustible and as soon as one problem is solved others come forth in its place. Permit me in the following to dwell on particular and definite problems, drawn from various departments of mathematics, whose discussion and possible solution may result in the advancement and progress of science.

1. Cantor's Problem of the Cardinal Number of the Continuum.
2. The compatibility of the Arithmetical axioms.
3. The equality of the volumes of two tetrahedra of equal bases and equal altitudes.
4. Problems of the straight line as the shortest distance between two points.
5. Lie's concept of a continuous group of transformations without the assumption of the differentiability of the functions defining the group.
6. Mathematical treatment of the axioms of physics.
7. Irrationality and transcendence of certain numbers.
8. Problems of prime numbers.
9. Proof of the most general law of reciprocity in any number field.
10. Determination of the solvability of a Diophantine equation.
11. Quadratic forms with any algebraic numerical coefficients.
12. Extension of Kronecker's theorem on Abelian fields to any algebraic realm of rationality.
13. Impossibility of the solution of the general equation of the 7th degree by means of functions of only two arguments.
14. Proof of the finiteness of certain complete systems of functions.
15. Rigorous foundations of Schubert's enumerative calculus.
16. Problem of the topology of algebraic curves and surfaces.
17. Representation of definite forms by squares.
18. Building up of space from congruent polyhedra.
19. Are the solutions of regular problems in the calculus of variations always necessarily analytic?
20. The general problem of boundary values.
21. Proof of the existence of linear differential equations having a prescribed monodromic group.
22. Uniformization of analytic relations by means of automorphic functions.
23. Further development of the methods of the calculus of variations.

The problems mentioned are merely samples of problems yet they will suffice to show how rich, how manifold and extensive the mathematical science of today is, and the question is raised whether mathematics can like other sciences split into separate branches, whose representatives can hardly understand one another and whose connection becomes ever more loose. I do not believe this nor wish it. Mathematical science is in my opinion, an indivisible whole, an organism whose vitality is conditioned upon the ties of its parts. For with all the variety of mathematical knowledge, we are still convinced of the similarity of the logical devices, the relationship of the ideas in mathematics as a whole and, the numerous analogies in its different departments.

But, we ask, with the extension of mathematical knowledge cannot it finally become impossible for a single person to embrace all the areas of this knowledge? In answer let me point out, that it is quite possible for the individual investigator to master and make all new sharper tools and methods his own and find his way more easily in the various parts of modern mathematics that it is possible in any other science. The organic unity of mathematics is inherent in the nature of this science, for mathematics is the foundation of all exact knowledge of natural phenomena. That it may completely fulfil this high mission, may (let) the new century bring it gifted master and many enthusiastic disciples.

ACTIVE VOCABULARY

1. to achieve
2. to appeal (to)
3. to approach
4. to approximate
5. to attain
6. to attract
7. to attribute
8. to award
9. to cease
10. to challenge
11. to conclude
12. to converge
13. to convince
14. to defy
15. to dwell (on)
16. to estimate
17. to exist
18. to exhaust
19. to extend
20. to fail
21. to generate
22. to guide
23. to identify
24. to impose
25. to intersect
26. to intrude
27. to investigate
28. to issue
29. to pursue
30. to recur
31. to reduce
32. to refer (to)
33. to refute
34. to remind
35. to restrict
36. to reveal
37. to reverse
38. to satisfy
39. to specify
40. to succeed
41. to suffice
42. to surmount

VOCABULARY EXERCISES

I. *Translate the sentences and compound words. Specify the "ing"-ending forms bold-faced. Consult the dictionary if necessary.*

drawing — черчение, вычерчивание; рисование; чертеж; рисунок, набросок, изображение; рисующий, чертежный; вычерчивающий

Model. The ruler is the simplest instrument for **drawing** (Ger., Attr.) Линейка — простейший инструмент для черчения. (Какой инструмент?

a) 1. The problem is **drawing** geometric figures with a high degree of accuracy. 2. Ruler-compass constructions are simply the **drawings** made by using only a straightedge and a compass. 3. We always refer to a **drawing** as a geometric object. 4. **Drawing** a picture of two intersecting lines helps the student discuss the idea of the interior and exterior of the angles obtained. 5. **Drawing** is his favourite subject. 6. Let us represent geometric figures by **drawings**.

b) drawing board, drawing compass, drawing knife, drawing master, drawing paper, drawing pen, drawing press.

failing — ошибка; слабость; недостаток; неудача, неуспех, провал; недостающий, слабеющий; за неимением, ввиду отсутствия, из-за отсутствия

Model. **Failing** the solution, the Greeks sought to find other means to effect the solution. (Prep., a structural word)
Так как решение не находилось (ввиду отсутствия решения), греки стремились найти другие средства для его достижения.

1. Persistence of the Greeks in their efforts to solve the three famous construction problems can't be considered as **failing**. 2. **Failing** to obtain the solution sought the Greeks did not stop raising some other important problems, e. g., of constructing with a ruler and compass a regular polygon of n sides. 3. **Failing** to construct the regular heptagon ($n=7$) can be explained by the same impossibility of the solution. 4. The **failing** tools of analytic geometry and modern algebra were some of the reasons of the Greeks' inability to solve the problems. 5. **Failing** the understanding of the theoretical character of the problems under study, mo-

dern angle-trisectors, cube-duplicators and circle-squarers issue faulty solutions.

II. *Consult the dictionary, if necessary, and give the Russian equivalent of the following.*

Nouns: meaning, reasoning, reading, writing, drawing, thinking.
Adjectives: striking, surprising, astonishing, tiring, exhausting, annoying, missing, lacking, exciting, startling, intriguing, tempting, misleading, convincing, encouraging, disappointing, running, appealing, inspiring, boring.
Adverbs: according, notwithstanding, running.
Prepositions: according to, concerning, regarding, respecting, relating to, pertaining, considering, touching, excepting, saving, pending, during, failing, following, owing to, depending on.
Conjunctions: providing, granting, supposing.

III. *Observe the multifunctional use of "ing"-ending forms and their possible translations.*

notwithstanding
- prep. in spite of *несмотря на*
- adv. nevertheless *тем не менее*
- conj. in spite of the fact *несмотря на то, что*

Adv. mod. **in solving problems** — *при решении задач; решая задачи; когда решаем задачи; если решать задачи.*

IV. *Don't mix these words up! Illustrate their meaning by the examples of your own.*

plain	value	extend	vain	universe	cause	overestimated
plane	volume	extent	vague	space	course	overshadowed

lateral	equation	expansion	completed	unworthy	unit
literal	equality	extension	complicated	worthless	unity

carry out	ellipse	ultimate	result from	describe
carry throuqh	eclipse	eventual	result in	ascribe

V. *Make up sentences following the models.*

Models. 1.
We **credit** Menachmus with discovering conic sections.
Menachmus **is credited** with discovering conic sections.
Menachmus **is given credit** for (with) discovering conic sections.

2.
We **attribute (ascribe)** discovering conic sections **to** Menachmus.
Discovering conic sections **is attributed (ascribed)** to Menachmus.

3.
Either Pythagoras **or** some of his pupils proved the famous theorem.
Neither the duplication of the cube **nor** the trisection of the angle can be performed under the conditions specified.

4.
It goes without saying that irrational number cannot be constructed by Euclidean methods.
It goes without saying that proof must be valid, rigorous and elegant.

VI. *Use the proper verb or one of its derivatives in brackets. Try to explain in your own words the difference of their meaning.*

a) **to define, to determine, to specify, to identify**

1. You must (определить) this term in a more precise and formal way. Your (определение) is too broad and unscientific. 2. Can you (определить) this geometric figure? Yes, I can, sure enough. It is a triangle. 3. She should (уточнить) the conditions for the construction. 4. There exists a specific formula to (определить) the volume of a sphere. 5. (Если противное не оговорено) the concept "congruence" is undefined in today's mathematics. 6. We may (определить) this ratio as constant as it is true and holds in all cases. 7. Their (решение) to solve this famous problem is definite and should be encouraged.

b) **to divide, to separate, to cut, to share, to split**

1. When we bisect a line segment, we (делим) it at the mid point. 2. Nine (деленное) by three equals three. 3. I (разделяю) your views on this point to some extent. 4. (Разделите) these numbers (раздельно) and then add together the quotients. 5. Let us (разделим) the whole work so that everyone should have an equal share of it. 6. How can we (разделить) all geometric figures? 7. Great distance (разделяет) us. 8. Does mathematics (разделена) into (отдельные) fields with nothing in common?

c) **to estimate, to evaluate, to appreciate, to appraise, to value, to assess**

1. Scientists can (оценить, вычислить) the size and the altitude of that distant star only approximately. 2. (оценить) the full significance of this work is difficult so far. 3. He can (оценить) this picture. He is a painter, after all. 4. I (ценю) this book very much. It is a present of my best friend. 5. Is it difficult to (оценить) the ingenuity of the Greek geometers? 6. You (оцениваете, рассчитываете) that the work may take three months. 7. The time and origin of that problem is hard to (оценить).

d) **to perform, to carry out, to fulfil, to execute**

1. First (выполните) the operation of division and then multiply the quotients. 2. You ought to (выполнить) the work and (выполнить) your promise. It is your duty. 3. (Выполнить) the construction of that famous unsolved problem proved hopeless of (выполнения). 4. He is given credit for the (выполнение) of the task so quickly and accurately. 5. The Greeks failed to (выполнить) the construction under the conditions specified.

e) **to offer, to suggest, to propose**

1. She often (предлагает) to help me, but I prefer to do my work without anybody's help. 2. Listen! He can (предложить) a good idea. It is worth discussing, to my mind. 3. The Greeks (предложили) a lot of theoretical questions to be solved by later generations of mathematicians. 4. The Soviet Union (предлагает) a new way of settling this international problem.

VII. *Give one Russian equivalent of the following groups of words.*

a) to draw — to picture — to portray — to depict — to paint / to get to — to reach — to achieve — to gain — to attain — to accomplish /to finish — to end — to stop — to cease — to terminate / to fix — to sign — to assign — to designate — to denote / to bound — to limit — to restrict — to confine — to restrain / to discuss — to argue — to debate — to dispute / to mislead — to deceive / to be sure — to be certain — to be convinced / to fit — to suit — to match / to do problems — to solve... — to resolve... — to settle... — to handle... / to select — to choose — to single out / to raise a question — to pose... — to state... — to formulate / to carry through — to realize — to put into practice.

b) bound — limit — boundary — border — frontier — verge — margin / top — peak — apex — vertex — summit / height — altitude — elevation — pitch — tallness — highness / name — title — label / source — origin — spring / absence — privation — lack — want — shortcoming / degree — power — extent / estimation — evalution — appreciation — appraisal — assessment / point of view — viewpoint — standpoint / method — means — device — technique — procedure.

c) futile — vain — fruitless / difficult — hard — complicated — involved / strange — peculiar — odd / valid — legal — lawful — legitimate — legislative / endless — infinite — nonterminating / countable — numerable / final — ultimate — terminal / ardent — passionate / different — various / flawless — perfect / rough — approximate.

d) in the end — at last — finally — eventually / at the same time — simultaneously / really — in fact — indeed — actually.

e) yet — however — nevertheless — nonetheless / likewise — in the similar way / also — moreover.

LAB. PRACTICE

Grammar Rules Patterns

Modal Verbs and their Equivalents

I. *Reword sentences according to the models.*

Model. **can → to be able to**

1. **I want to make a drawing of this figure.**
2. I **can make** ... I **am able to make**...
3. A drawing of this figure **can be made.**

1. He wants to define all possible types of triangles. 2. Mathematicians want to apply their theories in practice. 3. She wants to evaluate the area of the floor. 4. Scientists want to solve this famous problem. 5. We want to perform the construction with compass alone.

Model. **may → to be allowed to**

1. **Possibly (perhaps) geometers use other tools for the construction.**
2. Geometers **may use** ... Geometers **are allowed to use**...
3. Other tools **may be used** for the construction.

1. Possibly, they find a good approximation for the solution. 2. Perhaps, scientists handle the problem completely. 3. Possibly, they misunderstand the theoretical character of the issue. 4. Perhaps, he specifies the restriction on the instruments. 5. Possibly, mathematicians give perfectly legitimate solutions.

Model. **must → need, shall, should, to be to, to have to, ought to, to have got to, to be obliged to**

1. It is necessary (important) for you to make a drawing with high degree of accuracy.
2. You **must** (shall, should,...) make a drawing with high.

1. It is not necessary for him to pay so much attention to constructions. 2. It is important for us to seek the solution to this problem. 3. It is necessary for them to fulfil the demand. 4. It is important for you to draw three arbitrary circles in the plane. 5. It is not necessary for her to construct a regular polygon.

II. *Change the following sentences, using the equivalents of the modal verbs.*

Model. We **can** give a short "yes — no" answer only. (to be able)
We **are able to** give a short "yes — no" answer only.

1. The Greeks agreed to use only a straightedge and a compass in the construction. (to be to) 2. She works too much at her problems she must have a rest. (should) 3. They may take a "fixed" compass to perform the construction. (to be allowed to) 4. I failed to find the solution, I think I must try again. (ought to) 5. The existence or nonexistence of the proof must be developed. (have to)

III. *Answer the questions using the words suggested.*

Model. **Do I have to** make another drawing? (No, this one will do.)
No, you **needn't**. This one will do.

1. Must she make her own choice? (No, discuss other choices.) 2. Do we have to define conic sections? (No, familiar to everyone.) 3. Must you measure the perimeter? (No, not necessary.) 4. Does he have to refer to this issue again? (No, it is worthless.) 5. Do they have to reverse their approach to the problem? (No, try the old method.)

IV. *Choose the proper equivalent of the model verbs.*

Model. You (надо) construct a square with the area equal to the area of the given circle.
You **have got to** construct a square...

1. She (придется) to give a reason and possible justification for the restriction. 2. We (предстоит) to find a good approximation to the number π value. 3. He (следует) specify the conditions of the experiment. 4. They (разрешают) to use a dictionary if necessary. 5. I (в состоянии) to solve this difficult problem myself. 6. You (долг) to exercise all your ingenuity and fulfil the task. 7. They (нужно) to check all the calculations again. 8. We (следует) to satisfy the requirements for the solution. 9. She (не надо) to refer to her failure with the task now. 10. The students (долг) to appreciate the ancient mathematics in a proper way.

Indefinite Pronouns, Adverbs and their Derivatives

I. *Substitute* **no**-*forms for* **some**-*forms using the given suggestions.*

Model. They know **something** about that famous problem (I ...)
I know **nothing** about that famous problem.

1. **Somebody** can perform the drawing in a better way. (the work) 2. The teacher asks **someone** to find the area directly. (the volume) 3. Refer to this issue **somewhere** in your report. (if not asked) 4. There is **something** unfamiliar in his description. (of a procedure)

II. *Ask questions substituting* **any**-*forms for* **no**-*forms.*

Model. There are **no** equations in this text. (really?)
Are there really **any** equations in this text?

1. There is **nobody** to help you find the proof of the theorem (right now?) 2) **None** of us underestimate his contribution to science. (of the Greeks) 3. There is **nothing** in the text to justify the choice. (the failure?) 4. We can seek the solution **nowhere.** It is impossible. (the proof?)

III. *Reword the following sentences using the word "else" and the derivatives of some, any, no, body, one.*

Model. Make a drawing of a quadrilateral on **some** other piece of paper. (somewhere)
Make a drawing of a quadrilateral **somewhere else.**

1. Give that task to some other person. (some one). 2. They want to measure some other surface. (something) 3. There is no other figure to define. (nothing) 4. They needn't perform any other construction. (anything) 5. Somebody can draw figures with such high degree of accuracy. (no one)

CONVERSATIONAL EXERCISES

I. *Make the following sentences more emphatic by using the interrogative pronouns.*

Models. 1. We have deductive reasoning, we have mathematics. (Wherever).
Wherever we have deductive reasoning, we have mathematics.
2. He suggests this idea, he is wrong. (Whoever).
Whoever suggests this idea, he is wrong.

1. The famous unsolved problems of Antiquity we seek to solve, we fail to obtain the solution. (Whichever) 2. The measures of areas and volumes you take, they are indirect measurements. (Whatever) 3. He seeks the solution otherwise, his trials may not be futile. (Whoever) 4. The efforts they apply to handle the problem, they are in vain. (Whatever) 5. When he bisects an angle, the angles obtained are congruent. (Whenever) 6. The higher curves the ancient geometers discovered to effect the solution, the solutions were not valid according to the criterion. (Whichever) 7. He discovered conic sections, he is credited with great contribution to mathematics. (Whoever) 8. We refer to the three famous unsolved problems, we emphasize the ingenuity of the Green geometers. (Wherever) 9. When we employ the fomulae for areas and volumes, we give credit to the Greek mathematicians. (Whenever) 10. Modern circle-squares, angle-trisectors, cube-duplicators issue their solutions, the solutions involve some fallacies. (Whenever)

II. *Convince or refute.*

I am convinced that...
This is a convincing argument.
The statement is convincing by itself.
The assertion convincingly represents...
Athough no conviction serves as a proof...

I can't but refute it.
The statement must be refuted.
I have a counter-argument that may serve as a refutation.
One can refute it by...
Although no refutation is offered...

1. Construction problems are favourite now only with puzzle enthusiasts, not mathematicians. 2. Due to their seeming simplicity and recreational nature, the construction problems do not appeal to eminent

mathematicians. 3. The theoretical proof is unnecessary, only precise drawing is required for their solution. 4. A more severe restriction of the tools (e.g., with a compass alone) limit the number of possible constructions. 5. It is always possible to improve on earlier constructions by performing them in fewer steps. 6. The solution for the construction problems can be found by pure reason. 7. The ancient Greeks proved the impossibility of the solutions under the conditions specified. 8. The solution should be obtained by means of a finite number of steps. 9. The answers to famous unsolved problems are still not found. 10. There are no unsolved problems in modern mathematics. 11. People should not confuse the Greek letter π [pai] with the mathematical symbol π (pi:). 12. The history of the number π is associated with the oldest of unsolved construction problems "Quadrature of the Circle".

III. *Suppose that the statement seems insufficient to you and you want to add. Repeat the statement and add your own reasoning. Use the opening phrases. Summarize the whole topic.*

I may as well add that... **More than that...** **Moreover...**

M o d e l. To trisect any angle means to divide any angle into three equal parts.
That's right... But I may as well add that many special angles, e. g., an angle of 90° can of course be easily trisected.

1. Using only straightedge and compass the Greeks could easily divide any line segment into any number of equal parts. 2. The ease with which any angle can be bisected was the motivation to attempt the multisection of any angle under similar restrictions. 3. The problem of constructing a regular polygon of nine sides which requires the trisection of a 60° angle was the second source of the famous problem. 4. The Greeks could not solve the problem not because they were not clever enough, but because the problem is insoluble under the specified conditions. 5. The Greeks added "the trisection problem" to their three famous unsolved problems. 6. It is customary to emphasize the futile search of the Greeks for the solution. 7. However, a very important fact should not be overlooked. 8. The ancient Greek geometers realized that the allowable instruments were inadequate for trisecting any arbitrary angle. 9. They tried to find other means to solve the trisection problem. 10. They devised various methods by means of special geometric curves. 11. Nicomedes invented a special curve, **the conchoid**, with which he could trisect any angle. 12. Papus trisected an angle with the aid of a **hyperbola**. 13. Hippias used **quadratrix** to divide an angle in any given ratio. 14. Other Greek mathematicians used various mechanical solutions. 15. All these constructions produced good approximations to the trisection of an angle. 16. The solutions were not theoretically exact and didn't satisfy the criterion. 17. The search for the solution under the imposed restrictions led the Greeks to many great mathematical discoveries. 18. Mathematical discoveries and the use of higher geometric curves to effect the solution of the problem show the great ingenuity of the Greek geometers. 19. To give a valid proof they lacked the necessary tools of analytic geometry and algebraic theory. 20. Over a period of two thousand years mathematicians were sure that it is impossible to perform the construction under the conditions stated. 21. The first rigorous proof of the impossibility of the trisection of any given angle by compass and straightedge was given by **P. Wantzel** in 1837. 22. The proof is algebraic in nature and involves such concepts as domain of rationality, algebraic numbers and group theory.

23. The solution of the problem can be found by means of higher algebraic and transcendental curves. Otherwise no solution is possible. 24. This century geometers amuse themselves by imposing even more severe restrictions on instruments used in construction problems. 25. Nowadays it is proved that trisection problem can be solved by various uses of even more limited means, e. g., by compass alone. 26. In most cases the construction can be performed in more than one way. 27. The criterion is the simplicity and a small number of operations used in the construction.

IV. *Dispute the following statements. The hints may prove helpful.*

1. **Not all mathematical problems can be solved right away.**

(There are some problems that are impossible to solve (insoluble) because... Such problems challenge mathematicians of all periods as... The conviction of the solvability of every problem is a powerful stimulus to the researcher because...)

2. **Higher algebraic curves were invented by the Greeks to effect the solution of the famous construction problems.**

(The wonderful curves invented are... They have valuable properties such as... and they are still applied in...)

3. **The search for the solutions led the Greeks to the novel developments in maths.**

(The Greeks sought to devise a theory which... Menachmus's discovery of conic sections provided the foundations for...)

4. **Maths offers an abundance of unsolved problems.**

(A number of unsolved problems can be settled if one invents the right and often sophisticated technique or...)

V. *Discuss the main ideas of the following text.*

To solve a mathematical problem originally meant to find its complete numerical solution. Gradually it became clear that such explicit solutions are possible only in exceptional cases, that in general one must be satisfied with a scheme by which the solution may be determined approximately, though with any desired accuracy. Something quite different is very frequently offered as the solution of a mathematical problem, namely a representation of the solution in terms of the data of the problem; although it is in principle possible to devise a scheme for numerical calculation from such a representation, the question remains: **What actually is the solution?** Mathematicians, in their search for representations of solutions, often modified the meaning of "solution" even further: **to solve a problem is simply to prove the unique existence of a solution.**

Clearly, if a mathematical problem is the correct expression of a physical one, it has a unique solution, for the physical situation to be determined from given data does actually occur. Thus to know that certain mathematical problems have unique solutions may have no significance in mathematics. The statement that a unique solution exists may then serve as a partial verification of the correctness of the mathematical expression of the problem. If the solution is not unique the data given are not sufficient; if the solution does not exist, the data are incompatible.

A mathematical problem which possesses a unique solution is referred to as correctly posed or formulated. For a large class of mathematical problems, the way in which they are posed is never questioned, just because of their physical significance. These problems are mostly of a standard, rather regular, type. Doubts arise, however, when, for simplicity, the actual physical problem is replaced by an idealized problem. Such idealized problems may be considered as limiting cases of actual problems, arising when, for example, the domain is extended to infinity, forces are concentrated on surfaces, lines or points, or terms in the equations are simply omitted as insignificantly small. To the understanding of such idealized problems, purely mathematical existence and uniqueness considerations may still make valuable contributions.

As it is often emphasized, not only existence and uniqueness, but also a third abstract property of the solution should be required of the problem if it is to be called correctly posed: the property of continuous dependence on the data. Since physical data are not given with absolute precision, the mathematical problem is certainly not the appropriate expression of an actual physical situation if an arbitrarily small variation of the data may have a finite effect on the solution, or even destroy its existence or uniqueness. If the solution does not depend continuously on the data, it may be called unstable. It should, however, be noted that in the customary sense the term instability refers to problems in which the continuous dependence on the data breaks down only for exceptional values of the data. There are important problems, problems in transonic flow, for example, which possess solutions only for exceptional values of the data; thus the solutions do not depend continuously on the data even when they exist.

COMPOSITION

Abstract (Precis) Writing Practice

1. *Omit the unnecessary and extra information, compress and transform the following abstract into a) 8-sentences; b) 3-sentences long abstracts.*

Compass and Straightedge Constructions

A "construction" is drawing geometric figures with a high degree of accuracy. The construction performed constitutes both a **proof** of the existence of a geometric object and the **solution** of the problem. The ancient Greeks were convinced that all plane figures can be constructed with a compass and straightedge alone. Their methods of bisecting a line segment and an angle are ingenious and hard to improve on. They worked with all numbers geometrically. A length was chosen to represent the number 1 and all other numbers were expressed in terms of this length. They solved equations with unknowns by series of geometric constructions. The answers were line segments whose lengths were the unknown value sought. The Greeks imposed the restrictions of straightedge and compass for the construction of the problems. It is supposed that this tradition was started by Plato, Greece's greatest philosopher. He claimed that more complicated instruments called for manual skill unworthy of a thinker. The Greeks failed to obtain the solution of the famous problems. under the restrictions specified not due to the lack of ingenuity of the geometers. (The famous problems are insoluble because they involve irrational numbers that cannot be constructed by Euclidean methods.) The Greeks' persistent efforts to find compass-and-straightedge ways of trisecting an angle, squaring the circle and duplicating the cube were not futile for almost 2000 years. The Greeks made great mathematical discoveries on the way. The desire to gain full understanding of the **theoretical character** of the problems inspired many great mathematicians — among them Descartes, Gauss, Poncelet, Lindemann — to mention but a few. The long years of labour on these "impractical", "worthless" problems indicate the care, patience, persistence and rigour of mathematicians in their attempts to perform the constructions and justify them theoretically. The problems did not exhaust themselves. Even nowadays some authors of the scientific papers issued "solutions" containing some fallacies. The search for the rigorous solution resulted in great discoveries and novel developments in mathematics. It introduced new geometric concepts (e. g., conic sections), raised a number of important theoretical questions (e. g., to prove the impossibility of the solution) and suggested an entirely new direction for scientific research (e. g., the extension and further generalization of number concept).

II. *Write a generalizing sentence to characterize the stylistic peculiarities of Text Two or choose one among the given.*

D. Hilbert's report is:

1) dull, uninteresting prose; 2) dry, formal, boring, academic; 3) clear and vivid, notwithstanding scientific; 4) popular, pulsating with life and emotions; 5) quite within the grasp of even a layman; 6) pathetic, uncommon for the mathematician; 7) too involved, only specialists can appreciate it.

III. *Study the text carefully and answer (in writing) the following questions.*

1. Why does D. Hilbert think that the International Congress of Mathematicians is the appropriate and right place for a) the review and appraisal of the development and achievements of mathematics, and b) for formulating and posing the "open", "unsolved" or "challenging" questions and problems? 2. For how long can a particular field of mathematics be considered "alive"? 3. Specify in which paragraph D. Hilbert speaks about a) a good mathematical problem, b) the requirements for the solution of the problem, c) the sources mathematics derives its problems from, d) the unity of all fields of mathematics and the reasons justifying this unity, e) the interrelations of mathematics and the external world, f) the problems that are the product of the human mind alone.

Paragraph Writing

Study the problems themselves thoroughly and write a paragraph (5—7 sentences) while answering the questions.

1. What do the following groups of Hilbert's problems deal with? [1—6], [7—15], [16—23].

2. Were the problems formulated and stated in a good way? Were all the problems formulated by D. Hilbert alone? Were the problems really the most vital and significant for that time? Are the problems in question easy to solve?

3. Did D. Hilbert reveal great insight in selecting the problems? Does the XX century mathematics justify his choice?

4. Is there any problem in the list having anything to do with the unsolved problems of antiquity, with algebra, with number theory? Which of them consists, in fact, of several problems?

5. What directions and tendencies in the XIX c. mathematics gave implications and led D. Hilbert to raising those particular problems? How is the then level of mathematics reflected in the problems?

6. Do modern mathematicians give a short "yes-no" answer to the (3—7) problems?

7. Which problems are solved? For which the impossibility of the solution is proved?

8. How and to what extent did the problems listed make for the solution of more general and complicated problems of today's mathematics? Could they disclose the future development of mathematics?

9. How do XX c. mathematicians estimate the solution of any problem in D. Hilbert's list?

10. Does the a priori and exciting conviction of solvability or impossibility of the solution add anything appealing and reassuring to the mathematician?

11. What is the contribution of the Russian-Soviet mathematicans to the solution of the problems under study?

12. Are the unsolved problems the very essence of mathematics?

Composition Writing

Write a composition: "Unsolved mathematical problems of D. Hilbert". Try to give convincing arguments of the significance of both the Second Congress of Mathematicians 1900 and the report of D. Hilvert for the further development of mathematics. Your answers to the above given questions and your paragraphs should be included. Your composition must be 2 pages long. Consult the following books, if necessary.

Проблемы Гильберта. М., 1969; Демидов С. С. К истории проблем Гильберта. — Историко-математические исследования. Вып. XVII. М., 1966.

COMPREHENSION EXERCISES

Questions

1. What curves can be drawn in a single stroke, without retracing any line or lifting the pencil from the paper? 2. What is the triangle of the shortest perimeter that can be inscribed in a given triangle? 3. What is the smallest circle that encloses a finite set of points? 4. Under what conditions are two objects equal (or congruent) in size and shape? 5. If figures are not equal, what significant relationship may they posses to each other and what geometric properties can they have in common?

Discussion

1. In mathematics **the conviction** that a definite mathematical problem can necessarily be solved must be supported by a **proof** either in the form of a direct answer to the question posed or by the proof of the impossibility of the solution. What about other sciences?

2. Among professional mathematicians **asking questions** rates almost as high as answering them. Why?

3. There are **two kinds of mathematical problems**: one is so easy that it is not worth doing and the other so difficult that it can't be done. Give some examples.

4. It is one thing to say that a problem is not solved yet and another thing to say that it is impossible to solve it. How is it possible to prove a thing impossible?

5. What is more difficult to prove: **the possibility** (the existence) of a solution of some problem or the **impossibility** (the nonexistence) of the solution sought?

6. How is it possible **to prove that certain problem** cannot be solved?

7. What other branches of mathematics, besides geometry, have unsolved problems with seemingly simple nature failing the solution since Antiquity?

8. How can we estimate the new and novel developments in mathematics raised by the Greeks' famous unsolved problems?

9. What do the three famous problems have in common?

10. Your appreciation of "Squaring the circle", "Doubling and cube", "Trisecting the angle" problems.

11. Unsolved problems formulated by D. Hilbert in 1900. Which of them are solved? Choose one of them and explain why it is so difficult to solve it.

12. Unsolved problems of modern mathematics.

LESSON FOUR

INTRODUCTION TO GEOMETRY

Grammar:

1. Substitutes of the Noun: *it, one, that of, those of, the former, the latter.*
2. Emphatic Constructions.
3. Impersonal Sentences.

LAB. PRACTICE

Repeat the sentences after the instructors.

1. We honour ancient Greece as the cradle of modern science; it was in ancient Greece that the first mathematical, astronomical and physical **theories** originated and developed. 2. The Greeks' contributions to philosophy, art, literature and architecture are as significant today as they were in Antiquity. 3. Nevertheless, the contribution of the Greeks that determines most the character of the present-day civilization was their **mathematics**. 4. The Greeks must be credited with the founding of mathematics as a scientific discipline; even among the Greeks themselves mathematics was set up as the standard for all the sciences. 5. The Greeks were the first people to pursue mathematics as an art for its own sake. **Pure mathematics** emerged when the Greeks began to think of numbers as numbers and of shapes as shapes. 6. The Greeks were the first to formulate the two mental processes vital to all mathematical progress: **abstraction** and **proof**. 7. **Abstraction** is the art of perceiving common qualities in different things and forming a general idea therefrom. 8. **Proof** is the art of arguing from premises to a conclusion in such a way that no flaws can be picked in any step of the argument. 9. Using the information of the premises the Greeks proved by a reasoning process known as **deduction** the inescapable conclusion. 10. There are two main forms of thinking — **deduction** and **induction**. For the former we are chiefly indebted to the Greeks. They first saw clearly revealed the great power of anouncing general axioms or assumptions and deducing from these a useful array of implied propositions. 11. Inductive thinking proceeds in the opposite direction from deduction. Starting from the facts of experience, it leads us to infer general conclusions. Inductive reasoning produces in most cases an uncertain inference. 12. **Deductive reasoning is flawless, definite and absolute.** Its specific inferences follow inescapably from the general assumptions. 13. The Greeks converted mathematics from empirical science into a deductive system of thought. Greece is the mother of logic. A logical deductive system must start somewhere and according to the Greeks' criterion it must start with a **list of definitions, axioms** and **postulates**. 14. It is always better in pure science — the

Greeks claimed — to assume as little as possible at the start and from a few assumptions to deduce as much as one can. 15. The Greeks created the theory of the logical discourse and they embodied it in the first model of material axiomatic system — **Euclidean Geometry**. 16. Euclid was genius for system; his work "Elements" is a monument of the classical age mathematics. There were many "Elements" before Euclid; there was none after him. 17. Right up to and including the present time, Euclid's masterpiece serves as the highest standard of logic, rigour and perfect reasoning for all scientific treatments. 18. The Greeks had only one space and only one geometry; these were absolute concepts. 19. For more than twenty centuries mathematicans did not doubt the absolute truth of Euclidean geometry. Euclidean geometry was all of Geometry; it is no more. 20. The challenging idea of a non-Euclidean geometry originated in the XIX century simultaneously and independently in different countries. 21. Lobachevsky — one of the greatest Russian mathematicians — revolutionized the science of space and objects in space. 22. With the discovery of non-Euclidean geometry, mathematicians realized that there is more than one conceivalbe space and hence more than one geometry. 23. In the twentieth century Geometry lost its former intimate connection with physical space and the study of "abstract spaces" was inaugurated. 24. The creators of non-Euclidean geometry did not think of its practical applications. It was pure science. 25. Hilbert built a model of non-Euclidean geometry, thereby the pure science received its **theoretical** justification. 26. After the days of Lobachevsky it became the fashion to challenge axioms. 21. There developed the concept of **formal axiomatics** and postulate sets for a large variety of geometries were investigated. Axiomatics as a science came into being. 28. Whereas the axiomatic method was formerly used for explaining the foundations of mathematics, nowadays it is a tool for concrete mathematical research. 29. Einstein applied Riemann's and Minkovsky's non-Euclidean geometries in his Relativity Theory. Thus, pure science obtained its **practical** justification. 30. Einstein's geometry is four-dimensional. Space-time is its fourth dimension. Contemporary mathematicians speak of n'th dimensional geometries. 31. Euclidean geometry nowadays is only one applied science furnishing an interpretation of Hilbert's pure science. There are an **infinite** number of others besides. 32. Today mathematicians claim that geometry is not a separate mathematical discipline, but a particular point of view — a particular way of looking at a subject.

Key Grammar Patterns

Present and Past Indefinite Tense-Aspect Forms

I. *Give the forms of the Past Indefinite and the Participle II of the following irregular verbs. Note the type-forms.*

a)	to speak	to be	to lie	to grow	to draw
	to break	to do	to take	to know	to bear
	to choose	to give	to write	to show	to tear
b)	to deal	to keep	to lead	to make	to say
	to feel	to hear	to leave	to mean	to seek
	to find	to learn	to light	to read	to stand
c)	to beset	to cost	to hurt	to reset	to slit
	to bet	to cut	to let	to set	to split
	to cast	to hit	to put	to shut	to upset

II. *Put the verb in brackets in the Present or Past Indefinite tense-form of Active and Passive Voice and translate the text into Russian. Generalize the main ideas of the text.*

Greek Schools of Mathematics

Great minds of Greece such as Thales, Pythagoras, Euclid, Archimedes, Appolonius, Eudoxus, etc. (to produce) an amazing amount of first-class mathematics. The fame of these mathematicians (to spread) to all corners of the Mediterranean world and (to attract) numerous pupils. Masters and pupils (to gather) in schools which though they had few buildings and no campus (to be) truly centres of learning. The teaching of these schools (to dominate) the entire life of the Greeks.

Despite the unquestioned influence of Egypt and Babylonia on Greek mathematicians, the mathematics produced by the Greeks (to differ) fundamentally from that which (to precede) it. It (to be) the Greeks who (to found) mathematics as a scientific discipline. **The Pythagorean school** (to be) the most influential in determining both the nature and content of Greek mathematics. Its leader Pythagoras (to found) a community which (to embrace) both mystical and rational doctrines.

The original Pythagorean brotherhood (c. 550—300 B. C.) (to be) a secret aristocratic society whose members (to prefer) to operate from behind the scenes and, from there, to rule social and intellectual affairs with an iron hand. Their noble born initiates (to be taught) entirely by word of mouth. Written documentation (not to be permitted), since anything written (may) give away the secrets largely responsible for their power. Among these early Pythagoreans (to be) men who (to know) more about mathematics then available than most other people of their time. They (to recognize) that vastly superior in design and manageability Babylonian base-ten positional numeration system (may) make computational skills available to people in all walks of life and rapidly democratize mathematics and diminish their power over the masses. They (to use) their own non-possitional numeration system (=standard Greek alphabet supplemented by special symbols). Although there (to be) no difficulty in determining when the symbols (to represent) a number instead of a word, for computation the people of the lower classes had to consult an exclusive group of experts or to use complicated tables — and both of these sources of help (to be controlled) by the brotherhood. The Pythagoreans (can) tell the tradesmen how such tables and devices (e. g., Abaci) were to be **used** but never how to **make** them or what the hidden patterns (to be) which (to make) them possible.

For Pythagoras and his followers the fundamental studies (to be) geometry, arithmetic, music, and astronomy. The basic element of all these studies (to be) **number** not in its practical computational aspects, but as the very essence of their being; they (to mean) that the nature of numbers should (to be conceived) with the mind only. In spite of the mystical nature of much of the Pythagorean study the members of community (to contribute) during the two hundred or so years following the founding of their organization, a good deal of sound mathematics. Thus, in geometry they (to develop) the properties of parallel lines and (to use) them to prove that "the sum of the angles of any triangle is equal to two right angles". They (to contribute) in a noteworthy manner to Greek geometrical algebra, and they (to develop) a fairly complete theory of proportion though it (to be limited) to commensurable magnitudes, and (to use) it to deduce properties of similar figures. They (to be aware) of the existence of at least three of the regular polyhedral solids, and they (to discover) the incommensurability of a side and a diagonal of a square.

Details concerning **the discovery of the existence of incommensurable quantities** (to be) lacking, but it is apparent that the Pythagoreans (to find) it as difficult to accept incommensurable quantities as to discover them. Two segments (to be) commensurable if there (to be) a segment that "measures" each of them — that is, (to be contained) exactly a whole number of times in each of the segments. The fact that there (to reveal) pairs of segments for which such a measure (not to exist) provides the incommensurable case. It (to be) possible that the first pair of segments found to be incommensurable (to be) the side and diagonal of a regular pentagon, the favourite-figure of the Pythagoreans because its diagonals (to form) the star pentagon, the distinctive mark of their society. This same geometric procedure (can) also (to be adapted) to the side and diagonal of a square. Here there (to exist) an association with the so-called Pythagoreans' side and diagonal numbers. The ratio of associated pairs of these numbers (to give) successively closer and closer rational approximations to $\sqrt{2}$; in fact, they (to be) the approximations obtained by computing successive convergents of the continued fraction form of $\sqrt{2}$. This (to be reflected) in modern mathematics in the concept of irrational number, a number that (can) (not to be expressed) as the ratio of two integers, e. g., π, e, $\sqrt[3]{2}$. This devastating discovery (to be ascribed) to Pythagoras himself, but more probably it (to be made) by some Pythagorean. Since the philosophy of the Pythagorean school (to be) that whole numbers or whole numbers in ratio (to be) the essence of all existing things, the followers of that school (to regard) the emergence of irrationals as a "logical scandal". As the revelation of geometrical magnitudes whose ratio (can) (not to be represented) by pairs of integers (to lead) to the "crisis" in the foundations of their mathematics, the Pythagoreans (to try) to conceal their greatest discovery. A Pythagorean **Hippasus** (c. 400 B. C.) who first (to bring out) the irrationals from concelment into the open supposedly (to perish) in a shipwreck at sea. But great discoveries (can) (not to be suppressed)! The discovery of incommensurables (to be) a turning-point, a landmark in the history of mathematics and its significance (can) hardly (to be overappreciated). It (to result in) a need to establish a new theory of proportions independent of commensubarility. This (to be accomplished) by **Eudoxus** (c. 370 B. C.). The details of the gradual transition from a theory of proportions which (to include) incommensurable quantities to a clear realization of the concept of an irrational number (to cover) a wide range of sophisticated mathematical topics and this concept (to be fully clarified) only in the nineteenth century by R. Dedekind and G. Cantor. In mathematics of today the irrationals (to form) an important subset of real numbers, the basis of both algebra and analysis.

The "Pythagorean" theorem (to be) one of the most important proposions in the entire realm of geometry. There (to be) no doubt, however, that the "Pythagorean property": $c^2=a^2+b^2$ (to be known) prior to the time of Pythagoras; there (to exist) clay tablet texts which (to contain) columns of figures related to Pythagorean triples. The frequent textbook reference to Egyptian "rope-stretchers" and their knotted surveying ropes as proof that these ancients (to know) the theorem (to be) erroneous. While it (to be known) that the Egyptians (to realize) as early as 2000 B. C. that $4^2+3^2=5^2$, there (to be) no evidence that the Egyptians (to know) or (to be able) to prove the right angle property of the figure involved. Pythagoras (to be credited) with the proof of this property which (to be) true for all right trianges, and for all natural numbers. Although much of this information (to be known) already to the ancients of earlier times, **the deductive aspect of geometry (to be exploited and advanced) considerably in the work of the Pythagoreans.**

The mysticism of this celebrated school (to arouse) the suspicion and dislike of the people who finally (to drive) the Pythagoreans out of Crotona, a Greek seaport in Southern Italy and (to burn) their buildings. Pythagoras (to be murdered) but his followers (to scatter) to other Greek centres and (to continue) his teachings. **The Pythagoreans (to be credited) with giving the subject of mathematics special and independent status.** They (to be) the first group to treat mathematical concepts as abstractions and they (to distinguish) mathematical theory from practices or calculations. They (to prove) the fundamental theorems of plane and solid geometry and of "arithmetica" — the theory of numbers.

More widely known than the Pythagoreans (to be) the **Academy of Plato** which (to have) **Aristotle** as its most distinguished student. The latter then (to found) his own school at the time of Plato's death. Plato's pupils (to be) the most famous philosophers, mathematicians and astronomers of their age. Under Plato's influence they (to emphasize) pure mathematics to the extent of ignoring all practical applications and they (to add) immensely to the range of mathematics.

Substitutes of the Noun

a) **it**

It is necessary to do it.
It appears that there is a new meaning of this term.

b) **one**

One can define this term rigorously.
I can't solve this problem, let me try another **one.**

c) **that of**

Your proof is more elegant that **that of** the rest of the students.

d) **those of**

His results are much better than **those** of his friends.

e) **the former**

Of those two properties **the formre** is far more important.

f) **the latter**

Of the Greek mathematicians people know Pythagoras and Euclid best of all, **the latter** is the author of the "Elements".

Emphatic Constructions

a) it is (was) ... that (who)
Именно, как раз.

b) **do**+Predicate
Действительно, в самом деле, ведь, же.

Translate the emphatic constructions.

1. The major Near Eastern civilizations from which mathematics arose were the Egyptian and the Babylonian. Yet, **it was** the Greeks **who** formed mathematics as a scientific discipline. 2. **It was** with the Greeks of the classical period **that** our modern mathematics began. 3. **It was** from about 600 to 300 B. C. **that** the classical period lasted. 4. **It was** by deductive reasoning **that** the Greeks derived all mathematical conclusions. 5. **It was** Euclid **who** gave the summation of the mathematics of the clas-

sical period in his "Elements". 6. Although the Greeks **did regard** a straight line as being infinite in extent and defined parallel lines as lines which do not meet however far extended, they did not carry far enough the idea of geometrical infinity. 7. The Greeks **did perform** many constructions using only a straightedge and a compass.

Impersonal Sentences

(Не) Говорят (Говорится. Говорим)	**Можно сказать**	**Нельзя сказать**
It is said It is not said	It can / may be said	It cannot / may not be said
One says One does not say	One can / may say	One cannot / may not say
We / You / They say / do not say	We / You / They can / may say	We / You / They cannot / may not say
People say People do not say	People can / may say	People cannot / may not say
I am (we are) told / We (they) hear (coll.)		

Нужно (надо) сказать	**Не надо (не следует) говорить**	**Едва ли (вряд ли) можно сказать**
It must / should be said	It must not / should not be said	It can hardly / scarcely be said
One must / should say	One must not / should not say	One can hardly say
We / You / They must / should say	We / You / They must not / should not say	We / You / They can hardly say
People must / should say	People must / should not say	People can hardly say

I. *Translate the following impersonal sentences.*

1. **One must distinguish** between mathematical objects (e. g., numbers, sets of numbers, functions, mappings, transformations, etc.) and the mathematical method. 2. **It should be said** that a proof constitutes the principal part of the mathematical method. 3. **One may not know** that it was Thales of Miletus who introduced the concept of a rigorous mathematical proof. 4. **It is conjectured** that the crisis in Greek mathematics resulted from the discovery of irrational numbers led to the method of deriving theorems from axioms. 5. **You ought not to define** axioms today as self-evident or universally recognized truths, accepted without proof. 6. **People say** now that some terms must be taken as undefined and axioms are mere assumptions about these undefined terms (or variables). 7. **It can hardly be denied** that the choice of axioms for a logical system is a creative act. 8. When the axioms are selected, **one must** then **determine** whether or not the axioms chosen satisfy certain properties

among themselves. 9. **We can say** that axioms and their properties are important enough. Yet, this is only the beginning. 10. **One should determine** next what the relationship between the axioms and other statements called "theorems" is. 11. **They assert** that **if** such and such statements are granted as axioms, **then** such and such statements follow. 12. **It is known** that the process of reaching conclusions from axioms is called deduction. 13. **One says** that the relationship that holds between the statements chosen as axioms and those which are deduced from them is called implication. 14. **We can state** therefore that the axioms imply theorems. 15. In fact, **one can** in theory **prove** any theorem directly from the axioms; however, the theorems in turn, are usually a convenient short-cut to proofs. 16. **One should not confuse** the postulate with the definition.

II. *Read the text in class. Locate emphatic constructions and impersonal sentences in the text. Practice questions and answers on the text.*

The Alexandrian School of Mathematics

Most people think of ancient Greece in terms of the 3rd, 4th and 5th century B. C. The "Golden age" when the empire was at its height and the greatest artists, poets and writers lived, was the 5th century B. C. But the giants in mathematics came later, Eudoxus about 350 B. C., and Euclid, Archimedes and Appolonius between 300 and 200 B. C. The greatest mathematical centre of ancient world was neither Crotona nor Athens but **Alexandria.** It is with Alexandria that the names of Euclid, Archimedes, Appolonius, Ptolemy, Heron, Pappus, Hypatia, Diophantus, Hipparchus, etc. are connected.

For at least four thousand years the civilization of Egypt followed a rigid pattern. In religion, mathematics, philosophy, commerce, and argiculture each man imitated his forefathers. No external influence disrupted the calm life and fixed ways. Then, about 325 B. C. **Alexander the Great** conquered this vast land as well as Greece and the Near East. He founded the city of Alexandria and moved the capital of the ancient world from Athens to this new city. From a fusion of cultures, centered at Alexandria, a new civilization appeared and made its very significant and distinctive contribution to mathematics and to Western civilization. Two factors vitally influenced the character of the culture of Alexandria: the commercial interests of the Alexandrians with their geographical and navigational problems and the fact that the scholars became involved in the problems facing the people at large. Alexandria became the centre of the entire ancient world, for it was ideally located at the junction of Asia, Africa and Europe. On the streets of the city native Egyptians met and traded with Greeks, Persians, Syrians, Romans and Arabs. No city in the world ever embraced such a variety of peoples. It was to this important centre that traders and businessmen from all corners of the world directed their routes.

One must not forget that credit for making Alexandria the intellectual centre of the new world does not go to the founder of the city, who died while still engaged in conquests, but to the very capable **Ptolemy the First,** the general who took over control of Egypt on the death of Alexander. Aware of the cultural importance of the great Greek schools such as those founded by Pythagoras, Plato and Aristotle, Ptolemy decided that Alexandria should have such a school and that it should become the centre of Greek culture in this new world. He built a home for the Muses and adjacent to this Museum Ptolemy erected a library not only for the preservation of important manuscripts but also for the use of the general public. This famous library at one time contained 750,000 volumes. Together with the Museum, the library resembled a modern University, though no University of today can boast of possessing as many great intellects as were assembled there. Today, however, not the slightest trace remains of the famous library and Museum and even their exact locations are merely conjectural.

Scholars of all countries were invited to Alexandria by Ptolemy and were supported by grants from him. Consequently, there gathered at this Museum poets, philosophers, philologists, astronomers, geographers, physicians, historians, artists, and the most famous mathematicians of the Alexandrian age. The principal group of the scholars gathered at the museum was Greek, but distinguished members of many other nations also settled there. Among the non-Greeks the most celebrated was the learned Egyptian astronomer, Claudius Ptolemy. One can hardly doubt, of course, that mathematics had a most important place in the Alexandrian world, but it was not the mathematics that the classical Greek scholars knew. The civilization of Alexandria developed a kind of mathematics almost opposite in character to that produced by the classical Greek age. The new mathematics was **practical**; while the former was entirely unrelated to application; the latter measured the distance to the farthest stars, enabled men to travel over land and sea, etc. The great Alexandrian mathematicians Archimedes, Hipparchus, Ptolemy, Heron, Menelaus, Diophantus, etc. though they did display almost without exception the Greek genius for theoretical abstractions, nevertheless, they were quite willing to apply their talents to the practical problems necessarily important in their civilization. The man whose work best epitomizes the character of the Alexandrian age is **Archimedes**, one of the greatest intellects of antiquity.

In the field of mathematics proper the Alexandrians created and applied methods of indirect measurement. One ought not to underestimate this contribution of the Alexandrians. Their formulas for areas and volumes surprisingly are not in Euclid's "Elements" for though Euclid lived at the beginning of the Alexandrian age, his goal and the subject matter was really the summation and culmination of the mathematics of the classical period. Euclid's work is a monument both in original or in any epitome. As far as the Alexandrian age is concerned, the supreme achievement of the Alexandrians was the creation of the most accurate and most influential astronomical theory of ancient times developed by Hipparchus and Claudius Ptolemy.

Unfortunately, the intellectual life of the Greeks was cut short by polytical events beyond the control of mathematicians and philosophers. The Romans rolled over the Italian peninsula and then began to attack other lands bordering the Mediterranean. The fire swept in from the sea destroyed the great library (47 B. C.) at Alexandria. Two and a half centuries of book collecting and half a million manuscripts were wiped out. The fire at Alexandria was symbolic of the Roman contempt for abstract knowledge. The Romans were practical people and they boasted of their practicality. They left no worthy imprint in the history of mathematics. Though the Museum of Alexandria and the great library were destroyed and the scholars dispersed, Greek science eventually re-emerged, Greek culture did survice, and Europe did learn a lot from the Greeks.

THE INTRODUCTORY TEXT

(*Scanning Reading*)

THE HISTORY OF GEOMETRY

The story of the history of geometry, like that of many other growing and changing subjects, is composed of two intertwined strands. One strand narrates the growing content of the subject and the other the changing nature of the subject. The following is a brief outline of the birth and the development of Geometry.

Subconscious Geometry

The first geometrical considerations of man are unquestionably very ancient. They had their origin in simple observation stemming from human ability to recognize physical form and to compare shapes and sizes. The notion of distance was undoubtedly one of the first geometric concepts developed. Many observations in the daily life of early man led to the notion of simple geometric concepts such as rectangles, squares, triangles, curves, surfaces and solids. Such geometry may, for want of a better name, be called "subconscious geometry".

Scientific Geometry

In the beginning, man considered only concrete geometrical problems, which present themselves individually and with no observed interconnections. Later (but still before the dawn of recorded history), human intelligence evolved to the point where it was able to extract from a number of observations certain general properties and relationships. This introduced the advantage of ordering practical geometrical problems into sets such that the problems in a set can be solved by the same general procedure. One thus arrives at the notion of a geometrical law or rule. This higher stage of geometry may be called "scientific geometry", in view of the fact that induction, trial and error, and empirical procedures were the tools of discovery. Geometry became a collection of general rule-of-thumb and laboratory results, concerning areas, volumes, and relationships of various figures suggested by physical objects. No evidence permits us to estimate the number of centuries that passed before man was able to raise geometry to the status of a science, but the writers of antiquity unanimously agree upon the Nile Valley of ancient Egypt and Babylonia as the place where subconscious geometry first became scientific geometry. Geometry remained of this type until the great Greek period of Antiquity.

Demonstrative Geometry (Early Greek Geometry)

The economic and political changes of the last centuries of the secon millennium B. C. caused the power of Egypt and Babylonia to wane. New peoples came to the fore, and it happened that the further development of geometry passed over to the Greeks, who transformed the subject into something vastly different from the set of empirical conclusions worked out by their predecessors. The Greeks insisted that geometric fact must be established not by empirical procedures, but by deductive reasoning; geometrical truth was to be attained in the classroom rather than in laboratory. In short, the Greeks transformed the empirical or scientific geometry of the ancient Egyptians and Babylonians into what we may call "systematic" or "demonstrative" geometry. Greek geometry started in an essential way with the work of **Thales of Miletus** in the first half of the sixth century B. C. This versatile genius, one of the "seven wise men" of antiquity was a worthy founder of demonstrative geometry. He is the first known individual with whom the use of deductive methods in geometry is associated. He is credited with a number of very elementary geometrical results the value of which is not to be measured by their content but rather by the belief that he supplied them with a certain amount of logical reasoning instead of intuition and experiment. The next outstanding Greek geometer is **Pythagoras** who continued the systematization of geometry begun some fifty years earlier by Thales.

Later Greek Geometry

The three most outstanding Greek geometers of antiquity are **Euclid** (c. 300 B. C.), **Archimedes** (287—212 B. C.) and **Apollonius** (c. 225 B. C.) and it is no exaggeration to say that almost every subsequent significant geometrical development, right up to and including the present time, finds its seeds of origin in some work of these three great scholars.

With the passing of Apollonius the Golden age of Greek geometry came to an end. The geometers who followed did little more than fill in details and perhaps independently develop certain theories the germs of which were already contained in the works of the three great predecessors. Among these later geometers special mention should be made of **Heron** (or Hero) of **Alexandria** (c. A.D.75), **Menelaus** (c. 100) and Claudius **Ptolemy** (c. 85 — c. 165). In ancient Greek geometry both in its form and its content, we find the fountainhead of the subject.

Middle Ages

The closing period of ancient times comes when in 146 B. C. Greece became a province of the Roman Empire and a gradual decline in creative thinking set in. The period starting with the fall of the Roman Empire in the midde of the fifth century and extending into the eleventh century is known as Europe's Dark Ages, for during this period civilization in western Europe reached a very low ebb. Schooling became almost nonexistent, Greek learning all but disappeared, and many of the arts and crafts were forgotten. During this period of learning, the peoples of the East, especially the Hindus and the Arabs, became the major custodians of mathematics. Although the Hindus excelled in computation, contributed to the devices of algebra, and played an important role in the development of our present positional numeral system, they produced almost nothing of importance in geometry or in basic mathematical methodology.

It was not until the latter part of the eleventh century that Greek classics in science and mathematics began once again to filter into Europe. The fifteenth century, the early period of the Renaissance, witnessed the rebirth of art and learning in Europe. Many Greek classics, known up to that time only through Arabic translations, often quite inadequate, could now be studied from original sources. Mathematical activity in this century was largely centred in the Italian cities and in the central European cities of Nuremberg, Vienna and Prague. It concentrated on arithmetic, algebra, and trigonometry, under the practical influence of trade, navigation, astronomy, and surveying.

Projective Geometry

In an effort to produce more realistic pictures, many of the Renaissance artists and architects became deeply interested in discovering the formal laws controlling the constructions of objects on a screen, and as early as the fifteenth century a number of these men created the elements of an underlying geometrical theory of perspective. Some aspects of this subject which concerns a way of representing and analyzing three-dimensional objects by means of their projections on certain planes had their origin in the design of fortifications.

Analytic Geometry

Projective geometry was overshadowed by the more supple analytic geometry introduced by **René Descartes** and **Pierre de Fermat**. There is a fundamental distinction between the two studies, for the former is a **branch** of geometry whereas the latter is a **method** of Geometry. Analytic geometry is often described as the "royal road" in geometry that Euclid thought did not exist.

Differential Geometry

Many new and extensive fields of mathematical investigation were opened up in **the seventeenth century**, making that era an outstandingly productive one in the development of mathematics. Unquestionably the most remarkable mathematical achievement of the period was the invention of the calculus by **Isaac Newton** and **Gottfried Wilhelm von Leibnitz.** A fair share of its remarkable applicability lies in the field of geometry and there is an exceedingly vast body of geometry wherein one studies properties of curves and surfaces, and their generalizations, by means of the calculus. This body of geometry is known as "differential geometry". For the most part, differential geometry investigates curves and surfaces only in the immediate neighbourhood of any one of their points. This aspect of differential geometry is known as "local differential geometry" or "differential geometry in the small". However, sometimes properties of the total structure of a geometric figure are implied by certain local properties of the figure that hold at every point of the figure. This leads to what is known as "integral geometry" or "global differential geometry", or "differential geometry in the large". It is probably quite correct to say that differential geometry, at least in its modern dress, started in the early part of the eighteenth century with the interapplications of the calculus and analytic geometry. **Carl Friedrich Gauss** (1777—1855) introduced the fruitful method of studying the differential geometry of curves and surfaces by means of parametric representation of these objects. **Bernhard Riemann** introduced an improved notation and a procedure independent of any particular coordinate system employed. The tensor calculus was accordingly devised and developed. Here we find an assertion of the tendency of mathematics in recent times to strive for the greatest possible generalization.

Generalized differential geometries, known as **Riemannian geometries** were explored intensively, and this in turn led to non-Riemannian, and other, geometries. Much of this material finds significant application in relativity theory and other parts of modern physics.

Non-Euclidean Geometry

There is evidence that a logical development of the theory of parallels gave the early Greeks a lot of trouble. Euclid met the difficulties by defining parallel lines as coplaner straight lines that do not meet one another however far they may be produced in either direction, and by adopting as an initial assumption his now famous parallel postulate: "If a straight line intersects two straight lines so as to make the interior angles on one side of it together less than two right angles, the two straight lines will intersect, if indefinitely produced, on the side on which are the angles which are together less than two right angles". Actually, the postulate is the converse of Proposition 17 of Euclid's Book II and it seemed more like a proposition than a postulate. It was natural to ask if the postulate was really needed at all, or perhaps it could be derived as a theorem, or, at least, it could be replaced by a more acceptable equivalent. The attempts to devise substitutes and to derive it as a theorem from the rest of Euclid's postulates occupied geometers for over two thousand years and culminated in the most far-reaching development of modern mathematics — **non-Euclidean Geometry.**

Topology started as a branch of geometry, but during the second quarter of **the twentieth century** it underwent such generalization and became involved with so many other branches of mathematics that it is now more properly considered, along with geometry, algebra, and analysis, a fundamental division of mathematics. Today topology may roughly be defined as the mathematical study of continuity, though it still reflects its geometric origin. Topology is the study of those properties of geometric figures which remain invariant under so-called topological transformations, that is, under single-valued continuous mapping possessing single-valued continuous inverses.

The Erlanger Programme

In the middle of **the nineteenth century** a number of different geometries came into existence, and the time was ripe for some sort of codification, synthesis and classification to give a sense of order to these geometries. Such a scheme was announced in 1871 by **Felix Klein**, in his inaugural address upon appointment to the Philosophical Faculty and the Senate of the University of Erlanger. This address, based on work he and Sophus Lie did in group theory, set forth a remarkable definition of "a geometry", one that served to codify essentially all the existing geometries of the time and pointed the way to new fields of research in Geometry. This address with the programme of geometrical study advocated by it is known as **Erlanger Programme**. Somewhat oversimply stated, the **Erlanger Programme** claims that a geometry is the investigation of those properties of figures which remain unchanged when the figures are subjected to a group of transformations. It advocates the classification of existing geometries and the creation and study of new geometries, according to this scheme. In particular one should study the geometries characterized by the various proper subgroups of the transformation group of a given geometry, thereby obtaining geometries that embrace others.

For plane Euclidean metric geometry, the group of transformations is the set of all rotations and translations in the plane; for plane projective geometry, the group of transformations is the set of all so-called planar projective transformations; for topology, the group of transformations is the set of all topological transformations. Each geometry has its underlying controlling transformation group. In building up a geometry, then, one is at liberty to choose, first of all, the fundamental elements (point, line, etc.); next the manifold of these elements (plane of points, ordinary space of points, spherical surface of points, plane of lines, pencil of circles, etc.); and, finally, the group of transformations to which the manifold of elements is to be subjected.

Abstract spaces

In the twentieth century the study of "abstract spaces" was inaugurated and some very general geometries came into being. A "space" became merely a set of objects, for convenience called "points" together with a set of relations in which these points are involved, and a geometry becomes simply the theory of such a space. The set of relations to which the points are subjected is called the "structure" of the space, and this structure may or may not be explainable in terms of the invariant theory of a transformation group. Through set theory Geometry received a further generalization or metamorphosis. These new geometries find invaluable application in the modern development of analysis. Important among abstract spaces are the so-called metric spaces, Hausdorf spaces, topological spaces, Hilbert's spaces, and vector spaces.

Hilbert's Formal Axiomatics

The discovery by Lobachevsky, Bolyai and Gauss of a self-consistent geometry different from the geometry of Euclid liberated geometry from its traditional mold. A deep-rooted and centuries old conviction that there can be only one possible geometry is shattered and the way is opened for the creation of many different systems of geometry. With the possibility of creating such purely "artificial" geometries, it becomes apparent that geometry is not necessarily tied to actual physical space. The postulates of geometry become, for the mathematician, mere hypotheses whose physical truth or falsity need not concern him. The mathematician may take his postulates a suit he pleases provided they are consistent with one another. Whereas it is customary, in Euclidean geometry, to think of the objects that represent the primitive terms of an axiomatic discourse as being known prior to the postulates, now the postulates become regarded as prior to the specification of primitive terms. This new point of the axiomatic method is known as "formal axiomatics" in contrast to the earlier "material axiomatics". In a formal axiomatic treatment the primitive terms have no meaning whatever except that implied by the postulates, and the postulates have nothing to do with "self-evidence" or "truth" — they are merely assumed statements about the undefined primitive terms.

Many mathematicians now regard any discourse conducted by formal axiomatics as a "branch of pure mathematics". If for the primitive terms in such a postulation discourse we substitute terms of definite meaning which convert the postulates into true statements about those terms, then we have an "interpretation" of the postulate system. Such an interpretation may also, if the reasoning is valid, convert the derived statements of the discourse into true statements. Such an evaluation of a branch of pure mathematics is called a "branch of applied mathematics". Clearly, a given branch of pure mathematics may possess many interpretations and may thus lead to many branches of applied mathematics. From this point of view, we see that material axiomatics is the independent axiomatic development of some branch of applied axiomatics. In a formal axiomatic treatment one strips the discourse of all concrete content and goes to the abstract development that lies behind any specific application.

Formal axiomatics was first systematically developed by **David Hilbert** in his famous book "The foundations of Geometry" in 1899. This little work, which ran through nine editions, is today a classic in its field. Next to Euclid's "Elements" it may be regarded as perhaps the most influential work so far written in the field of geometry. Backed by the author's great mathematical authority, the work firmly implants the postulation method of formal axiomatics not only in the field of geometry but also in nearly every branch of mathematics of the twentieth century. The book offers a completely acceptable postulate set for Euclidean geometry, and it can be read by any intelligent person.

ACTIVE VOCABULARY

1. to accomplish
2. to approach
3. to assert
4. to assume
5. to conceive
6. to concern
7. to confide
8. to confine
9. to contradict
10. to convert
11. to deserve
12. to display
13. to doubt
14. to emerge
15. to furnish
16. to inaugurate
17. to infer
18. to proceed
19. to propose
20. to realize
21. to substitute
22. to survey
23. to survive
24. to trace
25. to unify
26. to yield

Analyze A. Einstein's citation with the teacher in class. Generalize the main ideas in it. Read the text and 1) find some evidence or proof to Einstein's statements, 2) some disproofs or refutations.

TEXT ONE

EUCLID'S ELEMENTS

"We honour ancient Greece as the cradle of western science. She for the first time created the intellectual miracle of **a logical system,** the assertions of which followed one from another with such rigour that not one of the demonstrated propositions admitted of the slightest doubt — **Euclid's geometry.**

This marvellous accomplishment of reason gave to the mind the confidence it needed for its future achievements. The man who was not enthralled in youth by this work was not born to be a scientific theorist" (*A. Einstein*).

«Мы почитаем древнюю Грецию как колыбель западной науки. Там была впервые создана **геометрия Евклида** — это чудо мысли, **логическая система,** выводы которой с такой точностью вытекают один из другого, что ни один из них не был подвергнут какому-либо сомнению.

Это удивительнейшее произведение мысли дало человеческому разуму ту уверенность в себе, которая была необходима для его последующей деятельности. Не рожден для теоретических исследований тот, кто в молодости не восхищался этим творением» (*А. Эйнштейн*).

When most people describe the Greeks' contribution to modern civilization they talk in terms of art, literature and philosophy. No doubt the Greeks deserve the highest praise in all these fields. Nevertheless, the contribution of the Greeks that determines most the character of present-day civilization was their **mathematics!**

In a relatively brief period (from about 600 till 300 B. C.) great intellects such as **Thales, Pythagoras, Euclid, Eudoxus, Archimedes** and **Appolonius** created an amazing amount of first-class mathematics. It is disappointing that unlike the situation with the ancient Egyptian and Babylonian geometry, there exist virtually no primary sources for the study of very early Greek geometry. We are forced to rely on manuscripts and accounts that are dated several hundred years after the birth of the original treatment.

Our principal source of information concerning very early Greek geometry is the so-called "Eudemian Summary" of **Proclus.** This summary constitutes several pages of Proclus's "Commentary on Euclid, Book I" and is a brief outline of the development of Greek geometry from the earliest times up to Euclid. Although Proclus lived in the fifth century A. D., a good thousand years after the inception of Greek geometry he still had access to a number of historical and critical works that are now lost to us except for the fragments and allusions preserved by him and others.

Although much of the information on plane geometry was known to the Babylonians of earlier times, the **deductive aspect** of geometry was introduced for the first time by **the Pythagoreans.** Chains of propositions in which successive propositions are derived from earlier ones began to emerge in the works of Thales. As the chains lengthen and some are tied to others, the bold idea of developing all of geometry in one long chain suggests itself.

During the first three hundred years of Greek mathematics, there developed the Greek notion of **a logical discourse** as a sequence of statements obtained by deductive reasoning from the accepted set of initial statements. Now, both the initial and the derived statements of the discourse were statements about the technical matter of the discourse and hence involved special or techinical terms. The meanings of these terms must be clear to the reader, and so, for the Greeks the discourse must start with a list of explanations and definition of these technical terms. After these explanations and definitions the initial statements, called "axioms" or "postulates" of the discourse, were to be listed. These initial statements, according to the Greeks, should be so carefully chosen that their truths were quite acceptable to the reader in the light of the explanations and definitions already cited. Certainly, the most outstanding contribution of the early Greeks to mathematics was the formulation of **the mathematical method** (400 B. C) for this method is the very core of modern mathematics. Unfortunately, we do not know with whom the mathematical method originated personally, but it evolved with the Pythagoreans as a natural outgrowth and refinement of the early application of deductive procedures to mathematics.

It is claimed in Proclus's "Summary" that a Pythagorean, **Hippocrates of Chios,** attempted, with at least partial success, a logical presentation of geometry in the form of a single chain of propositions based on a few initial definitions and assumptions. There followed other writers' attempts and then, about 300 B. C., **Euclid** produced his epoch-making effort, the "**Elements**", a single deductive chain of 465 propositions neatly and beautifylly comprising plane and solid geometry, number theory, and Greek geometrical algebra.

From its very first appearance this work was accorded the highest respect, and it so quickly and so completely superseded all previous efforts of the same nature that now no trace remained of the earlier systems. The work of many schools and isolated individuals was unified by Euclid in this most famous textbook on geometry. Euclid deduced all the most important results of the Greek masters of the classical period and therefore the "Elements" constituted the mathematical history of the age as well as the logical presentation of Geometry. The effect of this single book on the future development of geometry was enormous and is difficult to overstate.

The plan of Euclid's "Elements" is as follows. It begins with **a list of definitions** of such notions as point and line; for example, a line is defined as "length without breadth". Next appear various statements some of which are labelled **axioms** and others **postulates.** It appears that the axioms are intended to be principles of reasoning which are valid in any science (for example, one axiom asserts that "equals to the same thing are equal to each other"), while the postulates are intended to be assertions about the subject matter, that is geometry (for example, one postulate asserts that "it is possible to draw a line joining any two distinct points").

From a modern viewpoint it may be said that Euclid treats point and line essentially as primitive or undefined notions, subject only to the restrictions stated in the postulates, and that his definitions of these notions offer merely an intuitive description which helps one in thinking about formal properties of points and lines. Concerning the postulates, he probably believed that they were true statements on the basis of the meaning suggested by his definitions of the terms involved and the proofs acquired status of "self-evident truths".

The exioms chosen by Euclid state properties of points, lines and other geometric figures that are possessed by their physical counterparts. The properties in question are so obviously true of these physical objects that all mathematicians agreed on them as a basis for further reasoning. In the selection of axioms Euclid displayed great insight and judgement. Euclid chose a very limited number of axioms, **twelve** in all (later generations reduced this number to **ten**), and constructed the whole system of geometry.

From this starting point of definitions, axioms and postulates, Euclid proceeds to derive **propositions (theorems)** and at appropriate places to introduce further definitions (for example, an obtuse angle is defined as an angle which is greater than a right angle). **His method of proof is strictly deductive** that is, his theorems are proved by several deductive arguments, each employs unquestionable premises and yields an unquestionalbe conclusion.

A discourse developed according to the above plan is referred to as "**material axiomatics**" Certainly, the most outstanding contribution of the early Greeks to mathematics was the formulation of the pattern of material axiomatics and the insistence that geometry should be systematized according to this pattern. Euclid's Elements is the earliest extensively developed example of this use of the pattern available to us. In recent years, this pattern was significantly generalized to yield a more abstract form of discourse known as "**formal axiomatics**".

The creation of Euclidean geometry is more than the contribution of numerous useful theorems. It reveals the power of reason. No other human creation demonstrates how much knowledge can be derived by reasoning alone as have the hundreds of proofs in Euclid's "Elements". The necessity for accurate and exact definitions, for clearly stated assumptions and for rigorous proof became evident in Euclid's "Elements".

We know much of the material of Euclid's "Elements" through our high-school studies. By studying Euclid, hundreds of generations from Greek times learned how to reason, how perfect logical reasoning must proceed, how to master the procedure, how to distinguish exact reasoning from vague pretence of proof. Even nowadays this masterpiece of Euclid serves as a logical exercise and as a model of reasoning and the art of the mind.

Read the text in class. Analyze and translate those sentences (paragraphs) that may present some difficulty for understanding. Collect the information concerning N. Lobachevsy. Add your own comments and speak about his bold and great discovery.

TEXT TWO

NON-EUCLIDEAN GEOMETRIES

The man, who deserves the honour for the creation of non-Euclidean geometry, is the distinguished Russian mathematician **N. Lobachevsky.** Lobachevsky challenged the parallel axiom and substituted another: "Through a point outside a line L there are an infinite number of lines parallel to L". He built a new geometry on the basis of a parallel axiom contradicting Euclid's, it is a logically consistent geometry, one in which there are not contradictions. The most unbelievable hteorems to which he was led did not discourage him and he came to the conclusion: "There are geometries different from Euclid's and equally valid".

Lobachevsky succeeded in creating a new geometry with many surprising theorems. The most unexpected is the theorem that the sum of the angles of any triangle is always less than 180°. Moreover of two triangles the one with a larger area has a smaller angle sum, i. e., two similar triangles must be also congruent. As a final example is the following: the distance between two parallel lines approaches zero in one direction along the lines and becomes infinite in the other direction.

Which then is correct geometry? Which is the correct theory of the universe? Which is the most convenient theory? Which fits the observed data best and involves the least computation and the simplest mathematics? We may think that the new geometry cannot be applied to the physical world because, for example, it asserts that similar triangles must be congruent. The surprising revelation that emerged from all the attempts to decide which of the two geometries fits physical space is that **both** fit equally well.

The creation of non-Euclidean geometry brought into clear light a distinction between a mathematical space and physical space. The axioms of Euclidean geometry are true of physical space. With the creation of non-Euclidean geometry mathematicians appreciated the fact that systems of thought based on statements about physical space might be different from that physical space.

If both Euclidean and non-Euclidean geometry can represent physical space equally well, which one is the truth about space and figures in space? One cannot say. In fact, the choice may not be limited to just these two. Geometry is not the truth about physical space but the study of possible spaces. Several of these mathematically constructed spaces, differing sharply from one another, can fit physical space equally well as far as experience can decide.

We must give due credit also to other mathematicians who contributed much to the creation of non-Euclidean geometry, **J. Bolyai** a Hungarian mathematician worked out the notion of a non-Euclidean geometry simultaneously with Lobachevsky, but independently. Since Lobachevsky's publications precede Bolyai's it is customary to name Lobachevsky as the discoverer of the concept of non-Euclidean geometry.

K. F. Gauss, the great mathematical giant of the nineteeth century, discovered the same results as Lobachevsky and Bolyai **before either** and lacked the courage to publish facts so startling. After carefully considering the parallel axiom, Gauss gave a criterion for determining the truth of Euclidean geometry: measuring the angles of a triangle must decide which geometry fits the physical world in the particular case.

These radical departures from Euclid followed by **Riemann's** geometry with many striking theorems. The German mathematician **Riemann (1826—1866) postulated no parallels.** In other words, he substituted for Euclid's parallel postulate the assumption that: "Through a point *P* outside a line *L* there is no line parallel to it; that is, every pair of lines in a plane must intersect". In his geometry all the perpendiculars to a straight line meet in a point, the sum of the angles of any triangle is greater than 180°, etc.

To test the "truth" of all these theorems revealed in Gauss's criterion, mathematicians once tried to measure a huge triangle with vertices on three peaks in Germany. But all experiments failed to bring about a decisive conclusion. The sum of the angles found was always so close to 180° that the excess or deficit in each case could be made by the unavoidable imperfection in the measuring techniques. Even if the three theories fit experimetal facts equally well, they are not equal in convenience of computation. For ordinary everyday purposes, the Euclidean system is the simplest and hence we use it not because of the "absolute" and only truth, but because it makes our work easier. Riemann's system is the simplest for use in Einstein theory.

When the term non-Euclidean geometry is used in mathematical literature, the geometries of Lobachevsky and Riemann are always meant, although the term may well be applied generically to any geometry that denies one or more axioms of Euclid. After the days of Lobachevsky and Riemann it became the fashion to challenge axioms. To correct the defects in Euclid's "Elements" many axiom (postulate) systems were suggested and developed. Among these systems are those of **Pasch** (1882), **Peano** (1889), **Veblen** (1904), **Hilbert** (1909) and **Birkhoff** and **Beatley** (1940). Each is different; some have certain advantages over the others. Hilbert's system, perhaps because he is known as one of the outstanding mathematicians of the twentieth century, had the most profound effect. Perhaps too, this is because his system, as compared to the others, is most similar to Euclid's. Whatever the reasons, Hilbert's system was so widely used, revised, and refined over the years that many variations of his system — changes in statements and phraseology — are now in existence.

Euclidean geometry is only one applied science furnishing an interpretation of Hilbert's pure science; spherical geometry is another. There are an infinite number of others besides — some vital, many trivial, but all possible interpretations. It took unusual imagination to entertain the possibility of a geometry different from Euclid's, for the human mind was for two millennia bound by the prejudice of tradition to the firm belief that Euclid's system was most certainly the only way geometrically to describe physical space, and that any contrary geometric system simply could not be consistent. One may ask today whether a geometry is based on a set of consistent postulates, whether these postulates are independent of one another, or whether this geometry serves better than another geometry for a given application. But the question of whether a geometry is "true" has no place in pure science.

Read and translate the text in class. Practise questions and answers. Find the paragraph and the sentence which may be used as a short answer to the question: What is a modern view of Geometry?

TEXT THREE

A MODERN VIEW OF GEOMETRY

For a long time geometry was intimately tied to physical space, actually beginning as a gradual accumulation of subconscious notions about physical space and about forms, content, and spatial relations of specific objects in that space. We call this very early geometry "subconscious geometry". Later, human intelligence evolved to the point where it became possible to consolidate some of the early geometrical notions into a collection of somewhat general laws or rules. We call this laboratory phase in the development of geometry "scientific geometry". About 600 B. C. the Greeks began to inject deduction into geometry giving rise to what we call "demonstrative geometry".

In time demonstrative geometry becomes a material-axiomatic study of idealized physical space and of the shapes, sizes, and relations of idealized physical objects in that space. The Greeks had only one space and one geometry; these were absolute concepts. The space was not thought of as a collection of points but rather as a realm or locus, in which objects could be freely moved about and compared with one another. From this point of view, the basic relation in geometry was that of congruence or superposability.

With the elaboration of analytic geometry in the first half of the seventeenth century, space came to be regarded as a collection of points; and with the invention, about two hundred years later of the classical non-Euclidean geometries, mathematicians accepted the situation that there is more than one conceivable space and hence more than one geometry. But space was still regarded as a locus in which figures could be compared with one another. The central idea became of a group of congruent transformations of a space into itself and geometry came to be regarded as the study of those properties of configurations of points which remain unchanged when the enclosing space is subjected to these transformations, and a geometry is defined as the invariant theory of a transformation group. Geometry came to be rather far removed from its former intimate connection with physical space, and it became a relatively simple matter to invent new and even bizzare geometries.

At the end of the last century, Hilbert and others formulated the concept of **formal axiomatics**, and there developed the idea of branch of mathematics as an abstract body of theorems deduced from a set of postulates. Each geometry became, from this point of view, a particular branch of mathematics. Postulate sets for a large variety of geometries were studied.

In the twentieth century the study of abstract spaces was inaugurated and some very general studies came into being. A space became merely a set of objects together with a set of relations in which the objects are involved, and a geometry became the theory of such a space. It must be confessed that this latter notion of a geometry is so embracive that the boundary lines between geometry and other areas of mathematics became very blurred, if not entirely obliterated. It is essentially only the terminology and the mode of thinking involved that makes the subject "geometric".

There are many areas of mathematics where the introduction of geometrical terminology and procedure greatly simplifies both the understanding, and the presentation of some concept or development. This becomes increasingly evident in so much of mathematics that some mathematicians of the second half of the twentieth century feel that perhaps the best way to describe geometry today is not as some separate and prescribed body of knowledge but as a **point of view** — a particular way of looking at a subject. Not only is the language of geometry often much simpler and more elegant that the language of algebra and analysis, but it is at times possible to carry through rigorous trains of reasoning in geometrical terms without translating them into algebra or analysis. There results a considerable economy both in thought and in communication of thought. Moreover, and perhaps most important, the suggested geometrical imagery frequently leads to further results and studies, thus furnishing us with a powerful tool of inductive or creative reasoning. A great deal of modern analysis becomes singularly compact and unified through the employment of geometrical language and imagery.

VOCABULARY EXERCISES

I. *Consult the dictionary and translate the following phrases.*

1. **derivative** of an element, absolute **derivative**, approximate **derivative**, directional **derivative**, left-hand, right-hand **derivative**, total **derivative**;
2. **emergency** control, **emergency** signal, in case of **emergency**, the state of **emergency**, **emergency** door, **emergency** landing, **emergency measures**, **emergency** boat, **emergency** service, **emergency** work, on an **emergency** basis, **emergence** probability;
3. a **succession** of, in **sucession**, by **sucession**, **sucessive** carriers, **successive** derivatives, method of **successive** approximations, a **successor**, a worthy **successor**, **succeeding** ages, the **succeeding** period, to be a **success**, to turn out a **success**, among **successes**, **success** is never blamed, **successful** results, to be **successful** in doing smth, to be **successful** in everything (nothing), to deal **successfully** with a task, to undergo a test **successfully**;
4. (in)**consistent**, (in)**consistency**, (in)**consistently**, simple **consistency**, **consistency** of an estimation, **consistent system**, **consistent policy**;
5. **conscience** — **conscienceless**, **conscious** — **subconscious** — **subconsciously**, self-**conscious**, **conscientious** — **conscientiously**;
6. convert — converse — conversely — conversion, **conversion** factor, **conversion** frequency, **conversion** table, code **conversion**, digital **conversion**, analog-to-digital **converter**, binary-to-digital **converter**, data **converter**, pulse **converter**, radix **converter**.

II. *Don't mix these words up! Illustrate their meaning by the sentences of your own.*

to precede	proposition	approach	approximate	convert
to proceed	preposition	reproach	appropriate	converge
sufficient	changeable	conscience	emerge	experiment
efficient	challengeable	conscious	embrace	experience

III. *Give one Russian equivalent of the following groups of words.*

a) to finish — to complete / to fulfil — to achieve — to accomplish / to succeed in — to manage — to cope with / to conceive — to perceive / to cite — to quote / to change — to alter — to convert / to draw — to deduce — to derive — to extract — to infer / to supersede — to replace — to substitute / to be true — to be valid — to hold — to hold true — to be satisfied / to inaugurate — to start — to begin / to imagine — to fancy / to enunciate — to declare — to announce / to attribute to — to ascribe to / to proceed — to continue — to follow / to decrease — to reduce — to diminish / to give — to provide — to supply — to furnish — to yield / to stop — to cease / to challenge — to question — to doubt.

b) effort — attempt — endeavour / citation — quotation / reference — allusion / case — incident — accident — occasion / deduction — derivation — extraction — inference / defect — blemish / start — beginning — outset / change — alteration — conversion.

c) true — genuine / mistaken — erroneous — faulty / conscious — aware — knowing / tacit — unspoken / rough — approximate / practical — empirical — "thumb-rule" — "trial-and-error" methods / unchallengeable — undoubtful — incontestable — unquestionable.

IV. *Translate the following antonyms.*

implicit(ly)	tacit(ly)	consistent (ly)	rational (ly)
explicit(ly)	verbal(ly)	inconsistent (ly)	irrational (ly)
absolute	commensurable	practice	existence
relative	incommensurable	theory	non-existence
satisfaction	blemishes	predecessor	superior
dissatisfaction	merits	successor	inferior

V. *Fill in the blanks with one of the given below words.*

Deductive, to precede, physical counterparts, to be concerned, to reveal, to serve, reasoning, to derive, to deserve, to give rise, to proceed, to trace, to distinguish, to fail.

1. ... by induction is the fundamental method of ... in experimental science, but sometimes it does not guarantee the certainty of the conclusions. The Greeks ... all the mathematical conclusions only by ... 2. Despite the unquestioned influence of Egypt and Babylonia on Greek minds, the mathematics produced by the Greeks differs radically from that which ... it. 3. He ... highest praise for this work. No doubt, it's his masterpiece. 4. Apparently, one may find ... that ... to these mathematical abstractions. 5. The creation of Euclidean geometry ... the power of reason. 6. The deductive reasoning ... as follows. "All good people are honest and if I am good then I must be honest. And if I am not honest I am not then good". 7. Geometry, philosophy, logic and art are all expressions of one type of mind, one outlook on the Universe, and it is

reasonable ... the existence of common characteristics in all these phases of the classic Greek culture. 8. One must ... a rigorous proof from vague pretence of proof. 9. The trial and error methods ... during ages of experience to arrive at the simple mathematical formulae. 10. Elementary mathematics ... with the properties of numbers and of space, in other words, with algebra and geometry. 11. The development of non-Euclidean geometries is of special significance in showing why attempts to prove Euclid's parallel postulate ... .

VI. *Translate the sentences and analyze the bold-faced words.*

Verb to concern — to be concerned with. *Noun* concern. *Participles* concerned, concerning. *Preposition* concerning.

1. In the modern approach to elementary mathematics, we distinguish from the outset between numerals and numbers. One aspect of the number study is **concerned** with what is called "properties of numbers", the other deals with "number system". Geometry — the study of forms — was the special **concern** of the classical Greeks. 2. **Concerning** numeration and number, it may be conjectured that the classical Greeks were hardly **concerned** with numeration — if, indeed, they were interested in it at all. Their minds apparently were not inclined toward the mechanical aspect of elementary mathematics but rather were fascinated by the properties of numbers; their main **concern** were positive integers. For the mathematician the postulates of geometry today are mere hypotheses whose physical truth or falsity need not **concern** him. 3. Historically, **concern** with computation preceded by many centuries **concern** with properties of numbers. In turn, the latter preceded **concern** with number systems by almost two thousand years. 4. Writers of antiquity who **concerned** themselves with geometry unanimously agreed upon the Nile Valley of ancient Egypt as the place where subconscious geometry became scientific geometry. 5. Our chief sources of information **concerning** ancient Egyptian geometry are the Moscow and Rhind papiri, mathematical texts containing 25 and 85 problems respectively and dating from approximately 1850 B. C. and 1650 B. C. 6. Greek Mathematics **concerning** mostly with plane geometry buried algebra in geometry and developed the so-called "geometrical algebra". 7. Early Greek geometers developed notions **concerning** infinitesimals and limit and summation processes. 8. Gaspard Monge's projective geometry **concerning** a way of representing and analyzing three-dimensional projects by means of their projections on certain planes, had its origin in the design of fortifications. 9. Questions **concerning** Euclid's fifth postulate brought about startling development in mathematics. 10. Euclid did not consider the possibility **concerning** other postulates to define other geometries. 11. Where mathematics is **concerned**, no doubt I am only a layman. 12. As far as the membership was **concerned**, the Pythagoreans brotherhood had only male members, despite a doubtful anecdote **concerning** Pythagoras's beautiful girl friend admitted as a member.

LAB. PRACTICE

Grammar Rules Patterns

I. *Refer the following sentences to the Past. Mind the Passive Voice.*

Model. The Greeks **apply** deduction as a principal means of scientific reasoning.

The Greeks **applied** deduction as a principal means of scientific reasoning.

1. The primary purpose of the lesson **is to deal** with Geometry as a science. 2. The students **conceive** the concept of a geometry as a logical system based upon postulation thinking. 3. The lesson **introduces** students to a modern rigorous approach to the foundations of Geometry. 4. The students **trace** and **appreciate** the historical evolution of geometrical concepts. 5. The subject matter of the lesson **reveals** the relation of Euclidean Geometry to the space we live in. 6. The lesson **provides** an understanding of a formal and valid proof. 7. One **can say** that Geometry in fact consists of several geometries. 8. The students **infer** how Euclidean Geometry **is related** to many other geometries. 9. Mathematicians **claim** that it **becomes** a relatively simple matter to invent new geometries. 10. The students **do not fail** to discover the hierarchy of geometries and abstract spaces. 11. **There is** no logical reason why one geometry **should be preferred** to another. 12. Mathematicians **do not arrive** at any contradictions and inconsistencies in the alternative geometries. 13. The study **yields** a good understanding of a modern view of geometry. 14. Geometrical language, imagery and the mode of thinking **makes** any subject under study "geometric". 15. Geometry as a science never **exhausts** itself.

II. *Give short answers.*

Model. Did **the Greeks'** predecessors create the theory of a logical system structure?
No, they did not. But **the Greeks** did.

1. Did **the Greeks'** predecessors develop the theory of definition? (No,...) (the Greeks) 2. Was a definition for **the Babylonians** a phrase revealing a thing's essence? (No,...) (the Greeks) 3. Did the definitions enable **Egyptians** to associate names with elements and relations? (No,...) (the Greeks) 4. Should definitions be concisely stated? (Yes,...) (According to the Greeks' demand) 5. Must definitions give distinguishing characteristics of the element or relation involved? (Yes,...) 6. Did the definition of the Greeks contain any new element? (No,...) 7. Was a definition of a new term accepted if it comprised undefined terms? (No,...) 8. Did the Greeks' definitions assert the existence of the thing definied? (Yes,...) 9. Did the Babilonians develop a rigorous mathematical (as opposed to the dictionary) type of definitions? (No,...) (the Greeks) 10. Did Euclid preface his "Elements" with a list of definitions? (Yes,...) 11. Did Euclid actually use the definitions further in his work? (No,...) 12. Were Euclid's definitions criticized? (Yes,...) 13. Did mathematicians manage to improve Euclid's definitions? (No,...) 14. Can any mathematician give an explicit definition of every basic term of a logical system? (No,...) 15. Do continual definitions lead to a "vicious circle"? (Yes,...) 16. Do mathematicians nowadays accept basic terms as underfined? (Yes,...) 17. Do axioms specify basic terms in modern treatment? (Yes,...) 18. Do scientists interpret the undefined terms in any concrete way they please? (Yes,...) 19. Did the Greeks avoid the circularity of definitions by accepting undefined basic terms? (No,...) (modern scientists) 20. Did the Babylonians' empirical procedures have anything in common with the Greeks' material axiomatic method? (No,...) (Contemporary formal axiomatics)

III. *Give full answers.*

Model. Where **did** modern mathematics **begin**? (in Ancient Greece)
Modern mathematics **began** in Ancient Greece.

1. What mathematicians were the most distinguished in early Greek mathematics? (Undoubtedly Pythagoras and his followers) 2. Whose pupil was Pythagoras? (Thales', the founder of deductive methods) 3. What doctrines did the Pythagoreans teach? (A mixture of morality, astrology, music and genuine mathematics) 4. What ancient cultures did the Pythagoreans inherit mathematical knowledge from? (Babylonians, Egyptians and Indians) 5. What contribution did the Pythagoreans make to mathematics? (Abstracted the concepts of **Number**, Geometric **Form** and **Figure**; Developed and applied deductive reasoning) 6. What were their profoundest discoveries? (Pythagorean theorem, Incommensurables) 7. Who proved the general theorem? (Pythagoras is credited with; the proof is attributed to) 8. How did the Pythagoreans prove their great theorem? (Under the unchallenged assumption that "Numbers rule the Universe") 9 .What was the Pythagoreans' contribution to number theory? (The creation of the classical theory of natural numbers) 10. How did the Pythagoreans treat natural nymbers? (The essence of all the existing things) 11. How did the Pythagoreans come to recognize incommensurable quantities? (Failed to find a rational number for $\sqrt{2}$) 12. How did they regard the discovery of incommensurables? (Logical scandal. Crisis in the foundations of mathematics) 13. What did they call numbers $\sqrt{2}$, $\sqrt{3}$, etc.? (Irrationals) 14. How did they deal with irrationals? (By approximating them by means of ratios) 15. What did the discovery of incommensurables result in? (A need to establish a new theory of proportions independent of commensurability) 16. Who developed a theory of incommensurables? (Eudoxus) 17. Who made this theory popular? (Euclid) 18. How did mathematicians learn about the theory? (Euclid presented it in geometrical form in his "Elements") 19. How can Eudoxus' theory be estimated? (A masterpiece of Greek mathematics) 20. When did Eudoxus's theory become fully appreciated? (In the late nineteenth century) 21. Who constructed the first truly rigorous theory of irrational numbers? (Dedikind, Cantor, Weierstrass) 22. How did Greek mathematics benefit from its first classical crisis in the foundations of mathematics? (Ultimate influence was beneficial and considerable) 23. What did the Pythagoreans' discovery of "incommensurable" quantities bring about? (Ultimately dispelled the belief that the Universe was built on natural numbers)

IV. *Ask questions as in the model using the question words suggested.*

Model. The word "geometry" was derived from the Greek words for "earth measure". (Where ... from?)
Where was the word "geometry" derived from?

1. The ancients believed that the earth was flat (What?) 2. The early geometers dealt with measurements of line segments, angles, and other figures in a plane. (What ... with?) 3. Gradually the meaning of "geometry" was extended to the ordinary space of solids. (How?) 4. Greek mathematicians considered geometry as a logical system. (Who?) 5. They assumed certain properties and try to deduce other properties from these assumptions. (How?) 6. During the last century geometry was still further extended to include the study of abstract spaces.

(Why?) 7. Nowadays Geometry has to be defined in an entirely new way. (How?) 8. In contemporary science geometrical imagery (points, lines, planes, etc.) may be represented in many ways. (What?) 9. Any modern geometric discourse starts with a list of undefined terms and relations. (What... with?) 10. The set of relations to which the points are subjected is called the structure of the space. 11. Geometry today is the theory of any space structure. (What?) 12. Geometry multiplied from one to many. (How?) 13. Some very general geometries came into being. (What?) 14. Each geometry has its own underlying controlling transformation group. (What?) 15. New geometries find invaluable application in the modern development of analysis. (Where?)

V. *Make the sentences negative.*

Model. The early Greeks **dealt with** numeration systems and counting. (practically)
The early Greeks **did not practically deal with** numeration systems and counting.

1. The Pythagoreans inherited more superior Babylonian positional numeration system. (in spite of infinite contact) 2. Their minds were inclined toward the elementary mechanical aspect of mathematics. (apparently) 3. The Greeks' system of counting was simple. (obviously) 4. It was positional system based on 10 (non-positional without place value or symbol for zero) 5. They used special symbols for numbers. (letters of their alphabet for numbers) 6. The Pythagoreans investigated and discovered many operations on real numbers. (properties of natural numbers) 7. They represented numbers as algebraic structures. (geometric patterns) 8. They developed algebra. (buried algebra in geometry) 9. They created a grand structure of number-systems similar to that for Geometry. (since their general outlook was geometric rather than arithmetical) 10. The principal Greek contributions were number systems. (since they were fascinated by the **properties** and not the **operations** on numbers) 11. There still exists only the Greeks' classical number theory. (modern number theory) 12. Modern approach is definitely oriented toward the structural properties of numeration systems. (number systems, i.e., the properties of operations on numbers)

VI. *Make the sentences impersonal using the noun-substitutes and modal verbs.*

Model. **Mathematicians** claim that mathematics is man's greatest intellectual achievement. (accomplishment)
One can claim that mathematics is man's greatest intellectual achievement. (accomplishment).

1. **Historians** assert that Geometry — the study of forms — was the special concern of the classical Greeks. 2. **The scientists** failed to trace the exact date of the emergence of abstract notions of number and geometric figure. 3. **Tradition** credits the Pythagoreans with this greatest contribution. 4. **It is necessary** to emphasize that the appreciation of number as an abstract idea was one of the major advances in the history of thought. 5. **It is true** to say that pure abstract geometric form is the common property of all solids. 6. **Scientists do not** perform any experiments with pure forms. 7. **Mathematicians** deal exclusively with abstract pure forms in mathematics. 8. **The Greeks** introduced and applied extensively the pure deductive reasoning with abstract forms. 9. By means of deductive reasoning **geometers** reveal the essential properties and the relation-

ships of geometric forms. 10. By abstracting the concept of a geometric figure **the Greeks** achieved the greatest level of generalization then known. 11. Nowadays **mathematicians** gain a new understanding of the abstract foundations of mathematics.

Model. Objects have both **essential** and accidental properties.
the former — According to the Greeks only **the former** should enter.
the latter into definition of an object.

1. The classical Greeks neglected **experimentation** and **practical applications.** According to their principles ... was mechanical, and ... was vulgar. 2. Current mathematics is a method of inquiry known as postulation thinking and **a field for creative endeavour.** ... constitutes responces to purely intellectual challenges. 3. The Greeks had only **one space** and **one geometry**; these were absolute concepts. Today ... is the set of objects together with a set of relations in which the objects are involved, and ... is the theory of such a space. 4. The modern concept of **Geometry** is so embracive that the boundary lines between ... and the other areas of mathematics became very obliterated. 5. Some mathematicians claim that geometry is not a separate mathematical discipline but *a* **point of view.** ... implies a particular way of looking at a subject. 6. **The concept of axiomatic method** originated in Ancient Greece in the fifth century B. C. **The modern form of axiomatic method** was in a stage of evolution for more than half of the XIX century. The application of ... was embodied in Euclid's "Elements", while ... was exemplified by Hilbert's geometry. 7. Without doubt the most outstanding contribution of the early Greeks was **the formulation of the pattern of axiomatics.** Unlike Hilbert's pattern of "formal axiomatics" ... is usually referred to as 'material axiomatics". 8. **The modern form of axiomatic method** and **a very high level of abstraction** characterize today's mathematics. ... is the unifying principle for all the branches of mathematics, ... implies a higher order of abstraction compared to Euclid's; as the objects, relations and operations are already themselves abstractions. 9. From the axiomatic point of view mathematics is a storehouse of **abstract forms** and mathematical structures. Most of ... **had** originally a very definite intuitive content. 10. The interdependence in mathematics and **the internationalism of its appeal** are displayed by simultaneous discoveries in mathematics. The evidence of ... is the discovery of non-Euclidean geometry by a German, a Russian and a Hungarian, who had no connections with or knowledge of each other.

VII. *Make the sentences more emphatic. Turn active into Passive, if necessary.*

Model.

It	is ... that was ... who	The only numbers accepted by the Greeks were the **natural numbers.** It was **the natural numbers** that were the only numbers accepted by the Greeks.

1. **The Greeks** first appreciated the power of mathematical reasoning. 2. The Greeks gave **mathematics** a major place in their civilization. 3. The Greeks initiated **patterns of thought that are still basic in our civilization.** 4. The Greeks converted mathematics into **abstract, deductive and axiomatic system of thought.** 5. In constructing methods of proof, mathematicians employ **a high order of intuition, imagination and ingenuity.** 6. Though Thales of M. proved some geometric theorems deducti-

vely, **the Pythagoreans** applied this process of reasoning exclusively and developed it further. 7. Mathematical theory emerged and evolved first **in the mathematics of the early Greeks**. 8. **Euclid's "Elements" were** the first successful attempt to build all Geometry based on postulation thinking. 9. Euclid based his development of Geometry **on a logical system.** 10. Despite some shortcomings **the "Elements"** are a work of genius. 11. **The Ancient Greece** created the intellectual miracle of a logical system. 12. **Euclid's masterpiece** was the first magnificent and epoch-making application of the axiomatic method. 13. **Mathematicians** study Euclid's "Elements" to master the art of rigorous geometric reasoning. 14. The subject of Geometry was once almost synonymous **with the name of Euclid.** 15. Euclidean Geometry was **the only Geometry** for more than two thousand years. It is not any more. 16. "Elements" are no longer all Geometry, but **this masterwork** is the logical ideal of all science. 17. **"Elements"** is the most durable and influential textbook in the history of mathematics.

Model. Modern mathematics **ignores** the distinction between postulates and axioms.
Modern mathematics **does ignore** this distinction.

1. Gauss **conceived** the idea of non-Euclidean Geometry long before other creators of the subject. The term is due to him. 2. Gauss **claimed** that there exists a closed and consistent Geometry in which the Euclidean 5th postulate does not hold. 3. Euclid himself **realized** the impossibility of deriving the parallel postulate from the rest of the axioms and postulates. 4. Many mathematicians after Euclid **attempted** to prove the parallel postulate by an indirect method (i.e., reductio ad absurdum). 5. Mathematicians **tried** to construct a geometry in which the negation (the converse) of the parallel postulate holds. 6. The futile and fruitless efforts to produce a proof of the parallel postulate **led** to the idea of a geometry with more than one parallel. 7. The discoverers of non-Euclidean Geometry **wanted** to show that the 5th postulate is, in fact, deducible. 8. In this they failed. But they **succeeded** in finding a new world, a new consistent geometry with infinitely many parallel lines. 9. N. Lobachevsky **published** the first account dealing with non-Euclidean geometry and created the subject concerned. 10. Priority arguments are very important in science and that is why we **honour** N. Lobachevsky as the creator of non-Euclidean geometry. 11. Gauss and Bolyai independently **drew** the same conclusions from the impossibility of proving the parallel postulate. 12. Their contemporaries **paid** almost no attention at first to their novel and grand ideas. 13. New non-Euclidean geometry **lacked** intuitive appeal and so was almost ignored. 14. Euclidean Geometry **rooted** so firmly in thought that their contemporaries hardly recognized and appreciated the latter. 15. A few decades **passed** before mathematicians took notice of this grand discovery. 16. The new geometry **gained** intuitive appeal as a result of the "models" for non-Euclidean Geometry constructed by F. Klein, Poincare and Hilbert.

VIII. *Translate the sentences identifying the grammar rules patterns involved.*

1. **One knows** that Geometry started far back in antiquity from some very modest beginnings and gradually grew to its present enormous size. 2. **People are aware** that the nature, or inherent character of the subject had different connotations at different periods of its development. 3. Many observations in the daily life of early man **did lead** to the geometric

concepts of curves, surfaces and solids. 4. **It was** the construction of houses **that** suggested the notion of vertical, of parallel, and of perpendicular. **One can multiply** such examples almost indefinitely. 5. While **we must not be certain**, it **does seem safe** to assume that scientific geometry arose from practical necessity. 6. **It was the Greeks who** transformed Geometry into something vastly different from the set of empirical conclusions worked out by the ancient Egyptians and Babylonians. 7. Induction and deduction are different ways of reasoning. **The former** is mainly used in experimental science, while **the latter** is the major method of reasoning in mathematics. 8. **People can hardly overemphasize** the importance of Euclid's "Elements". 9. The theory of parallels **did occupy** geometers for over two thousand years. 10. Euclidean and non-Euclidean geometries are alternative ways geometrically to describe physical space, both **the former** and **the latter** are consistent.

REVISION

(Similar revision of all the Grammar rules studied should be done at the end of each term)

Summarize all the Grammar rules and the verb "to do" functions so far studied.

I. **Imperative.**

Suppose (Let us suppose) we have a theorem. Prove it deductively. Let her (him, them) do it.

II. **Дедуктивно и строго доказывают (должны доказывать) теоремы математики.**

Indefinite Tense-Aspect forms

Pressent
- Mathematicians **prove (must prove)** theorems deductively and rigorously.
- Theorems **are proved (must be proved)** deductively and rigorously.

Past
- Mathematicians **proved (were to prove, had to prove)** theorems deductively.
- Theorems **were proved (were to be proved, had to be proved)** deductively.

A deductive proof **is (was)** much **spoken and written about.** A rigorous and elegant deductive proof **is (was) looked** at with admiration.

III. **Questions. Who** proves (must prove) theorems? Who proved (was to prove, had to prove) theorems? What do (did) mathematicians do? How do (did) mathematicians prove theorems? Mathematicians prove(d) theorems, don't (didn't) they? Do (Did) mathematicians prove theorems deductively or inductively? What is a deductive proof? Is it difficult to prove theorems deductively?

IV. **Negations. Don't prove** theorems that way. **Don't let him** (her, them) prove theorems that way. **Let us not prove** theorems that way. Mathematicians **do not (did not) prove** theorems that way. **No** mathematician **proves** theorems that way. **Don't (Didn't)** mathematicians **prove** theorems deductively? Mathematicians prove(d) **no** theorem that way. Mathematicians prove(d) **nothing** that way. Mathematicians prove(d) theorems that way **nowhere.**

V. **В математике много думают, рассуждают, доказывают, обосновывают.**

There is much thinking, reasoning, proving and justifying in maths. Is there? There is **no** arguing (**not any** argument) in this theory. **There exists** (**emerged**) a new proof of this theorem. **Does there exist**...? There **does not exist**... **Did there emerge**...? There **did not emerge** a new proof of this theorem.

VI. **Impers. (Пред)полагается (можно, нельзя, вряд ли следует ...)**

It is (pre)supposed that mathematicians prove(d) theorems that way.

One (does not) **suppose(s)** (can hardly suppose)...

We (you, they) **must (not) suppose**...

People should (not) suppose...

VII. **Emph. Именно (как раз) математики (в самом деле, ведь) до- доказывают теоремы.**

It is (was) mathematicians **who** prove(d) theorems. **It is (was)** deductively that mathematicians prove(d) theorems. **Do prove** theorems deductively! Mathematicians **do (did) prove** theorems deductively. **Whatever (Whichever)** Euclid's proof you take, it is deductive. **The earlier** you master the procedure of a deductive proof, **the sooner** you appreciate mathematical rigour.

VIII. **Noun Subst.** The proof(s) by deduction is (are) much more rigorous than **that of (those of)** by induction. **Deduction** and **rigour** are essentials of a mathematical proof. **The former** and **the latter** are essentials of a mathematical proof. These proofs are valid but try to establish more rigorous **ones.**

IX. **Verb. Subst.** Mathematicians prove theorems inductively rather rarely but physicists **do** it regularly. Mathematicians prove what they **do** (=prove) deductively and rigorously.

X. **The verb "to do" functions.**

1. These students **do maths.** 2. What **do** these student **do?** 3. They **do prove** theorems. **Do prove** this theorem deductively! 4. They **do not prove** theorems but we **do**. They prove what they **do** deductively.

CONVERSATIONAL EXERCISES

I. *Use the following sentences in your oral description of the ancient practical (empirical, experimental) geometry.*

1. The word "geometry" refers to the branch of mathematics. 2. Geometry arose from practical necessities and appeared several thousand years before our era. 3. The need to bound land led to the notion of simple geometric concepts. 4. The notions of vertical, of parallel, and of perpendicular were suggested by the construction of buildings. 5. Many observations of the physical forms and shapes in the daily life led to the conception of curves, surfaces, and solids. 6. Empirical geometry was developed in certain areas of the ancient Orient — **Egypt, Mesopotamia, India, China**. 7. Geometry appeared as a science to assist surveying, agricultural and engineering pursuits. 8. The river basins (the Nile, the Tigris and Euphrates, the Indus, and Ganges, the Yangtze) cradled advanced forms of society known for their engineering skill. 9. Surveying and engineering projects required the creation of much practical geometry. 10. Trial-and-error methods and empirical procedures were the tools and means of discovery. 11. Geometry was a collection of laboratory results, some correct, others only approximate, concerning areas and volumes. 12. The earliest existing records of man's geometrical activity were found

in **Mesopotamia** (=**Babylonia** once situated on the territory of modern Iraq) and dated from Sumerian times about **3000 B. C.** 13. The ancient Babylonians used the imperishable baked-clay tablets to record their results. 14. The Babylonians of 2000—1600 B. C. were familiar with the general rules for computing land areas and volumes of solids. 15. The Pythagorean theorem was also known to them (without any proof, of course) as far back as 2000 B. C.

II. *Express your appreciation of the ancient Egyptian mathematics.*

Our chief sources of information concerning ancient **Egyptian** geometry are the Moscow (1850 B. C.) and Rhind (1650 B. C.) papiri. A Scottish scholar and antiquary, **A. M. Rhind** discovered in 1858 in Egypt and bought an ancient Egyptian papyrus found in some ruins in Thebets. The Rhind papyrus is a collection of arithmetical, geometrical and miscellaneous problems, including some area and volume applications. The papyrus is a copy of 1650 B. C. of much earlier writings of the latter part of the 1900 B. C. The entire work emphasizes the two concepts that particularly characterize the mathematics of the early Egyptians: 1) the consistent use of additive procedures and 2) computations with fractions. Most problems are of practical nature. Some problem may present a challenge even to the modern student; e. g., "Find the volume of a cylindrical granary of diameter 9 and height 10 cubets."

The Moscow papyrus also referred to as the Golenischev papyrus for the man who owned it before its acquisition by the Moscow Museum of Fine Art, was probably written about 1850 B. C. Although it contains only 25 problems, it is similar to the Rhind Papyrus. This work shows that the Egyptians were familiar with the formula for the area of a hemisphere and the correct formula for the volume of a truncated square pyramid' $V = \frac{a^2 + ab + b^2}{3} h$. The solution is expressed only in terms of the necessary computational steps for the given numerical values: height of 6 and the bases of sides 4 and 2. There are various conjectures about how the Egyptians could develop this procedure, but the papyrus offers no help. This formula is often referred to as the Egyptians' "greatest pyramid". A challenging and exciting discovery of more than a century ago of these two mathematical texts gives fascinating exercise to the student of mathematics, both modern and ancient. Of the 110 problems in the papiri 26 concern the computation of land areas and volumes. The ancient Egyptians recorded their work on stone and papyrus resisting the ages because of Egypt's dry climate. There is no documentary evidence that the ancient Egyptians were aware of the Pythagorean theorem. Nevertheless, early Egyptian surveyors realized that a triangle with sides of lengths 3, 4 and 5 units is a right triangle. Egyptian geometry arose from necessity. The annual inundation of the Nile Valley forced the Egyptians to develop some systems for redetermining land markings; in fact, the word "surveying" means "measurement of the earth". The Babylonians likewise faced an urgent need for mathematics in the construction of the great engineering structures (marsh drainage, irrigation and flood control) for which they were famous. Similar undertakings and geometrical accomplishments occurred in India and China. The ancient Indians and Chinese, however, used very perishable writing materials (bark bast and bamboo) and due to the lack of primary sources we know next to nothing about mathematics in ancient India and China. To a great extent the earliest geometries were little more than a practically workable empiricism — a collection of rule-of-thumb procedures and trial-and-error methods that gave results of sufficient acceptability for the simple needs of those early civilizations. In spite of the empirical nature of ancient oriental geometry, with its complete neglect of proof and the lack of difference between exact and approximate truth, mathematicians are nevertheless struck by the extent and the diversity of the problems so successfully attacked. One ought not to underestimate the contributions of these ancient civilizations to the development of Geometry.

III. *Substantiate and supply the main ideas (claims) of the text with your examples, illustrations and proofs.*

The ability to think out answers to new problems is a defining characteristic of human beings. People are not always reasonable, that's for sure. But people do reason. They do try to figure things out. **Reasoning is thinking** in which one step leads to the next step and the whole process draws finally to a conclusion. **Human reasoning** is, in fact, a step-by-step process of reaching conclusions by a connected logical train of thought. Reasoning can follow many routes among which the most commonly used are: reasoning by **analogy, induction** and **deduction. Deductive reasoning** has many advantages, and it is this kind of thinking that reaches a particularly refined level in mathematics. Its precision in mathematics depends upon several things — the unambiguous way in which terms are defined, the restraint with which the definitions are obeyed, and the care with which all the rules of procedure are set out and made clear. This kind of thinking is **logical thinking**; and logic itself can be defined as the systematic study of the conditions and procedures which permit **valid inference** — that is to say, which permit one to start with one or more statements (or propositions) and **derive** from these or **infer** from these, one or more new statements which are **valid** in the sense that they **are justified** by, and are in strict fact consequences of, the starting (initial) statements. Thus what is important about a logical inference is not its **truth** but rather its **validity.** A logical conclusion can properly merit the adjectives **correct**, **sound**, or **accurate** but not **true.**

The type of reasoning used in geometry and algebra was developed in classical (Aristotelian) logic. It involves three steps. **First**, you start out with a **set of premises** (statements of postulates) which are to be accepted without question. In modern mathematics these initial premises are recognized as pure assumptions without further discussion. **The second step** involves the application, to the starting premises, of rules of logic. The core of deductive logic is the so-called **syllogism**, which consists of a **major premise**, a **minor premise** and a **conclusion.** In mathematical terms a syllogism runs like:

Every M is P	Either A or B or C
S is M	Not A., Not B
Therefore S is P	Therefore C

But the most important aspect — **the third** and culminating **step** of the classical procedure that one first learns in geometry — is precisely that **there is a definite conclusion** and one can be sure that the conclusion is, under the given premises, **completely valid,** and that its opposite is completely false. This type of reasoning produces nothing more than an "if-then" kind of result. That is, **if the assumption hold, than the result is valid.** Indeed it is then unquestionably valid, and its opposite is unquestionably false. This process enables one confidently to decide that certain statements, on the basis of the given start, are **true** or **false.** To certain questions one can confidently respond "yes" or "no". But this process in its classical form has an absolutely **devastating limitation.** The trouble is that there are so very many interesting and important questions to which this logical method does not apply at all. There are many questions which cannot possibly be answered with either "yes" or "no". There are so very many cases in which our starting knowledge is simply not sufficiently extensive, or sufficiently reliable, to lead to completely negative "no". The answer should be "a definite may be".

There are other limitations of classical logic. One of the most fundamental and exciting intellectual feats of this century is the discovery by **Kurt Gödel** that logic has some built-in limitations which were previously unsuspected. When one wishes to develop any logical system (say the logical basis for arithmetic, or the logical basis for some field of physics) he has to start with a set of assumptions or postulatas. It is naturally a matter of the greatest concern to be sure that the set of postulates chosen is internally **consistent** so that they do not in some complicated and concealed way contradict each other. Gödel proved the absolutely stunning result (stunning in all senses) that it is **impossibe** — actually impossibe, not just unreasonably difficult — to prove the consistency of any set of postulates which is rich enough in content, that is, in the sense of leading to a useful body of results. The question, "Is there an inner flaw in this logical system?" is a question which is **unanswerable!** The business of classical logic is to put all statements into ony two categories: **true** (1) and **false** (0). A third, and embarrassingly large, category contains all the statements which classical logic cannot handle at all, i. e., statements that are presumably neither completely true nor completely false. Gödel also proved that this is not so! He demonstrated that it is always possible with a logical system, to ask questions which are **undecidable!** The limitation to only two truth values (true or false) does in fact keep most of us, the most of the time, from getting a great deal of practical help from logic. It is at just this point that logic of probability comes to our rescue.

IV. *Reproduce the text.*

The Pythagorean Theorem

"The Pythagorean" theorem is one of the most important propositions in the entire realm of geometry. Despite the strong Greek tradition that associates the name of Pythagoras with the statement that "the square on the hypothenuse of the right-angled triangle is equal to the square on the sides containing the right angle", there is no doublt that this result was known prior to the time of Pythagoras.

It is possible that Pythagoras gave the proof of the theorem based on the proportionality of similar figures. With the later realization that all lines are not necessarily commensurable, this proof became invalid. Thus, at the time of Euclid's "Elements" there was no need for a more adequate proof. Euclid's Proposition 1,47 is the Pythagorean theorem, with a proof universally credited to Euclid himself. Proclus's speculation was simply that Euclid rewrote the proof in order that he might put the proposition in his first book to complete it. There is also considerable evidence that the first book was written to lead to the climax of this theorem and its converse.

In 1907 L. S. Loomis published his book "The Pythagorean Proposition", a work that contained **370 proofs** of this theorem. Probably no other theorem in mathematics can be demonstrated by such a wide variety of algebraic and geometric proofs. The Pythagorean theorem and the proof are so important in mathematics that Loomis writes in his book: "I noticed two or three American texts on Geometry in which Euclid's proof of the Pythagorean theorem does not appear. I suppose the author wishes to show his originality or independence — possibly up-to-dateness. He shows something else. The leaving out of Euclid's proof is like the play of "Hamlet" with Hamlet left out".

V. *Choose the proper alternative or give your own answers.*

Is Pythagorean theorem general or special?

1. General. 2. Special. 3. Fundamental.

What role do fundamental theorems play in mathematical reasoning, in proofs, justification and in science?

1) every fundamental theorem is a landmark in the history of maths. 2) they are precise and concise arguments. 3) they are convenient shortcuts to proofs. 4) their application is the best justification. 5) they serve as points of reference in maths. 6) they are the main guiding threads in scientific theories.

Why do mathematicans re-prove fundamental theorems. (e. g., Pythagorean theorem)?

1) they enjoy doing it. 2) it's their hobby. 3) re-proving theorems is mental gymnastics. 4) it is the "food" for the mind. 5) Euclid gave the proof of a special case of the Pythagorean property. 6) it's simple to give a proof of this theorem. 7) the first proof is, as a rule, not rigorous and elegant. 8) they want to become more famous. 9) to display their ingenuity. 10) to broaden the range and scope of the theory, where it was originally proved.

Every highschool leaver remembers the Pythagorean theorem for the rest of his life. Why is it unforgettable?

1) because of the legendary fame of its creator — Pythagoras. 2) because according to the legend for the proof Pythagoras sacrificed 100 oxen to the Gods. 3) due to the mastery of highschool teachers' presentation of the theorem. 4) thanks to the simplicity of its proof. 5) because there exist too many proofs. 6) Geometry begins with this theorem. 7) the theorem runs "like golden thread" throughout mathematical history. 8) because of the beauty and elegance of its proof. 9) the theorem is an obvious consequent of lots of other theorems. 10) it holds for all right triangles and for all Pythagorean triples (=a set of three positive whole numbers x, y and z such that $x^2+y^2=z^2$, e. g., 3, 4, 5 and 5, 12, 13). 11) the theorem leads directly to the famous Fermat's theorem $x^n+y^n=z^n$.

VI. *Suppose you are to prove the Pythagorean theorem. What proof (geometric, algebraic, etc.) do you prefer? Study the models of proofs in books devoted to the Pythagorean theorem available in the library, choose one up to your liking and demonstrate it in class, expressing all the formalized statements of the proof in words.*

VII. *Translate the text into English.*

Коренное преобразование математики по традиции единодушно приписывают Пифагору. Пифагору принадлежит первое построение геометрии как дедуктивной науки. К сожалению, до нас не дошли не только открывки сочинений Пифагора, но даже их переложение другими авторами. Мы мало знаем о самом Пифагоре, личность которого уже в древности стала полулегендарной. Он основал знаменитый пифагорейский союз, который преследовал не только научные, но и религиозно-этические и политические цели. Деятельность союза была окружена тайной, и все научные открытия, сделанные пифагорейцами, приписывались самому Пифагору. В начале V в. до н. э. после неудачного выступления на политической арене пифагорейцы были изгнаны из городов Южной Италии, и союз прекратил свое существование. Однако и после этого в городах Великой Греции встречались замечательные ученые, которые называли себя пифагорейцами. Что касается творчества ранних пифагорейцев (до разгрома союза), то в настоящее время невозможно отделить сделанное самим Пифагором от работ его учеников. Поэтому обычно говорят о математике пифагорейцев. Пифагорейцы занимались астрономией, геометрией, гармонией (теорией музыки) и арифметикой (теорией чисел). В их школе возникло представление о шарообразности Земли и существовании множественности миров.

Мы не знаем, какие геометрические предложения пифагорейцы выбрали в качестве исходных и насколько велика была эта первая система аксиом. Содержание их геометрии сводилось в основном к планиметрии прямолинейных фигур (изучались свойства треугольников, прямоугольников, параллелограммов, сравнивались их площади и т. д.). Венчало эту систему доказательство знаменитой «теоремы Пифагора», которая до этого была известна только для частных случаев.

Трудно переоценить значение этой теоремы, обобщение которой и до сих пор лежит в основе определения всех метрических пространств. Об этой части учения пифагорейцев можно судить по первой книге «Начал» Евклида. Первоначально пифагорейцы полагали, что все отрезки соизмеримы, т. е. отношение любых двух отрезков (а значит, и площадей прямолинейных фигур) можно выразить отношением целых чисел; таким образом метрическая геометрия, по их мнению, сводилась к арифметике рациональных чисел. Это привело пифагорейцев к мысли, что «все закономерности мира можно выражать с помощью чисел, что элементы чисел являются элементами всех вещей и что весь мир в целом является гармонией и числом». Отсюда исключительный интерес пифагорейцев к основе основ — арифметике, с помощью которой можно выразить все отношения между вещами и построить модель мира. Открытие несоизмеримых отрезков явилось поворотным пунктом в развитии математики. Оно разрушило раннюю систему пифагорейцев и привело к созданию новых глубоких теорий. Значение этого открытия можно, пожалуй, сравнивать только с открытием неевклидовой геометрии в XIX в. или теории относительности в XX в.

VIII. *Ask question for which the following statements may serve as the answers. Work in pairs.*

1. Precise definitions of the logical system concepts are all important foundation for the whole structure. 2. In giving verbal definitions one has to begin somewhere; it is impossible to define every single term. 3. Axioms are assertions about the undefined and defined terms accepted without proof. 4. Axioms are the sole basis for any conclusion that may be drawn about the concepts under discussion. 5. Mathematicians accepted Euclid's axioms because experience with similar physical figures guaranteed and supported these axioms. 6. The axioms of any branch of mathematics must be consistent with each other, or else only confusion re-

sults. 7. Consistency means also that the axioms must not give rise to theorems contradicting each other. 8. Any interpretation of nature may not only be wrong, but it may also be inconsistent. 9. The axioms should be simple and independent of each other. 10. There are many sources of possible theorems. The subject matter of mathematics itself, experience and scientific problems are by far the most fruitful. 11. Pure chance, quesswork, imagination, intuition and insight of creative genius are valuable sources of possible theorems as well.

IX. *Dispute the following statements.*

1. Knowing **what** to prove precedes knowing **how** to prove it. 2. The mathematician may be convinced of the possibility to prove a certain theorem. 3. But until he can give a deductive proof of this theorem he cannot assert or apply it. 4. The distinction between conviction that a theorem should hold and proof of the theorem is examplified by classical unsolved problems. 5. Mathematicians literally work thousands of years to obtain such proofs. 6. What goes on in the mathematician's mind while he works on the problem we do not know. 7. We can say only that creative ability in mathematics calls for mental qualities of unusual excellence. 8. Mathematics advances by the interplay of many devices and approaches. 9. The proof should be valid, rigorous and elegant. 10. It is easy to establish rigorous proofs in modern mechanics.

X. *Agree with the following statements and develop the ideas further where possible. Begin your answer with:*

It's right. Quite so. I quite agree to it. I cannot agree more. I share this viewpoint. Absolutely correct.

1. Design is not merely accidental in mathematics; it is necessarily present in any logical structure. 2. Euclid produced his geometry only through conscious design. 3. Euclid's "Elements" ranks with the greatest works of all times. 4. It sets the pattern for characterizing abstract mathematical objects by means of axioms and postulates. 5. Euclid's postulates have intuitive appeal because they apply ,at least approximately, to the physical objects identified with points, lines, triangles, circles, etc. 6. Euclid formulated a set of basic postulates and proceeds to use them in giving the proofs of hundreds of important theorems. 7. It is probable that "Elements" is, for the most part, a highly successful compilation and systematic arrangement of earlier writers' works. 8. No doubt Euclid had to supply a number of the proofs and to perfect many others. 9. The chief merit of Euclid's work lies in the skillful selection of the propositions and in their arrangement into a logical system. 10. Euclid's masterpiece serves as a model for all pure mathematical theories.

XI. *Confirm or deny the statements.*

1. The parallel postulate is difficult to justify on intuitive grounds. 2. For many centuries mathematicians tried to derive it from other postulates and convert it into a theorem. 3. They never accomplished this goal; as by-products they proved many interesting results. 4. It was not until the XIX c. that it became apparent why all these attempts had to fail. 5. Their failures did not result from a lack of ingenuity on the part of the mathematicians. 6. The parallel postulate cannot be derived as a consequence of the other postulates. 7. This was demonstrated dramatically with the construction of non-Euclidean geometries in which Euclid's parallel postulate does not hold. 8. The most significant fact is that by

substituting alternative postulates the mathematicians did not arrive at any contradictions or inconsistencies. 9. The other departure from the parallel postulate by Riemann led ultimately to the theory of relativity.

XII. *Agree or disagree with the given statements. You may add whatever you like as well, if you think that the statement is too concise or insufficient.*

1. "Elements" consist of nothing but propositions and proofs. 2. To learn "Elements" is to learn the art of geometry. 3. The Greeks were the first mathematicians who are still "real" to us today. 4. Synthetic approach to Geometry was first employed by Euclid. Metric approach is of ancient origin as well. 5. Greek mathematicians spoke the language which one cannot understand today. 6. One can still believe Plato's statement that "geometry draws soul towards truth". Mathematics is a body of knowledge, but it contains no truths. 7. The method of moving figures and putting one on top of the other to see whether or not they coincide is called superposition. Intuition tells us that it is simple to compare sizes of geometric figures by superposability. 8. Congruence, similarity and equivalence are major themes of Euclidean geometry. 9. Non-Euclidean Geometry creators challenged Euclid. Euclidean Geometry is useless today. 10. Language habits change and die. Basic geometric ideas are always up-to-date.

XIII. *Express consent, doubt or disagreement in responce to the statements describing Euclid's errors (flaws, defects, omissions). Use the following phrases.*

I quite agree that ...	**I can't agree that ...**
I doubt that ...	**Not quite so.**
It's too much to say that ...	**It's not a defect, in my opinion.**

Tacit or unstated assumptions which Euclid introduces in his work by means of diagrams or otherwise.

1. The existence of points and lines. 2. Uniqueness of certain points and lines. 3. The infinitude of a straight line. 4. The continuity of his figures. 5. The existence of order relations on a line. 6. The concepts "inside", "outside", "between", "interior", "exterior". 7. Contraversial method of superposition. 8. The lack of axiom justifying superposability. 9. Euclid's geometry is 1) closed and finite, 2) static.

XIV. *Summarize the topic: Euclid's "Elements" is a work of genius. The following statements may prove helpful.*

Euclid's "Elements" are the first remarkable attempt to build **all** geometry. Euclid succeeded in basing his development of geometry on a system. A logical self-sufficient system must start somewhere. To be precise about what his abstract terms include Euclid begins his logical system with the first principles of definitions, axioms and postulates. Euclid's definitions were rightly criticized. "A pont is that which has no part" is the definition we are not told what a point is but rather what it is not. After centuries of vain effort it was realized that one must give up definitions of the kind attempted by Euclid. These must be a foundation on which to build, i. e., undefined terms. Euclid's fundamental propositions from which further statements follow logically are divided into "postulates" and "axioms". Modern mathematics ignores the distinction between these terms. A derivation of a theorem involves a **proof.** The precise and rigorous sense which the Geeks gave to this word may be understood by studying Euclid's "Elements". This sense is not changed because what constituted a proof for Euclid is still a proof for us. It is to Euclid's "Elements" that mathematicians turn for models of proof. However, intensive and sufficiently exhaustive study of "Elements" revealed some tacit assumptions that converted some of his proofs into invalid demonstrations. Besides, Euclid's first principles are insufficient for the derivation of all the 465 propositions. The fact of the impossibility of deriving the proposition on parallels from the rest of postulates and axioms was clearly recognized by Euclid himself. Despite some shortcomings and the inadequacy of Euclid's definitions, the "Elements" are a work of genius. There is no textbook in the history of mankind which retained a position of prominince for as long time as this work of Euclid. Nowadays highschool geometry is based principally on Euclid's accomplishment. Even the emergence of non-Euclidean geometries did not spoil the "**image**" of Euclid or of his "Elements".

XV. *Translate the text into English.*

Первые математические теории греков, абстрагированные из конкретных задач или из совокупности однотипных задач, создают предпосылки для осознания самостоятельности математики как науки. Это осознание самостоятельности математики как науки в свою очередь создает у античных математиков стремление систематизировать факты математики и логически последовательно изложить ее основы. Геометрия греков — это логическая последовательность теорем и задач на построение, использующая минимум исходных положений. Сочинения, в которых в античности излагаются первые системы математики, называются «Началами». В математической литературе встречаются упоминания о «Началах» Гиппократа Хиосского и других авторов. Однако все эти сочинения забыты и утеряны практически с тех пор, как появляются «Начала» Евклида. Последние получают всеобщее признание как система математических знаний, логическая строгость которой остается непревзойденной в течение более двадцати веков. При написании «Начал» Евклид, по-видимому, стремился изложить основы математики в виде совершенной математической теории, исходящей из минимума исходных положений. В этом смысле «Начала» Евклида являются ранним предшественником современного способа аксиоматического построения математических наук. «Начала» состоят из тринадцати книг, каждая из которых состоит из последовательности теорем. В первой книге вводятся определения, аксиомы и постулаты. Определения — это предложения, с помощью которых автор вводит математические понятия путем их пояснения, например: «точка есть то, что не имеет частей». Эти определения Евклида, в ходе истории критикуются с точки зрения их полноты и логической определенности. Однако другой равноценной или более совершенной системы нет. В наше время при аксиоматическом построении математической теории единственным способом описания объектов этой теории и их свойств является сама система аксиом, а объекты вводятся как первичные неразъясняемые сущности. Что касается определений Евклида, то их следует рассматривать как исторически сложившиеся к его времени абстракции реальных вещей. Это наиболее часто встречающийся в истории способ введения математических определений. Пять аксиом, или общих понятий, у Евклида — это предложения, вводящие отношения равенства или неравенства величин. В число исходных положений «Начал» входят пять постулатов (требований), т. е. утверждений о возможности геометрических построений. С их помощью Евклид обосновывает все геометрические построения и алгоритмические операции. В различных изданиях «Начал», а также переписчиками и комментаторами, система аксиом и постулатов Евклида видоизменяется и дополняется. «Начала» — система основ античной математики. В нее входят: элементарная геометрия, основы теории рациональных чисел, общая теория отношений величин и опирающиеся на нее теория пропорций и теория квадратичных и биквадратичных иррациональностей, элементы алгебры в геометрической форме и метод исчерпывания.

XVI. *Correct the following.*

1. Euclid's Geometry can be mastered in ten easy lessons. 2. Every concept or a term can be defined explicitly without entering upon an endless succession of dèfinitions. 3. Axioms are assertions about undefined terms which we cannot accept without a proof and construction. 4. The use of logic adds new information to the statements of Euclid's postulates. 5. Euclid's system is, most certainly, the only way geometrically to describe physical space. 6. Lobachevsky, Gauss and Bolyai drew different conclusions from the impossibility of proving the parallel postulate. 7. The projections of three-dimensional objects and the "principle of duality" are the main concern of Euclidean geometry.

XVII. The plan of Euclid's Elements proceeds as follows: definitions, axioms, postulates, propositions, construction, proof, conclusion. Euclid's method of proof is strictly deductive, that is, his theorems are proved by several deductive arguments, each employs unquestionable premises and yields an unquestionable conclusion. *Illustrate Euclid's procedure by proving one of his theorems.*

XVIII. *Suppose you are asked to explain the sense and meaning of a certain approach in mathematics or in science. Read the explanations given in the models. If they seem too concise or insufficient to you, develop the information available further and emphasize the difference (distinction), merits, (dis) advantages of each approach.*

Models. **What does it mean?**

Formal synthetics approach. This phrase is used to name (to describe) the approach to geometry first employed by Euclid and later used by many others. The phrase does not refer to a deductive system as such but rather to such a pure system when it is developed **without the use of numbers**. The former adjective "formal" means official, i. e., the then accepted in science. Contrasted to this approach are "analytic" and "metric".

Empirical approach means the formulation of conclusions based upon experience or observation of special cases, coincidences, good quessing and flashes of intuition. No real understanding is involved, and the logical element does not appear. The rule-of-thumb procedures and trial-and-error methods are employed. The results may be faulty; guessing supersedes deductive logic; patience replaces brilliance. Empirical conclusions are generalizations based upon limited number of observations and experiments.

Metric approach is that in which one **does employ numbers** and uses them to define real-valued functions (distance and angular measures) in terms of which the concept of congruence and "betweenness" can then be defined. It is of comparatively recent origin (XVII c.). The word "metric" may have other uses in today's mathematics when the numbers are not employed at all.

Formal axiomatic approach is embodied in a discourse conducted by **formal axiomatics methods.** In a formal axiomatic treatment primitive or basic terms are undefined and postulates have nothing to do with "truth" or "self-evidence". This approach implies that one strips the discourse of all concrete content and goes to the abstract development that lies behind any specific application.

Analytic? Genetic? Historical? Pragmatic? Geometrical? Topological? Algebraic? Scientific? Popular?

XIX. *Dispute the following problems trying to prove your point. The following phrases may come in handy.*

I have to admit that ...
I have reason to believe that ...
My point is that ...
It seems reasonable to assert that ...

1. **Every line is a set of points.** Is it an axiom or a definition? Does it imply that points exist? Does the statement say anything about the existence of points? Compare: **Any line is determined by two points.**

2. **There exist at least 3 points on a line. There exist at least one pair of nonintersecting lines.** Are these the statements of the existence theorems? Make a drawing to help with your reasoning.

XX. *Express your personal view about the statements given below and summarize orally the topic: "The discovery of non-Euclidean geometry". Use the introductory phrases.*

As for me ... As concerns ...
As far as I am concerned ...
What I mean to say is ...
Summing up the discussion ...
In conclusion, I may say ...
To summarize the topic ...

1. Fundamental propositions accepted without proof are called postulates and axioms. 2. Mathematicians for years thought of Euclidean geometry as the true and ideal system abstracted from physical points and lines. 3. Its axioms were viewed as necessary and self-evident, probably on the basis of observations of the physical world and geometric diagrams. 4. The source and the role of axioms are viewed differently today. 5. Such phrases as "It is axiomatic that", "It is a fundamental postulate of ..." are often used to signify statements beyond all logical opposition. 6. Within maths this viewpoint concerning the nature of axioms and postulates altered radically. 7. The change was gradual and it accompanied the full understanding of the discovery of a non-Euclidean geometry. 8. The parallel postulate is the most famous postulate in mathematical history. 9. For more than twenty centuries mathematicians struggled for proofs of the parallel postulate. 10. The problem of finding the proof furnished the same challenge to mathematicians as the famous unsolved problems of antiquity. 11. Mathematicians sought to deduce the postulate concerned from the rest of Euclid's postulates. In this they failed; their efforts however, were not in vain. 12. Many "proofs" of the postulate were offered, but each was sooner or later shown to rest upon a tacit assumption equivalent to the postulate itself. 13. It is reasonable to say that Gauss, Lobachevsky and Bolyai independently drew the same conclusion because the time was ripe for the idea of a non-Euclidean geometry. 14. Nevertheless, priority arguments are very important in maths and we honour **Lobachevsky** as the discoverer of a non-Euclidean Geometry. 15. Their contemporaries paid almost no attention to the new challenging ideas. 16. The new Geometry lacked intuitive appeal and so was almost inconceivable. 17. A few decades passed before mathemati-

cians took notice of their work. 18. The new geometry gained recognition due to the "models" constructed by F. Klein and Poincare. 18. The "model" concept itself is quite modern, but it has an extensive historical background. 20. With the creation of purely "artificial" geometry, it became apparent that geometry is not necessarily tied to physical space. 21. Non-Euclidean geometry can be presented in various ways. 22. It became a relatively simple matter to invent new and even bizzare geometries. 23. Non-Euclidean geometry is of great importance in the study of the foundations of maths. 24. Lobachevsky was the father of the most famous revolution in mathematics, but the Tsarist goverment erected no monument to commemorate the revolution for which Lobachevsky was responsible. 25. Instead the goverment relieved him of his job as head of the University of Kasan at the age of fifty-four — this with no explanation whatsoever, to a mathematician so great and well-known throughout the world. Lobachevsky survived this disgrace but his health failed and he became blind.

XXI. *Say it in English.*

Основные свойства пространства были изложены еще в «Началах» Евклида в III в. до н. э. В них дано безупречное для того времени построение геометрии. На протяжении более чем двух тысяч лет «Начала» являлись образцом логической строгости. По ним учились все математики до настоящего времени. Школьная геометрия и теперь в основном излагается по Евклиду. Однако с точки зрения современной математики в «Началах» содержатся существенные недостатки. В частности, Евклид не выделяет основных понятий. Он стремится определить все понятия. Именно поэтому часть определений «Начал» оказалась логически бездействующей. С выходом «Начал» встала проблема пятого постулата — доказать его на основании остальных четырех постулатов и девяти аксиом. Эта проблема, по существу, была поставлена еще до Евклида. Не случайно поэтому постулат о параллельных занимал в списке последнее место и при выводе теорем в первой книге его применение отодвигалось по возможности далее. Евклид стремился сначала обойтись без постулата о параллельных, надеясь доказать его и перевести из постулатов в теоремы.

Проблемой пятого постулата математики занимались более двух тысяч лет. Впервые проблему решил в 1826 г. великий русский математик Н. И. Лобачевский. Он принял вместо пятого постулата допущение, согласно которому на плоскости через точку A, не лежащую на прямой a, можно провести по крайней мере две прямые, не пересекающиеся с a. Дальнейшие рассуждения привели его к новой безупречной геометрической системе, называемой в настоящее время геометрией Лобачевского. Исследования Н. И. Лобачевского способствовали коренной ломке прежних представлений о пространстве. Они показали, что наряду с геометрией Евклида, считавшейся единственной геометрической системой, имеет место другая, логически безупречная система. Эти исследования привели математиков к дальнейшим абстракциям в свойствах геометрических понятий, строгому доказательству непротиворечивости геометрии Лобачевского и получению полного списка аксиом геометрии. Вопрос об аксиоматическом обосновании геометрии был впервые решен Гильбертом в 1899 г. Получение аксиом евклидовой геометрии, из которых логическим путем следовали бы все теоремы, является одной из главных задач оснований геометрии. Эта совокупность всех аксиом называется системой аксиом. Теоремы в геометрии базируются на аксиомах, определениях и ранее доказанных теоремах. Система аксиом Гильберта описывает восемь основных понятий. Основные понятия — точки, прямые, плоскости — называются основными образами. Понятия инцидентности (синонимы — принадлежности, лежать на, проходить через) точки и прямой, точки и плоскости, а также понятия «лежать между» или просто «между» для трех точек, инцидентных прямой, конгруэнтности (равенства) отрезка отрезку и угла углу называются основными отношениями. В других системах аксиом евклидовой геометрии принимаются другие основные понятия. Можно построить еще одну геометрию, в которой вовсе нет параллельных прямых. Она в определенном смысле двойственна геометрии Лобачевского и называется геометрией Римана (не путать с римановой геометрией) или эллиптической геометрией. Система аксиом этой геометрии отличается не только аксиомой параллельности, но и другими аксиомами. В этой геометрии сумма углов треугольника больше двух прямых, две окружности могут пересекаться в четырех различных точках. Промежуточное положение между геометриями Лобачевского и Римана занимает геометрия Евклида.

XXII. *What is meant by the following statements?*

1. In mathematics all roads lead to Greece, "the morning-land of civilization". 2. Number rules the Universe (the Pythagoreans). 3. The mathematical theory results from the interplay of the two things: a set of postulates and a logic. 4. Men accepted the axioms of Euclidean Geometry because experience with physical figures vouched for those axioms. 5. Certainly Euclid's geometry was a grand abstraction from physical world but the current mathematical abstractions are of even higher order. 6. With the creation of non-Euclidean geometries mathematicians inaugurated the interpretation of nature. 7. Nature is consistent, mathematics also has to be consistent. 8. Absolute consistency is unattainable. 9. In a modern deductive proof from explicitly stated axioms the meaning of the undefined terms is irrelevant. 10. Mathematics does not deal with objects, relations and phenomena of the external world, but, strictly speaking, only with objects and relations of its own imagery. 11. A mathematics absolutely divorced from reality soon becomes sterile. 12. The question of whether a geometry is "true" has no place in pure science. 13. Pure geometry is far richer in meaning, vaster in scope, and more fruitful in application than it was suspected before. 14. Hyperbolic geometary is one of many non-Euclidean geometries. It is particularly important. 15. The best way to describe geometry today is to display a geometrical way of looking at any subject.

COMPOSITION

I. *Write a dialogue based on the given questions.*

1. What was the cradle of mathematics? 2. How can the mathematics preceeding Greek times be characterized? 3. How did the Egyptians and the Babylonians produce their best mathematical results? 4. What are the contributions of the Greeks to mathematics? 5. What is the advantage of mathematics being abstract? 6. What is usually meant by a deductive system? 7. What are the basic means of Geometry? 8. Why did the Greeks turn toward Geometry and did not create number systems or algebra? 9. How did mathematical theories originate in Greek mathematics? 10. What was the result of the Greeks' discovery that the number $\sqrt{2}$ is irrational? 11. What are the earliest mathematical theories of antiquity? 12. What is the most outstanding contribution of the early

Greeks? 13. What do you mean by "material axiomatics"? 14. Who of the Greek mathematicians created the best example of material axiomatics? 15. What do Euclid's "Elements" deal with? 16. What is the origin of the creation of non-Euclidean geometry? 17. Euclid recognized that a postulate (an assumption, undemostrable statement) was needed to specify the nature of parallel lines, didn't he? 18. Was parallel postulate Euclid's great contribution or a blemish in his theory? 19. Why did parallel postulate give so much trouble? 20. When did dissatisfaction with Euclid's parallel postulate appear? How long did geometers' attempts last to find a better statement of this postulate or to prove it as a theorem dependent on other postulates? 21. What geometry may be called non-Euclidean? 22. Who created non-Euclidean geometry? 23. Did the creators of non-Euclidean geometry have any real understanding of the results obtained? 24. What developed out of their invention?

II. *Write a two pages long composition: "Modern Geometry".*

COMPREHENSION EXERCISES

Questions

1. Why is it customary to refer to **Thales of Miletus** as a) one of the "seven wise men of antiquity", b) a worthy founder of demonstrative geometry? What do you know about the rest six "wise men of antiquity?" What were Thales's great innovations and contribution to mathematics and science?

2. The residents of Crotona appreciated **the Pythagoreans' activities and their mathematics,** didn't they? Can you characterize Pythagoras as a person (leader, mathematician, teacher, athlet, speculative natural philosopher)? What legends about Pythagoras do you know? Why are the Pythagoreans' works unavailable in the libraries nowadays? Who(m) do we owe the preservation of the Pythagoreans' mathematics to?

3. **What is Euclid's definition?** What types of definition do you know? What is the distinction between explicit and implicit definitions? How can one distinguish between an implicit definition and a tacit assumption?

4. **What is an axiom?** How did Euclid himself differentiate between the axioms which referred to general mathematical ideas and those which referred specifically to geometric objects? Can you give a simple non-circular and uniquely characterizing definition of an axiom? Why does one choose the axioms one does? Is there any technique, mechanical process or algorithm to help us select out of infinite variety of statements those that should be used for axioms? Or is it an art that demands genius?

5. **What is a deduction?** How does Euclid draw all his conclusions? Is deduction of any value in actual practice? Can one deduce a cure for cancer from some definitions and axioms? **Deductive reasoning** is carried on as follows: "All good people are honest and if I am good then I must be honest. And if I am not honest I am not then good". Illustrate deductive reasoning in maths. Prove a theorem or solve some problem applying mathematical deduction.

6. **What is a theorem?** How does one discover what theorems one can prove from a particular set of axioms? Is proving theorems difficult? Can a theorem once proved be disproved? Does a proved theorem ever become old, old-fashioned or out-of-date? When does a proved theorem

become a particular case of a more general principle? Can you give some examples?

7. **What is a proof?** What constituted a proof for Euclid is still a proof for us. Why? When do mathematicians call a proof rigorous, valid, elegant? Priority arguments concerning proofs are not very important (Euclid), are they? Are there any proofs in applied mathematics?

8. **Geometric knowledge.** What does this phrase mean? Do geometric ideas die? Was geometric knowledge submitted to re-examination and re-evalution? Can geometric knowledge be decreased, reduced or diminished?

9. **What is an axiomatic approach?** What is the role of the axiomatic approach in current mathematics? What properties must an axiomatic system possess? Is the ordering of theorems of any importance in the axiomatic system?

10. Why do we call **D. Hilbert's geometry formal?** Do there exist different viewpoints (standpoint) concerning Geometry today? Does Geometry exhaust itself? What is the distinction between abstract and formal geometry.

Discussion

1. It is customary to speak about the ancient Egyptians', Babylonians' and Greeks' geometry. Does there exist any national geometry today? Why?

2. The empirical nature (thumb-rule procedures, trial-and-error-methods) of pre-Hellenistic mathematics and its accopmlishments.

3. The Pythagoreans' mathematics.

4. The first genuine stride of mathematics as a science was taken by Geometry and not by number or algebra. Why?

5. The importance and formal nature of Euclid's "Elements". Deduction versus Induction.

6. In Euclid's "Elements" each proposition stands by itself; its connection with others is never indicated; the leading ideas contained in his proofs are not stated explicitly; general principles do not exist. Modern tendency is toward generalization. What does it mean?

7. Professional mathematicians admire Euclid's work, nevertheless none of them likens him to, say, Archimedes. Why? Historians display extensive testimonials to the greatness of Archimedes. There is nothing like this in the case of Euclid. Why?

8. The best mathematical proofs are usually short, direct and penetrating. Some examples.

9. Every proof of Euclid calls for some new, often ingenious approach. Is this lack of a general procedure a merit or shortcoming of Euclid? Advantages and disadvantages.

10. There are no motivations, explanations or justifications in Euclid's "Elements". He never mentions the name of a person, he never makes a statement about or even an allusion to genetic development of mathematics. He has a fixed pattern for the enunciation of a proposition and never deviates from or reverses the procedure. In short, Euclid's "Elements" is the work of a dull unsufferable pedant and martinet. Your viewpoint?

11. Greek mathematics is "permanent", more permanent than Greek literature or art. Some historians of mathematics assert that mathematics was created by the ancient Greeks and very little was added since their time. Prove it or disagree.

12. The Parallel postulate is rich in implications. Its implications are drastic. What do these statements mean? What may happen if one discards or neglects the Parallel postulate? Can a geometry be consistent without it?

13. The essence of Euclid's, Lobachevsky's, Riemann's Parallel postulates. How many non-Euclidean Geometries do there exist (are studied and developed) today? Was non-Euclidean Geometry invented or discovered?

14. The creation of non-Euclidean Geometry brought about momentous innovations and novel developments in mathematics. What are they?

15. Geometry that is 1) true, 2) worthy of investigation, 3) the most convenient, 4) the most up-to-date.

16. There are unexpected reverse movements in mathematics in which a specialized theory (such as the theory of real numbers) lends indispensable aid in the construction of a more general theory (like topology or integration). Some examples of reverse movements (approaches) in Geometry.

17. Material and Formal axiomatics. The meaningfullness of axiomatic approach in mathematics. There are three important concepts, usually associated with any axiomatic system: consistency, independence and completeness. What do they all mean?

18. D. Hilbert's famous book "The foundation of Geometry". Its role in the development of modern mathematics.

19. Throughout the history, in fact, improvements in notation always succeed and parallel the progress in mathematics. Did there appear any improvements in geometrical notation?

20. A modern view of Geometry as 1) a branch of mathematics which is the invarient theory of a transformation group (the Erlanger Programme), 2) a point of view, a particular way of looking at a subject (Abstract Spaces). Modern Geometry is the royal road that Euclid thought did not exist.

21. Geometry is the only field of mathematics in which two-thousand-year-old traditions and theories are still valid and there is always a flood of fresh ideas. Today the fastest growing and most radically changing of all the branches of mathematics is **Geometry.** What are some of the new aspects (parts, divisions) of Geometry that claim the attention of contemporary researchers?

22. The sense of beauty is very personal and subjective. Nevertheless mathematicians unanimously agree that a certain mathematical result is elegant and beautiful. Do validity and truth alone suffice to make a mathematical theorem beautiful? How can one recognize "beauty" in a mathematical theorem? Some examples of beautiful theorems in Geometry.

23. "A geometry is the study of those properties of a set S which remains invariant when the elements of the set S are subjected to the transformations of some transformation group Γ" (F. Klein). Does there exist a still more general concept of Geometry nowadays?

LESSON FIVE

INTRODUCTION TO ANALYTIC GEOMETRY

Grammar:
1. Future Indefinite Tense-Aspect Forms.
2. Different Means of Expressing Future Actions.
3. Nouns of Latin and Greek Origin.

LAB. PRACTICE

Repeat the sentences after the instructors.

1. By the seventeenth century mathematics was still essentially a body of geometry and the heart of this body was Euclid's contribution. 2. Euclidean geometry confines itself to figures formed by straight lines and circles. 3. In the seventeenth century the advances of sciences and technology produced a need to work with many new configurations and new curves. 4. The great mathematicians of the Age of Genii (=the seventeenth century) were much concerned with the study of curves. 5. Ellipses, parabolas and hyperbolas became important because they have a host of practical applications. 6. The classical works and methods on conic sections became inadequate when dealing with practical problems. 7. Euclidean synthetic methods were too limited to deal with the problems of projectile paths, map-making and the study of lenses. 8. All these problems not only increased the need for knowledge of properties of familiar curves but also introduced new curves. 9. Of two great thinkers **R. Descartes** (the Latin version of his name is **Renatus Cartesius**) and **P. Fermat** who founded Analytic Geometry, the former was a profound philosopher, the latter was a scientist in the realm of ideas. 10. Analytic Geometry is a general method of geometry and the basis of all modern applied mathematics. 11. Analytic Geometry is often appreciated as the logical basis for mechanics and physics. Such appraisals are wholly verified. 12. Descartes saw as the objective of his work the cooperation of algebra and geometry so that mathematics might have the best aspects of both branches. 13. In the end it turned out that geometry lost popularity in the partnership. 14. Geometry was arithmetized and the beautiful geometrical reasoning was abandoned. 15. Geometry was submerged in a sea of formulas. The spirit of geometry was banished for more than 150 years. 16. Geometers remained in the shadows. In the XIX century, however, Projective Geometry revived spirit and vitality of pure geometry. 17. The idea of identifying numbers with points originated in Antiquty. 18. The discovery of incommensurables led to the definition of the real numbers with which we assign a number to each point of a line. 19. Logically speaking this is the basis of Analytic Geometry because it enables one to identify points (the most basic objects of geometry) with

numbers (the most basic objects of arithmetic and algebra). 20. A system with which one coordinates numbers and points is referred to as coordinate system or frame of reference. 21. Thanks to Descartes and Fermat points became pairs of numbers, and curves became collection of pairs of numbers expressed in equations. 22. The properties of curves can be deduced by algebraic processes applied to the equations. 23. Analytic Geometry replaced curves by equations through the device of a coordinate system. 24. Coordinate system locates points in a plane or in space by numbers. 25. The association of equation and curve, the combination of the best of algebra and the best of geometry was a revolutionary new thought. 26. Descartes and Fermat created a new method for studying geometric figures and curves, both familiar and new. 27. The heart of Descartes' and Fermat's idea is the following: To each curve there belongs an equation, that uniquely describes the points of that curve and no other points. 28. Conversely, each equation involving x and y can be pictured as a curve by interpreting x and y as coordinates of points. 29. The equation of any curve is an algebraic equality which is satisfied by the coordinates of all points on the curve but not the coordinates of any other points. 30. In Analytic Geometry of a three-dimensional space a plane is characterized by a linear equation. 31. A quadratic surface, e. g., a sphere or an ellipsoid, is characterized by a quadratic equation. 32. A quadratic equation is the one in which the highest power of an unknown is its square. 33. It is not difficult to generalize the basic ideas to include also points in space. 34. To coordinate numbers to points in space we shall employ a coordinate system consisting of three mutually perpendicular axes — x, y, z axes. 35. The algebraization of geometry permits one to speak of a space of more than three dimensions, say n-dimensions.

Key Grammar Patterns

Indefinite Tense-Aspect Forms

	Active Voice	*Passive Voice*
Present	Mathematicians commonly **use** this frame of reference to locate a point in the plane.	This frame of reference **is** commonly **used** to locate a point in the plane.
Past	Mathematicians **used** this frame of reference to solve that problem.	This frame of reference **was used** to solve that problem.
Future	Mathematicians **will use** this frame of reference to locate a point in space.	This frame of reference **will be used** to locate a point in space.

Translate the following sentences into Russian, analyzing the predicates.

1. As recently as three centuries ago the main fabric of mathematical thought **was supplied** by Geometry, inherited from the ancients and only perfected during the intervening 20 centuries. 2. Then **began** a radical and rapid transformation of mathematics. 3. It **is true** that the deductive method starting from axioms provides a shortcut for covering a large territory and general theories. 4. But the constructive method that **proceeds** from the particular to the general **leads** the way more surely to independent productive thinking. 5. The rigorous axiomatic, deductive style of Geometry **yielded** to inductive, intuitive insights and pure geo-

metric notions **gave way** to concepts of number and algebraic operations which **are embodied** in Analytic Geometry and calculus. 6. Few academic experiences **will be** more **thrilling** to the students of mathematics than an introduction to this new and powerful method of attacking geometrical problems — Analytic Geometry. 7. The task of establishing a theorem in Geometry **will be** cleverly **shifted** to that of establishing a corresponding theorem in aglebra. 8. Since many students **are** considerably more **able** as algebraists than as geometers, Analytic Geometry **can be described** as the "royal road" in Geometry that Euclid **thought did not exist.** 9. R. Descartes's claim to the invention of Analytic Geometry **rests** on one of the three appendices to his famous philosophical treatise on universal sciense: "Discourse on the Method of Rightly Conducting Reason and Seeking Truth in the Sciences", which **was published** in 1637. 10. P. Fermat's claim to priority **rests** on a letter written in 1636 in which it **is stated** that the ideas of the writer **were** even then **seven years old**. 11. Although Solid Analytic Geometry **was mentioned** by R. Descartes, it **was not elaborated** thoroughly and exhaustively by him. 12. A century later the whole subject matter of Analytic Geometry **was** well **advanced** beyond its elementary stages by L. Euler.

Different Means of Expressing Future Actions

1. Means of expressing **ability**, **capability** and **permission** in **Future**:

can → shall/will be able to   may → shall/will be allowed to.

The students **will be able to solve** geometrical problems in terms of algebra in Analytic Geometry. They **will be allowed to apply** this coordinate system in the following problems.

2. Means of expressing **obligation** or **necessity** in **Future**:

must → shall/will have to,   shall/will have got to.

We **shall have (got) to plot** the graph more accurately. They **will have (got) to verify** their results once more.

3. Means of expressing **intention**, **willingness**, **readiness**, **expectation** in **Future**:

to be + Inf.,   **to be going** + Inf.,   **to be about** + Inf.

We **are to consider** one-to-one correspondence as the main principle of Analytic Geometry. **Are** you **going to use** this method of reasoning again? He **was about to perform** the construction and then changed his mind.

4. Means of expressing Future Actions in subordinate clauses of **time**, **condition** and **concession**.

Future Indefinite → Present Indefinite

Conjunctions that may introduce a clause respectively:

when, while, till, until, before, after, as soon as, once, if, unless, on condition (that), provided (providing) that, in case, even if, even though, no matter how, whenever, whatever, however.

When the student **studies** calculus, he **will find** that radian measurement of anges is the natural and convenient system for use in theoretical developments. **Providing** the angle **is measured** in radians there **will be** several useful geometric relations. **Whenever** we **turn** to general methods of graphing a function, we **shall discuss** in detail the most common two-dimensional representation of a plane.

Nouns of Latin and Greek Origin

Singular	Plural	Singular	Plural
	I. **on(um)→a** [ə]		
Continuum континуум	continua	Maximum максимум	maxima
Criterion критерий	criteria	medium среда	media
Curriculum учебный план	curricula	minimum минимум	minima
Datum данная, величина	data	momentum количество движения	momenta
Equilibrium равновесие	equilibria	phenomenon явление	phenomena
Infinitum бесконечность	infinita	Polyhedron многогранник	polyhedra
Latus rectum фокальный параметр	latera recta	quantum квант	quanta
symposium симпозиум	symposia	vacuum вакуум	vacua
spectrum спектр	spectra	stratum слой	strata
	II. **is** → [is], **ix** → [iks] } **es** → [i:z]		
analysis анализ	analyses	emphasis эмфаза	emphases
axis ось	axes	hypothesis гипотеза	hypotheses
basis базис	bases	index указатель	indices
crisis кризис	crises	matrix матрица	matrices
directrix директриса	directrices	parenthesis скобка	parentheses
vertex вершина	vertices	phasis фаза	phases
thesis тезис, диссертация	theses	synthesis синтез	syntheses
	III. **us**→ [əs] → **i** → [ai]		
calculus исчисление, мат. анализ	calculi	modulus модуль	moduli
focus фокус	foci	nucleus ядро	nuclei
genius гений	genii	radius радиус	radii
locus геометрическое место точек	loci	rhombus ромб	rhombi

IV. Similar Forms

an apparatus аппарат, прибор	apparatus	a means средство	means
a headquarters штаб	headquarters	a series ряд	series
news новость	news	a species вид	species

V. a → [ə] → ae → [i:]

		Modern Forms	
abscissa абсцисса	abscissae	abscissas	criterions
hyperbola гипербола	hyperbolae	formulas	hyperbolas
formula формула	formulae	geniuses	indeces
corona корона	coronae	radiuses	terminuses
lacuna пустота	lacunae	mediums	nucleuses
nebula туманность	nebulae	indexes	spectrums
		rhombuses	vacuums
		lacunas	maximums

Translate the following sentences into Russian.

1. The area of an ellipse equals $\pi/4$ times the product of the long and the short diameters or π times the product of the long and the short **radii**. 2. If a curve is symmetric with respect to both **axes** is it symmetric with respect to the origin? 3. Analytic methods give us a **means** of finding the equations of **loci**; all these **loci** (a circle, an ellipse, a hyperbola, a parabola) are called conic sections or simply conics. 4. The notion of a four-dimensional geometry is a very helpful one in studying physical **phenomena.** 5. A chord drawn through either focus of the ellipse and perpendicular to the principal axis is called a **latus rectum.** Find the equation of the ellipse with **foci** at the points (0,4), if the length of its minor axis is **6**. Find the end points of its **latera recta** and sketch the ellipse. 6. In each of the following **hyperbolae**, locate the **vertices**, **foci**, and end of the **latera recta**; draw the asymptotes and the curves. 7. All these facts may serve as **reference data**. 8. Complete surfaces formed with regular polygons such as a complete surface of cube built up by joining six squares along their edges are called reqular **polyhedra.**

THE INTRODUCTORY TEXT

DESCARTES'S AND P. FERMAT'S COORDINATE GEOMETRY

Every student of mathematics meets the remarkable subject called Analytic Geometry, and he can hardly fail to be impressed by the powerful idea behind it. The essence of the idea as applied to the plane, it will be recalled, is the establishment of a correspondence between pairs of real numbers and points in the plane, thereby making possible a correspondence between curves in the plane and equations in two variables, so that for each curve in the plane there is a definite equation $f(x, y) = 0$, and for each such equation there is a definite curve in the plane.

A correspondence is similarly established between the algebraic and analytic properties of the equation $f(x, y) = 0$, and the geometric properties of the associated curve. The task of proving a theorem in geometry will cleverly be shifted to that of proving a corresponding theorem in algebra and analysis.

There is no unanimity of opinion among historians of mathematics concerning who invented Analytic Geometry, nor even concerning what age should be credited with the invention. Much of this difference of opinion is caused by a lack of agreement regarding just what constitutes Analytic Geometry. There are those who, favouring Antiquity as the era of the invention, point out the well-known fact that the concept of fixing the position of a point by means of suitable coordinates was employed in the ancient world by the Egyptians and the Romans in **surveying**, and by the Greeks in map-making. And, if Analytic Geometry implies not only the use of coordinates but also the geometric interpretation of relations among coordinates then a particularly strong argument in favour of crediting the Greeks is the fact that **Appolonius** (c. 225 B. C.) derived the bulk of his geometry of the conic sections from the geometrical equivalents of certain Cartesian equations of these curves, the idea which originated with **Menaechmus** about 350 B. C.

Others claim that the invention of Analytic Geometry should be credited to **Nicole Oresme**, who was born in Normandy about 1323 and died in 1382 after a career that carried him from a mathematics professorship to a bishopric. **N. Oresme** in one of his mathematical tracts, anticipated another aspect of Analytic Geometry, when he represented certain laws by graphing the dependent variable against the independent one, as the latter variable was permitted to take on small increments. Advocates for N. Oresme as the inventor of Analytic Geometry see in his work such accomplishments as the first explicit introduction of the equation of a straight line and the extension of some of the notions of the subject from two-dimensional space to three, and even four-dimensional spaces. A century after N. Oresme's tract was written, it enjoyed several printings and in this way it may possibly exert some influence on the succeeding mathematicians.

However, before Analytic Geometry could assume its present highly practical form, it had to wait the development of algebraic symbolism, and accordingly it may be more correct to agree with the majority of historians, who regard the decisive contributions made in the seventeenth century by the two French mathematicians, **R. Descartes** (1596—1650) and **P. Fermat** (1601—1663), as the essential origin of at least the modern spirit of the subject. After the great impetus given to the subject by these two men, we find Analytic Geometry in a form with which we are familiar today. In the history of mathematics a good deal of space will be devoted to R. Descartes and P. Fermat, for these men left very deep imprints on many subjects. Also, in the history of mathematics, much will be said about the importance of Analytic Geometry, not only for the development of Geometry and for the theory of curves and surfaces in particular, but as an indespensable force in the development of the calculus, as the influential power in molding our ideas of such far-reaching concepts as those of "function" and "dimension".

Thus, applied mathematics in the modern sense of the term was not the creation of the engineer or the engineering-minded mathematician. Of the two great thinkers who founded this subject one was a profound philosopher, the other was a scientist in the realm of ideas. The former **René Descartes** devoted himself to critical and profound thinking about the nature of truth, and the physical structure of the universe. The latter **Pierre Fermat**, lived an ordinary life as a lawyer and civil servant, but in his spare time he was busy creating and offering to the world his famous theorems. The work of both men in many fields will be immortal. R. Descartes proposed to generalize and extend the methods used by mathematicians in order to make them applicable to all investigations. In essence, the method will be an axiomatic deductive construction for all thought. The conclusions will be theorems derived from axioms. Guided by the methods of the geometers Descartes carefully formulated the rules that would direct him in his search for truth. His story of the search for method and the application of the method to problems of philosophy was presented in his famous "Discourse on Method". The method Descartes abstracted from mathematics and generalized he then reapplied to mathematics; with it he succeeded in creating a new way of representing and analyzing curves. This creation, now known as coordinate geometry, is the basis of all modern applied mathematics.

P. Fermat, despite the brief amount of time he was able to spend on mathematics and the pleasure-seeking attitude with which he approached it, established himself as one of the truly great mathematicians of all times. His contributions to the calculus were firstrate though somewhat overshadowed by those of Newton and Leibnitz. He shared with Pascal the honour of creating the mathematical theory of probability, and shared with Descartes the creation of coordinate geometry, and founded the theory of numbers. In all these fields this "amateur" produced brilliant results. Though not concerned with a universal method in philosophy, Fermat did seek a general method of working with curves and here his thoughts joined company with those of Descartes's.

One must understand why it was that the great mathematicians of the time were so much concerned with the study of curves. In the early part of the seventeenth century mathematics was still essentially a body of geometry and the heart of this body was Euclid's contribution. Euclidean geometry confines itself to figures formed by straight lines and circles, but by the seventeenth century the advances of science and technology produced a need to work with many new configurations. Ellipses, parabolas and hyperbolas became important because they described the paths of the planets and projectiles such as cannon balls.

Both Descartes and Fermat recognized that geometry supplied information and truth about the real world. They also appreciated the fact that algebra could be employed to reason about abstract and unknown quantities; and it could be used to mechanize the reasoning process and minimize the effort needed to solve problems. Therefore they proposed **to borrow** all that was best in geometry and algebra and correct the defects of one with the help of the other. In Descartes's general study of method he decided to solve all problems by proceeding from the simple to the complex. Now, the simplest figure in geometry is the straight line. He therefore sought to approach the study of curves through straight lines and he found the way to do this.

To discuss the equation of a curve he introduced a horizontal line called the X-axis, a point O on the line called the origin, and a vertical line through O called Y-axis. If P is any point on a curve, there are two numbers that describe its position. The first is the distance from O to the foot of the perpendicuar, from P to the X-axis. This number, called X-value, is the **abscissa** of P. The second number is the distance from P to the Y-axis, called Y-value or **ordinate** of P. These two numbers are called the coordinates of P and are generally written as $P(x, y)$. The curve itself is then described algebraically by stating some equation which holds for x and y values of points on that curve and only for those points.

The heart of Descartes's and Fermat's idea is the following. To each curve there belongs an equation that uniquely describes the points of that curve and no other points. Conversely, each equation involving x and y can be pictured as a curve by interpreting x and y as coordinates of points.

Thus formally stated: the equation of any curve is an algebraic equality which is satisfied by the coordinates of all points on the curve but not the coordinates of any other point.

Since each of these pairs of coordinates represents a point on the curve we can plot these points and join them by a smooth curve. The more coordinates we calculate, the more points can be plotted and the more accurately the curve can be drawn.

Beyond the analysis of properties of individual curves, the association of equation and curve makes possible a host of scientific applications of mathematics. Among the practical applications of mathematics we shall merely mention that all the conic sections possess the properties that make these curves effectively employed in lenses, telescopes, microscopes, X-ray machines, radio antennas, searchlights and hundreds of other major devices. When Kepler introduced the conic sections in astronomy they became basic in all astronomical calculations including those of eclipses and paths of comets.

To summarize, it was not so much the use of coordinates that made the work of Descartes and Fermat so important; coordinates were used effectively in antiquity, especially in the geometry of Appolonius, and again in the fourteenth century in a more primitive form in the latitude of forms of Oresme. Descartes saw as the objective of his work the cooperation of algebra and geometry to the end that mathematics might have the best aspects of both branches. In the end, however, it turned out that geometry lost popularity in the partnership. Pure geometry was so overshadowed that it made little progress during the next century and a half, during which time infinitesimal analysis went through a progress of arithmetization that amounted almost to a revolution.

Analysis Incarnate — Leonard Euler

Though P. Fermat and R. Descartes founded Analytic Geometry they did not advance the subject far enough and did not elaborate it purely analytically either. A century later **L. Euler** (1707—1783) a Swiss mathematician who lived the greater part of his life in Russia, engaged in scientific research, lecturing and textbook writing in St. Petersburg Academy, developed the subject matter of both Plane and Solid Analytic Geometry far beyond its elementary stages. Euler's mathematical career opened when Analytic Geometry (made public in 1637) was ninety years old, the calculus about fifty. In each of these fields a vast number of isolated problems were solved, but no systematic unification of the whole of the then mathematics, pure and applied, was made. In particular, the powerful analytic methods of Fermat, Descartes, Newton and Leibnitz were not exploited to the limit of what they were capable, especially in Calculus, Geometry and Mechanics, where Euler proved himself the master.

In the XVIII c. the Universities were not the principal centres of science in Europe. The lead in scientific research was taken by the various royal academies. In Euler's case St. Petersburg and Berlin furnished the sinews of mathematical creation. Both of these foci of creativity owed their inspiration to the restless ambition of Leibnitz. These academies were like some of these today: they were research organizations which paid their leading members to produce scientific research. Euler became famous for his great output of original mathematics and for the wide range of subjects he covered. He contributed new ideas to Calculus, Geometry, Algebra, Number Thery, Calculus of Variations, Probability and Topology. He also worked in many areas of applied mathematics, such as Acoustics, Optics, Mechanics, Astronomy, Ballistics, Navigation, Statistics and Finance. His industry was as remarkable as his genius. Euler was the most prolific mathematician in history; his scientific heritage is vast, a list of some 850 works of which 550 were published in his lifetime. Euler wrote his great memoirs quite easily and even total blindness during the last seventeen years of his life did not retard his unparalleled productivity. He overcame the difficulty of blindness chiefly by means of his remarkable memory. Indeed, if anything, the loss of his eyesight sharpened Euler's perception of the inner world of his imagination.

Euler first gave the examples of those long analytic procedures in which conditions of the problem are first expressed by algebraic symbols and then pure calculation resolves the difficulties. He skillfully applied his analytic method to Geometry and Mechanics. Where the synthetic methods of Euclidean geometry required elaborate and complicated constructions and furnished lenghts that could be measured only approximately, algebraic equation of Analytic Geometry is a much simpler tool and furnishes answers to as many decimal places as individual cases require. Euler improved the basic concepts of mathematical analysis, promoted differential and integral calculus, fathered the theory of linear differential equations and devised methods for their approximate solution. His treatises "Introduction to the Analysis of Infinities", "Differential Calculus" and "Integral Calculus" which for the most part present Euler's own results served as an encyclopedia in mathematical analysis of the period. Euler's contemporaries called him "Analysis Incarnate". One of the most remarkable features of Euler's universal genius was its equal strenght in both of the main currents of mathematics, the **continuous** and the **discrete**. "**Read Euler, he is teacher of us all...**", Laplace so amptly assessed his worth. But Euler was far more than a textbook writer. He enriched mathematics with beautiful new results. Differential Geometry got its first real start in Euler's study of lines of curvature (1760) and the Calculus of Variations took an independent status, when Euler (1736) gave his differential equation expressing a necessary condition for a minimizing curve.

Curiously enough, in arriving at his theoretical conclusions, by working on practical tasks in different fields, Euler sought to "rid" mathematical analysis of geometrical, mechanical and physical interpretation and couch it in purely analytic form. Thus he wrote, "**Here the entire exposition is limited to pure analysis and, hence, not a single drawing was needed to set out the rules of this calculus**". In an effort to replace synthetic methods by analytic Euler was succeeded by Lagrange, who dealt not with special concrete cases and tasks, but sought for abstract generality. Nevertheless, Euler was never excelled either in productivity or in the skillful and imaginative use of algorithmic devices for the solution of problems.

The contribution that this illustrious scientist made to mathematics is truly enormous. We have every right to entitle him **the 18th century Mathematician Number One** whose works left their imprint on almost all branches of mathematics. L. Euler was buried in 1783 near Lomonosov's grave in the old cemetery of the Alexandro-Nevsky Monastery in Leningrad's (then St. Petersburg) necropolis. Even when he was compelled to emigrate to Germany, Russia ever remained in Euler's heart and mind and he never ruptured ties with the St. Petersburg Academy. To this day the great mathematician's descendants live in this country, whose people will always revere his name. In 1983 scientists around the world extensively commemorated the 275th birthday and death bicentennial of this great scientist.

Nomography

The use of graphic techniques for computation and solution of equations goes back thus to Antiquity. In the time of Hipparchus (150 B. C.) graphic solutions of spherical triangles was very popular. During the Middle Ages, Arab mathematicians used geometric means to solve quadratic equations, and in the seventeenth century **W. Oughtred** used graphic methods for solving spherical triangles. However, the key to general application of graphic methods to the solution of algebraic problems was Analytic Geometry, introduced by R. Descartes and P. Fermat. The theory of nomograms rests largely on Analytic Geometry.

In 1842 **L. Lalanne** pointed out that by altering the scales along the Cartesian axes it is often possible to simplify graphs of equations in two variables. Furthermore, he noted that if these changes are subject to certain minimal restrictions, the new graph is essentially equivalent to its Cartesian counterpart. Lalanne called his new theory "geometrical anamorphosis" and further advances were made in this theory by **J. Massau** and **C. Lallemand** during the 1880's. Those and other developments were foreshadowings. The real creator of nomography was the French mathematician **M. d'Ocagne** (1862—1938). D'Ocagne was the first to describe the "alignment chart" (1884), and he applied this chart to many engineering formulas. In 1899 he published **Traite de Nomographie**, in which he brought together both the general theories and many applications of the subject. Since that time numerous texts on the subject were issued and many nomograms appeared in technical journals.

It is interesting that the original impetus for study of nomography came from problems that arose during the construction of railroads in France. Thus most nineteenth-century contributors to the subject were engineers. In fact, nomography remains essentially a branch of applied mathematics with uses in engineering, industry, physical and natural sciences.

ACTIVE VOCABULARY

1. to abandon
2. to align
3. to anticipate
4. to assign
5. to associate
6. to coordinate
7. to correspond
8. to designate
9. to elaborate
10. to embody
11. to exceed
12. to facilitate
13. to favour

13. to fit
15. to frame
16. to handle
17. to innovate
18. to locate
19. to perceive
20. to preclude
21. to publish
22. to scale
23. to submerge
24. to unite
25. to urge
26. to utilize

Read and translate the text in class. Try to guess the meaning of all the bold-faced words; paraphrase them or give a definition, a synonym or an explanation of their meaning in English.

TEXT ONE

HISTORY OF THE TERMS "ELLIPSE", "HYPERBOLA", AND "PARABOLA"

The evolution of our present-day meanings of the terms "ellipse" "hyperbola", and "parabola" may be understood by studying the discoveries of history's great mathematicians. As with many other words now in use, the original application was very different from the modern.

Pythagoras (c. 540 B. C.), or members of his society, first used these terms in connection with a method called the "application of areas". In the course of the solution (often a geometric solution of what is equivalent to a quadratic equation) one of three things happens: the base of the constructed figure either **falls short of**, **exceeds**, or **fits** the length of a given segment. (Actually, additional restrictions were imposed on certain of the geometric figures involved.) These three conditions were designated as **ellipsis** ("defect"), **hyperbola** ("excess") and **parabola** ("a placing beside"). It should be noted that the Pythagoreans were not using these terms **in reference to** the conic sections.

In the history of the conic sections Menaechmus (350 B. C.), a pupil of Eudoxus, **is credited with** the first treatment of the conic sections. Menaechmus was led to the discovery of the curves of the conic sections by a consideration of sections of geometrical solids. Proclus in his "Summary" reported that the three curves were **discovered** by Menaechmus; consequently they were called the "Menaechmian triads". It is thought that Menaechmus discovered the curves now known as the ellipse, parabola and hyperbola by **cutting** cones with planes perpendicular to an element and with the vertex angle of the cone being **acute**, **right**, **obtuse**, respectively.

The fame of Apollonius (c. 225 B. C.) rests mainly on his extraordinary "Conic Sections". This work was written in eight books, seven of which are **preserved.** The work of Apollonius on the conic sections differed from that of his predecessors in that he obtained all of the conic sections from one right double cone by **varying** the angle at which the intersecting plane cuts the element.

All of Apollonius' work was presented in regular geometric form, without the aid of the algebraic notation of the present-day Analytic Geometry. However, his work can be described more easily by using modern terminology and symbolism. If the conic is **referred** to a rectangular **coordinate system** in the usual manner with point A as the origin and with (x, y) as coordinates of any point P on the conic, the standard equation of the parabola $y^2=px$ (where p is the length of the latus rectum, i.e., the length of the chord that passes through a focus of the conic perpendicular to the principal axis) is immediately **verified**. Similarly, if the ellipse or hyperbola is referred to a coordinate system with vertex at the **origin**, it can be shown that $y^2 < px$ or $y^2 > px$, respectively. The three adjectives "hyperbolic", "parabolic", and "elliptic" are **encountered in** many places in mathematics, including projective geometry and non-Euclidean geometries. Often they are **associated with** the existence of exactly two, one, or none of something of **particular relevance**. The relationship arises from the fact that the number of points in common with the so-called line at **infinity** in the plane for the hyperbola, parabola and ellipse is two, one and zero, respectively.

Read the text in class. Make drawings of conic sections and supply them with their equations of Analytic Geometry. Discuss their properties and the roles they play in modern mathematics and engineering.

TEXT TWO

ANALYTIC GEOMETRY

The rectangular coordinate system provides a one-to-one correspondence between number pairs and points; that is, corresponding to a number pair (X_1, Y_1) there is always one and only one point P_1; and corresponding to a point P_2 there is one and only one number pair (X_2, Y_2). This one-to-one correspondence is the starting point of the plane Analytic Geometry.

The notion of a correspondence between a point in the plane and a pair of numbers can be extended to a more general kind of correspondence, namely, between a geometric locus and an equation. The graph of an equation is the locus of the points whose coordinates satisfy the equation. Conversely, the equation of a given curve is an equation satisfied by the coordinates of every point on the curve and by the coordinates of no other points.

This correspondence between equations and geometric loci, will indeed, form the central subject of our study. That is to say, our main investigation will take the form of one or the other of the problems:

1. Given an equation, to obtain the corresponding geometric locus (the graph of the equation) along with its properties.
2. Given a geometric locus whose points possess some common property (shared by no other points), to find the corresponding equation.

In the latter case the equation, in turn, will help us in studying other properties of the locus.

Thus, we define a curve as composed of points whose coordinates satisfy a certain equation. We may think of a curve as a locus or a path traced by a moving point according to certain specified conditions. From these conditions it is possible to derive the equation of its curve and then discuss the curve in detail from the equation. The locus of an equation in X and Y is defined as the totality of points whose coordinates satisfy the equation. There exists no definite rule for finding the equation of the locus. As a matter of fact the problem is to translate the geometric definition of the locus into an algebraic form with a suitable choice of a coordinate system.

We shall proceed to the discussion of particular species of loci — namely, the straight line, a circle, a parabola, an ellipse, and a hyperbola.

The problem of finding **the equation of the straight line** is the simplest case of the general problem of finding the equation of a curve. The equation of a straight line is determined by two points $P(X_1, Y_1)$ and $P_2(X_2, Y_2)$. This equation will be obtained from the fact that the point $P(X, Y)$ is on the straight line, if and only if, the slopes of the segments P_1P and P_1P_2 are equal. This condition is $(Y - Y_1)/(X - X_1) = (Y_2 - Y_1)/(X_2 - X_1)$, $X_1 \neq X_2$. We shall refer to this as the two-point form of the equation of the straight line. Thus any straight line may be represented by an equation of the first degree in X and Y. Conversely, every equation of first degree $Ax + By + C = 0$ represents a straight line.

The following loci lead to particular type of second degree equations, in two variables.

The **Circle** is the locus of a point, which moves so that its distance from a fixed point, called a centre, is constant. The distances from its centre to the locus are radii of the circle. Thus, $x^2+y^2=r^2$ is the equation of the circle with the centre at the origin and a radius r.

The **Parabola** is the locus of points which are equidistant from a fixed point and a fixed straight line.

The fixed point is the focus, the fixed line is the directrix. The line perpendicular to the directrix and passing through the focus is the axis of the parabola. The axis of the parabola is, obviously, a line of symmetry. The point on the axis halfway between the focus and the directrix on the parabola is the vertex of the parabola. The parabola is fixed when the focus and the directrix are fixed. The equation of the parabola, however, depends on the choice of the coordinate system. If the vertex of the parabola is at the origin and the focus is at the point (O, P) its equation is $X^2=2PY$ or $Y^2=2PX$.

The **Ellipse** — is the locus of a point which moves so that the sum of its distances from two fixed points called the foci is constant. This constant will be denoted by $2a$, which is necessarily greater than the distance between the foci (the focal distance). The line through the foci is the principal axis of the ellipse; the points in which the ellipse cuts the principal axis are called the vertices of the ellipse. If the centre of the ellipse is at the origin but the foci are on the y-axis its equation is

$$\frac{Y^2}{a^2}+\frac{X^2}{b^2}=1,$$

where a and b represent the lengths of its semimajor and semiminor axes.

The **Hyperbola** is the locus of a point which moves so that the difference of its distances from two fixed points is a constant $2a$. Its equation is

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1.$$

This equation shows that the hyperbola is symmetric with respect to both coordinate axes and also the origin. It intersects the X-axis but does not cut the Y-axis. Hence, the curve is not contained in a bounded portion of a plane. The curve consists of two branches. The line segment joining the vertices is called the transverse axis of the hyperbola; its lenght is $2a$. The point midway between the vertices is a geometrical centre and is called the centre of the hyperbola.

Read and translate the text in class. Discuss the problems involved.

TEXT THREE

HIGHER DIMENSIONS

One advantage of treating geometrical problems with analytic methods is that it becomes easier to generalize concepts beyond those dealing with three dimensions. In other words, the methods of Analytic Geometry make it easier to study geometric objects for which our power of visualization fails.

To illustrate this let us show how the notion of a distance can be generalized from one and two dimensions to three, four, five and even higher dimensions. One knows that the distance between (x_1, y_1) and

(x_2, y_2), namely, the distance between two points in a plane is $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$. In order to generalize this formula to three dimension we have only to consider a coordinate system consisting of **three** mutually perpendicular axes. With the use of this system we can locate any point in space by starting at the origin, going a certain distance to the right or left, then to the front or back, and finally, a certain distance up or down. With perpendicular axes like these we can identify each point with three coordinates x, y and z, and one can show that the distance between points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$. As in the two-dimensional case, the derivation of this formula is based on the Pythagorean theorem. If one wants to express these two distance formulas in words, one can say that "the distance between two points is given by the square root of the sum of the squares of the differences of the respective coordinates". Since this rule applies to one, two and three dimensions, it is tempting to let it apply also in the case where there are more than three.

In order to make such a generalization it will be necessary to explain first what a mathematician means when he speaks of four-dimensional, five-dimensional, or n-dimensional space. Evidently, the word "space" can no longer be interpreted in its colloquial sense standing for "physical space" or "the space we live in". Taking this term in its colloquial sense one can hardly go beyond the customary three dimensions of "left and right", "front and back", and "up and down". Indeed, when a mathematician talks about a space he is referring to a **collection of mathematical objects** which (for the sake of convenience) may be referred to as points. He then defines the dimension of such a space as the number of coordinates needed to determine each point.

A **line** or a curve constitutes a **one-dimensional space** since each point can be identified with one real number. Similarly, a **plane** or a **surface** constitutes a **two-dimensional space** since each point can be identified with two real numbers, namely, its two coordinates. Since three numbers are needed to locate a point in ordinary **space** one says that these points constitute a **three-dimensional space.** So far the mathematical concept of a dimension agrees with the intuitive notion which one ordinarily associates with this term. However, the analogy breaks down the moment one says that a collection of points constitutes a **four-dimensional space** because four numbers are needed to determine each point or if one says that a collection of points constitues an **n-dimensional space** because n numbers are needed to determine each point.

It must be understood, therefore, that when a mathematician speaks of a four-dimensional space, he does not refer to some mysterious generalization of the intuitive notion of a three-dimensional space. He refers to a set of mathematical objects which are **individually** determined by means of **four** numbers. To give an example of such a four-dimensional space, let us consider the position of an airplane at two different times: at 1:15 p.m. and 1:17 p.m. One says that the position of the airplane is in each case given by a point in three-dimensional space. Although the position of airplane is given by three coordinates x, y, z, it cannot be found unless one also knows the **time**, i.e., unless one knows a fourth variable t. In other words, to specify the location of the airplane one has to give four numbers x, y, z, t. It is in this sense, that one says that the location of the airplane is a point in a four-dimensional space. Naturally, it will be unreasonable to expect that one can visualize this in the same way in which one can visualize a point in one-, two-, or three-dimensional space. As a second example of a higher dimensional space let us consider five rolls of a die which yielded the numbers 3, 6, 1, 2, 5. When taken individually, these rolls of a die are given as single numbers, i.e., they are points in a one-dimensional space, when taken together they can be looked upon as a single point in a **five-dimensional space.** In other words, the experiment **as a whole** is characterized by five numbers, and it is therefore a point in a five dimensional space. In the given example the rolls of the die correspond to the point (3, 6, 1, 2, 5). If the rolls are all 2's, they will correspoind to the point (2, 2, 2, 2, 2).

Statisticians often refer to a sample of five measurements as a point in a **five-dimensional sample space**, and more generally, they refer to a sample of n measurements as a point in an **n-dimensional sample space.** This is just another way of saying that the sample as a whole consisted of n numbers. Physicists often consider systems of molecules in which the position of each molecule is determined by three coordinates x, y, z. If a gas consists of 1,000,000 molecules we shall need 3,000,000 numbers to describe the gas as a whole and we therefore refer to the state of this gas as a point in a **3,000,000-dimensional space**. If we also want to describe the **motion** of each particle, three more numbers will be needed for each molecule and the state of the gas will be a point in a **6,000,000-dimensional space.**

Let us now return to the problem which originally motivated this brief excursion into higher dimensions, namely the problem of generalizing the formula which measures the distance between two given points. Applying the rule that the distance between two points is the square root of the sum of the squares of the differences of the respecive coordinates to four-dimensions, one can write the distance between points (x_1, y_1, z_1, u_1) and (x_2, y_2, z_2, u_2) as a formula $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2+(u+_2-u_1)^2}$. It is important to note that this formula actually **difines** what one means by the distance between two points in a four-dimensional space. Proceeding in the same way, one can also define the distance between two points in a five-dimensional space as $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2+(u_2-u_1)^2+(v_2-v_1)^2}$ and if one introduces a suitable notation one will be able similarly to define the distance between two points in n-dimensional space. In the same way, in which we generalized the distance formula we are able to generalize many of the other formulas and equations of Analytic Geometry. For example, where in two dimensions $ax+by+c=0$ is the equation of a **line,** we shall say that in three dimensions $ax+by+cz+d=0$ is the equation of a **plane**, and that in four dimentions $ax+by+cz+du+e=0$ is the equation of a **hyperplane.**

In Analytic Geometry one insists that it takes **two** numbers to locate a point in the plane, i.e., it takes two coordinates to identify a point in a two-dimensional space. This is correct only as long as we restrict ourselves to **real numbers**, there is a way in which one can identify each point of plane by means of a **single complex number**. This is done simply by identifying the point whose coordinates are (x, y) with the complex number $x+iy$. This means that one can identify the points (1,4), (—2,5) and (3, —7) with the complex numbers $1+4i$, $-2+5i$, $3-7i$ and that one can vice versa plot the points $2+3i$ and $4-2i$. (They are the points whose coordinates are (2,3) and (4, —2).) One can thus identify the x- and y-coordinates of a point with the real and the imaginary parts of a complex number and represent each point by means of a unique complex number. Incidentally, this provides a "concrete" interpretation of the complex numbers. Complex numbers, thus, represent the points of a plane.

VOCABULARY EXERCISES

I. *Translate into Russian the following phrases, consulting the dictionary of mathematical terms.*

Alignment error, aligned sample, anticipatory control, each element is assigned two indices, assignable causes, assignment problem, associate elements, associated prime ideal, coefficient of association, associative algebra, associative law for multiplication, associativity relation, coordinate curve, coordinate paper, incidence correspondence, excess of nine, excess of triangle, excess-six code, coefficient of excess, favourable event, goodness of fit, least square fitting, guidance computer, hold circuit hold rele, data handling, influence function, influence line, domain of influence, local parameter (-field, -stability), localized vector, localization theorem, locally compact group, measure of location, storage location, locus of an equation, preservation of angles, norm-preserving mapping, providing bank, character recognition, pattern recognition, survey design, pilot survey, unified field theory, unique solution, unique factorization theorem, to vary directly, to vary indirectly, calculus of variations, function of bounded variation, reducible variety, minimal variety.

II. *Give the corresponding plural form of the following nouns and their Russian equivalents.*

a) **continuum → continua**
Datum, medium, vacuum, spectrum, quantum, infinitum, stratum, minimum, extremum, maximum, momentum, polyhedron, criterion, phenomenon, equilibrium, latus rectum, trustrum.

b) **axis → axes**
Thesis, emphasis, analysis, basis, appendix, crisis, phasis, synthesis, hypothesis, parenthesis, index, matrix, vertex, radix, directrix, bisectrix, trisectrix, tractrix, separatrix, indicatorix.

c) **focus → foci**
Calculus, genius, locus, modulus, nucleus, stimulus, radius, rhombus, abacus, regulus, torus, syllabus.

d) **formula → formulae**
Abscissa, hyperbola, nebula, corona, lacuna, coma, tessera.

e) **a means → means**
A series, a species, news, an apparatus, a headquarters.

III. *Give one Russian equivalent of the following groups of words.*

a) to fix — to plot — to locate a point / to encounter — to come across / to draw — to sketch — to produce a line / to fix — to denote — to assign an axis / the relation is satisfied — is true — holds / to elaborate — to work out / to stem from — to arise from / to coordinate — to arrange — to range — to align / to create — to produce — to set up — to establish / to understand — to comprehend — to perceive / to give — to provide — to supply — to furnish — to yield.

b) task — assignment / alignment chart — nomogram / scale — rule / scale — balance / scale — numbering / correspondence — mail / impetus — stimulus / survey — inspection / survey — summary / change — alteration — modification — variation / variable — unknown / locus — graph / looking glass — mirror.

c) cartesian — rectangular coordinates / deep — profound / chief — main — principal / huge — grand — immense / visual — visible / gifted — talented — able.

IV. *Translate the following antonyms.*

Clockwise — counterclockwise / dependent — independent / positive — negative / to differ — to liken / mortal — immortal / rectangular — oblique / initial — terminal / coincident — intersecting / to lend — to borrow / to exceed — to fall short / to generate — to degenerate / equality — inequality / preceding — subsequent / to be aware — to be ignorant / minimize — maximize / civil — military / truth — falsity.

V. *In the following sentences explain the use of one of the synonyms.*

to recognize, to acknowledge, to admit, to confess

1. R. Descartes did **recognize** some values of studing traditional philosophy, nevertheless he **acknowledged** that it could not serve as the foundation for the precise sciences as "real or veritable truth was noticeable in it by its absence". 2. "The long chains of simple and easy reasonings by means of which geometers are accustomed to reach the conclusions of their most difficult demonstrations" he **admitted** led him to believe that "all things to the knowledge of which men is competent are mutually connected in the same way". 3. He claimed that a sound body of phyilosophy could be deduced only by the methods of geometers, for only they, he **acknowledged**, were able to reason clearly and to arrive at indubitable truths. 4. He **confessed** that the method of establishing new truths came to him in a dream while he was on one of his military campaigns.

VI. *Practise (back) translations of the following sentences.*

1. The equivalent equations have the same loci. 2. The points that are common to two loci form their interesection. 3. The graph of an equation in X and Y is a drawing that pictures the corresponding locus. 4. The construction of a graph is called curve plotting. 5. The locus of every equation of the first degree in X and Y is a straight line. 6. Any point P which satisfies the geometric conditions must satisfy the algebraic equation and conversely. 7. When a right circular cone (including both its upper and lower nappes) is cut by a plane a conic section will result. 8. If the plane does not pass throught the vertex of the cone the section is a parabola, an ellipse, or a hyperbola. 9. If the plane does pass through the vertex the section may be 1) a single point, 2) a straight line, 3) two intersecting lines, 4) two coincident lines. 10. All these loci are called conic sections or simply conics. 11. We may use the term **regular conics** to designate sections cut by the planes, that do not pass through the vertex. 12. We may use the term **degenerate conics** to denote those cut by the planes through the vertex. 13. The conic sections will be defined with reference to a focus and a directrix. 14. If the cutting plane is perpendicular to the axis of the cone the section will be a circle. 15. Coordinate systems are used to locate points in the plane or in space. 16. There are two systems of plane coordinates in general use, rectangular and polar. 17. The coordinates of a point are numbers that determine the position of a point in reference to a fixed figure (the frame of reference). 18. The projection of a point upon a straight line is the foot of the perpendicular dropped from the point to the line. 19. If the point lies on the line it is its own projection. 20. A point will be plotted when it is located by means

of its coordinates. 21. A pair of coordinates determines one and only one point P of the plane and conversely. 22. This correspondence between number pairs and points of the plane is called one-to-one correspondence. 23. It provides a means of passing from the analytic form of an expression to the geometric, and vice versa. 24. The graph of an equation is the locus of the points whose coordinates satisfy the equation. 25. Conversely, the equation of a given curve is satisfied by the coordinates of every point on the curve and by the coordinates of no other points. 26. A **line segment** is a part of a line which is terminated by the two points given on it. 27. A **directed line segment** is a line segment to which either a positive or negative direction is assigned. 28. By the **inclination** of a line in the plane of a rectangular coordinate system is meant the smallest angle, positive or zero, measured from the positive X-axis to the line. 29. The tangent of the inclination is called **the slope** of the line. 30. Analytic Geometry is a branch of mathematics in which one studies geometry by means of algebra. 31. The first systematic treatment of the subject was published by René Descartes in 1637.

LAB. PRACTICE

Grammar Rules Patterns

I. *Say the following sentences using Future Indefinite Tense-Aspect forms (Active or Passive Voice).*

Model. Analytic Geometry **unifies** Geometry and Algebra.
Analytic Geometry **will unify** both sciences.

1. Analytic Geometry **creates** an algebraic approach to Geometry. 2. Many problems of geometry **are solved** with the methods of algebra. 3. Scientists **had to study** the properties of curves referred to as conic sections. 4. The path of heavenly bodies and projectiles **directed and quided** the research and applications of conics. 5. Advances of science and technology **influence** the investigation and introduction of new curves. 6. Scientists **sought** for a general method to represent curves algebraically. 7. The devices of Analytic Geometry **replace** curves by equations through a coordinate system. 8. A coordinate system **locates** points in a plane by means of numbers. 9. In a plane the coordinate system assigns two numbers to a point viz., an abscissa and an ordinate. 10. The abscissa **denotes** the distance of the point from a fixed vertical reference line, called the Y-axis. 11. The ordinate **fixes** the distance of the point from a horizontal reference line, called the X-axis. 12. Distances to the right of the Y-axis, or above the X-axis **are designated** as positive; distances in the opposite directions **are denoted** as negative. 13. In Analytic Geometry points **become** pairs of numbers and curves **become** collections of number pairs expressed in equations. 14. The coordinates of the points that lie on the curve **satisfy** the equation. 15. The properties of curves **are deduced** by algebraic processes applied to the equations. 16. The properties common to all the curves and special characteristics wherein they differ from each other **are discussed**. 17. The continuity with which the curves pass into each other **appear** from the definition of a conic section as a Locus. 18. Analytic methods **hold good** in problems dealing with more than three dimensions. 19. In such problems there **exists** the difficulty in visualizing geometric objects. 20. Euclidean synthetic methods **fail to cope** with such problems. 21. Analytic Geometry **is appreciated** as the basis of all modern applied mathematics.

II. *Use the proper (Present, Past, Future) Indefinite Tense-Aspect form according to the time indicator.*

1. **Analytic methods of proof (to replace) classical geometrical reasoning**. (in this problem, last century, with the creation of Analytic Geometry, in future, at present). 2. **Three-dimensional coordinate system (to relate) algebraic equations and geometric figures in space.** (In three-dimensional geometry, in the next problem, in the theory of relativity, with the inauguration of abstract spaces, in the nineteenth century). 3. **Pure geometry (to be arithmetized) through the devices of coordinate geometry.** (nowadays, next decade, in the seventeenth century).

III. *Turn from Active into Passive.*

1. Analytic geometry founded an algebraic approach to geometry. 2. Analytic geometry replaced curves by equations through the device of a coordinate system. 3. The coordinate system will locate points in a plane or in space by numbers: an abscissa and an ordinate. 4. Kepler introduced effective methods of working with the conic sections in astronomy. 5. The coordinates of any point that lies on the curve will satisfy the equation. 6. The mathematicians can deduce the properties of the curves involved by algebraic processes applied to the equations. 7. The classical Greeks embodied algebra in geometry. 8. As solid Analytic Geometry fails to cope with slope and curvature — the fundamental properties of the curves — mathematicians must employ the differential calculus to deal with curves and surfaces. 9. Mathematicians created a new term to designate the study of calculating the rates of change of slope and curvature. 10. We refer to the study which yields such rates as differential geometry.

Means of Expressing Future Actions

I. *Join the sentences. Express action in the clause by using Present Indefinite Tense-Aspect forms after the conjunctions: if, when, after, before, till, until, unless, as soon as.*

Model. **We shall take** two reference lines intersecting at right angles.
We shall obtain a Cartesian coordinate system.
(Providing)
Providing we **take** two reference lines, we **shall obtain**...

1. You'll find the pair of values (x, y) — the abscissa and the ordinate of the point. You'll obtain the rectangular coordinates of the point. (as soon as) 2. They'll not indicate the position of the point. They'll plot the point. (until) 3. The coordinates of the point will be plotted. The point in the plane will be located by a pair of numbers. (when) 4. A pair of coordinates will not determine one and only one point of the plane. A given point of the plane will not be determined by one and only one pair of coordinates. (unless) 5. We'll apply the polar coordinate system. The system will locate a point by means of pair of values (ρ, θ). (provided) 6. You'll take the polar axis, the pole, and the radius vector. A more convenient and useful frame of reference will result. (as soon as) 7. Any collections of points and lines will not be used to set up a coordinate system. A frame of reference will not be obtained. (until) 8. We'll introduce a still more general frame of reference for the plane called a triangle of reference. Rectangular and oblique axes will be only special cases of it. (if) 9. Just as in a plane or in space we shall assign coordi-

nates to a point. We'll use a frame of reference. (when) 10. An equation will be given. The corresponding geometric locus along with its properties will be obtained. (before) 11. A geometric locus will be given whose points possess some common property. The corresponding equation will be found. (after) 12. The rectangular coordinate system in a plane will be established. It will provide a one-to-one correspondence between number pairs and points of the curve. (as soon as) 13. We'll write the equation of a certain line as $y=mx+b$. We shall refer to m and b as constants and to x and y as variables. (providing) 14. The correspondence pairing off values of x and y will be called a functional relationship. An equation in two variables x and y will estabish a certain correspondence between the numerical values. (if)

II. *Translate the sentences into English.*

1. Как только мы определим кривую, мы сможем найти уравнение геометрического места. 2. Когда будет необходимо, они выберут подходящую систему координат. 3. Пока это не будет сделано, точка на плоскости не будет определяться парой чисел. 4. В случае, если значение одной тригонометрической функции угла A будет дано, остальные функции будут однозначно определены.

III. *Paraphrase the following sentences using the constructions "to be going to", "to be to" to express the near future actions and obligation resulting from some convention or instruction.*

Model. The problem **will be** difficult.
to be going to The problem **is going to be** difficult.

1. **Will** they **start** a new series of experiments? 2. **Will** the work **be** very complicated? 3. They **will not take** this frame of reference for the problem. 4. What subject **will** she **specialize in**? 5. He **will make** a report on analytical methods. 6. We'**ll finish** the article concerning the study of the phenomenon. 7. The professor **will present** his viewpoint at the conference. 8. The authors **will not publish** the results of the experiment. 9. The study of such complex problems **will involve** exceeding difficulty. 10. They **will explain** the methods by which such results were obtained.

Model. We'll **coordinate** numbers with points.
to be to We **are to coordinate** numbers with points.

1. They'll **establish** a correspondence between numbers and the points of a given line. 2. We'**ll choose** the point which'**ll correspond** to zero and indicate the unit of length. 3. There **will be** only one specific point corresponding to each positive and negative real number. 4. The directions in which measurements **will be made** may be clockwise or counterclockwise. 5. A system with which **we'll coordinate** numbers and points depends on the type of the problem concerned. 6. In most cases they'**ll employ** either rectangular or polar system. 7. Following the usual convention **we'll** first **choose** the point from which to start — the origin. 8. Then **we'll indicate** the directions in which the distances **will be measured** by means of coordinate axes. 9. The numbers which **will correspond** to a point **will be called** its coordinates. 10. These coordinates **will tell** us how far we must go in the direction of the x and y axes until the point is reached. 11. The adding of a third number **will specify** the location of a point in space. 12. To coordinate numbers to points in space we'**ll employ** a coordinate system consisting of three mutually perpendicular axes.

I. *Emphasize the revolutionary character and innovations of Analytic Geometry. The given statements may prove helpful.*

1. The Greeks' invention of pure forms and abstract shapes (e.g., cubic curves are often *S*-shaped) laid the basis for Euclid's geometry. 2. The Greeks thought of curves as tracings made by moving points. 3. Analytic Geometry merged all the arithmetic, algebra and geometry of ages past in a single technique. 4. Analytic method is a means of visualizing numbers as points on a graph, equation as geometric shapes and shapes as equations. 5. Thanks to Analytic Geometry every equation can be converted into a geometric shape and conversely. 6. Some shapes can be represented only by indefinitely long equations and some equations represent shapes hard to visualize. But every equation has its equivalent in algebraic form. 7. Out of Cartesian system emerged concepts fundamental to all higher maths: the ideas of "**variables**" and "**functions**". 8. If an x and y can be related through an equation or graph they are called "variables", i. e., one changes in value as the other changes in value. The two have a functional relationship. 9. In an ordinary algebraic equation y is a function of x if y's value changes when the value of x changes. 10. Cartesian system's basic contribution to maths was essentially a philosophical one. 11. By allowing a broad interchangeability of viewpoints, it gave rise to Analysis which encompasses much of higher maths. 12. Analytic Geometry was to grow far beyond Descartes's original brief presentation and was to touch nothing in maths without transforming it. 13. Descartes's association of equation and curve uncovered a new world of curves. 14. Since the number and the variety of equations is unlimited, so is the range of curves useful in various applications. 15. Descartes's association held forth prospects of new higher-dimensional spaces. 16. Nowadays Greek and Cartesian geometries are special cases in generalized geometries of n-dimensions.

II. *Answer the questions, using the words and phrases suggested. Work in pairs in class.*

Model. How did Analytic Geometry originate?
(as a new way of representing and analyzing curves)
Analytic Geometry originated as a new way of representing and analyzing curves.

1. Where did the words "ellipse", "hyperbola" and "parabola" first appear? (in the Pythagorean school) 2. What did these words mean at the outset (three different conditions resulted from a method of geometric solution) 3. What did these terms designate? (defect, excess, a placing beside, respectively) 4. Who invented Conic Sections? (Menaechmus, 350 B. C.) 5. How was Menaechmus led to discover conic sections? (trying to find a solution of the "Duplication of the Cube" problem) 6. What did Menaechmus do to obtain the curves? (cut cones with planes at different angles) 7. What were the curves discovered by Menaechmus called? (Menaechmian triads) 8. What was the main innovation introduced by Appolonius in his extraordinary "Conic Sections"? (obtained all the conics from one double cone or conical surface) 9. Who supplied the terms "ellipse", "parabola" and "hyperbola" referring to conic sections? (Appolonius) 10. Was Appolonius the only geometer in antiquity who fixed the position of a point in a plane by means of suitable coordinates? (the Egyptians and the Romans used them in surveying and the Greeks in

map-making) 11. Do we have to cut cones with planes to obtain conic sections nowadays? (By no means) 12. What is the simplest coordinate system or frames of reference we can obtain conic sections with? (a fixed straight line, a fixed point and a generator will do) 13. When were coordinate systems used in middle ages? (in the fourteenth century by Oresme in his "Latitude of Forms") 14. What did Oresme represent by means of a coordinate system? (certain laws by graphing both dependent and independent variables) 15. When did treatment of the conic sections once again claim the attention of researchers? (in the seventeenth century) 16. Why was the time ripe for present-day Analytic geometry only in the seventeenth century? (it had to await the development of algebraic symbolism) 17. Who devised the method of plotting graphs with x and y coordinates? (the French mathematicians R. Descartes and P. Fermat) 18. What will the adjective of the name R. Descartes be? (Cartesian) 19. How did the creators of Analytic Geometry approach the study of curves? (through plotting points and straight lines) 20. Why are the graphs so useful and helpful? (give a way of revealing a relationship between the measured things) 21. Why did Kepler introduce and apply conic sections in astronomy? (the curves describe the paths of celestial bodies better than the circle) 22. Which conic assumed the role of guide in mathematical astronomy? (the ellipse) 23. What motivated the engineering applications of the conics? (problems of projectile paths, the study of lenses for telescope, microscope, x-ray machines and other devices) 24. What was the objective the creators of Analytic Geometry set for themselves? (the cooperation of the best aspects of both algebra and geometry) 25. Why did they apply algebra for solving geometric problems? (algebra enables to reason about abstract and unknown quantities) 26. Was it the only gain available? (it mechanizes the reasoning process and minimizes the effort needed to solve problems)

III. *Agree or disagree with the following statements. Use the opening phrases suggested. Repeat the statement and devetop it further.*

That's right.	**Not quite so, I am afraid.**
Exactly. Certainly.	**I don't think this is just the case.**
This is the case.	**I doubt it. Far from that.**
I fully agree to it.	**Just the other way round.**
I accept it fully.	**Not at all. Quite the reverse.**

1. The association of curve and equation was the brand new thought. 2. R. Descartes and P. Fermat created a new method of solving geometric problems. 3. One does not generally distinguish between science and engineering. 4. Practically all applications of mathematics to the physical world depend on the coordinate geometry. 5. The theory of nomograms rests largely on Analytic Geometry. 6. Analytic Geometry accomplishes everything R. Descartes envisioned and expected. 7. Modern Analytic Geometry solves all geometric problems whatever. 8. Through Analytic Geometry the importance of mathematics was considerably decreased and diminished. 9. Drawings are the subject matter of Analytic Geometry. 10. Graphical methods are hardly known to the public at large. 11. Graphs have more visual appeal than formulas and tables. 12. Visual pictures permit mathematicians to reason from them and prove. 13. Analytic Geometry presents the basic ideas in a straightforward, colourless and matter-of-fact manner. 14. In higher dimensional geometry visualization is of great help. 15. There exists a frame of reference for n-dimen-

sional space. 16. The notion of a four-dimensional geometry is very helpful in studying physical phenomena. 17. The physical world should be regarded as four-dimensional. 18. The four numbers x, y, z and t, sometimes more than four, specify any physical event. 19. The notions of dimension and of higher-dimensional geometry are fascinating branches of mathematics. 20. They are the basis of the most sophisticated of modern scientific developments, including the theory of relativity.

IV. *Agree with the following negative statements, develop your answer further and keep the conversation going where possible.*

Model. P. Fermat was not concerned with a universal method in philosophy.
No, he wasn't. Nevertheless, he sought general methods in mathematics, and he did establish a general procedure of working with curves. But it's only one of his great accomplishments in mathematics.

1. P. Fermat's life was not adventurous. 2. P. Fermat was not a professor of mathematics at the University of Toulose. 3. P. Fermat did not practise teaching mathematics. 4. P. Fermat's leisure was not devoted to jurisprudence. 5. P. Fermat did not possess broad knowledge of medicine or military art. 6. P. Fermat enjoyed classical literature and even wrote verse himself but literary studies were not his real love and passion. 7. P. Fermat did not publish anything personally. 8. Many of P. Fermat's correspondents were not like he himself, but professional mathematicians. 8. P. Fermat's books, notes and his voluminous correspondence were not lost after his death. 9. P. Fermat did not acknowledge, due to his modesty, the outstanding value of his mathematical achievements. 11. There is no doubt that P. Fermat was the inventor and codiscoverer of coordinate geometry. 12. P. Fermat's challenging problems are not forgotten or abandoned. 13. One cannot overestimate the influence of P. Fermat's famous theorems in the development of modern mathematics. 14. The solutions of many difficult P. Fermat's problems are not established. 15. The proofs of his famous theorems are not obtained. 16. One cannot say that modern mathematicians reject and abandon P. Fermat's challenges as their goal and a constant source of new efforts.

V. *Explain in your own words the meaning of the statements and answer the questions posed.*

Model. Descartes was not the original creator of rectangular coordinates, was he? No, he was not. Why are they, then, usually referred to as **"Cartesian"**?
They bear this name because of many innovations and improved algebraic notation introduced by Descartes into his coordinate Geometry.

1. In the third appendix to his book "Discourse of the Method" Descartes deals with many basic ideas for solving equations that arise in connection with geometric problems — primarily the study of conic section by algebraic methods. He was the first to introduce the concepts of **a variable** and **a function.** Explain why Descartes had to introduce these concepts.

2. Descartes's variable possesses dual nature: it presents a) a line segment of varying length and constant direction or a continuous curve traced by a moving point coordinate; b) a continuous numerical variable that involves the set of all numbers expressing the line segment. Explain

how this dual image of a variable helped Descartes unite and interrelate Geometry and Algebra.

3. Though the appendix is entitled "Geometry", Descartes made in it his greatest contribution to Algebra and the whole of maths. It was celebrated Descartes's Rule of Signs "We can determine the number of true (positive) and false (negative) roots that the equation can have as follows: An equation can have as many true roots as it contains changes of sign from + to —, or from — to +; and as many false roots as the number of times two + signs or two — signs are found in succession". When applied to Cartesian coordinates this rule enables one to determine the sign (plus or minus) of coördinates. How? Explain.

4. As is often the case with the promulgation of a significant mathematical result this first statement of the relation between changes in signs of the successive terms of the polynomial and the nature of roots was not complete. Explain what Descartes's statement of the rule lacks.

5. Analytic Geometry was brought to bear on Number Theory. Negative numbers were not readily accepted by mathematicians. Thanks to Descartes's coordinate Geometry and his Rule of Signs negative numbers became legitimate in maths as directed ordinates. Explain.

6. Descartes anticipated many successive important discoveries and proofs in maths. Gauss gave (1797) the first proof of the Fundamental Theorem of algebra: Every algebraic equation of degree n has n roots. The insights of Descartes on this matter are of special interest because they are related to his famous Rule of Signs: "Every equation can have as many distinct roots (values of the unknown guantity) as the number of dimensions (i. e., degree) of the unknown quantity in the equation. It often happens, however, that some of the roots are false, or less than nothing". Explain the importance of this statement to Descartes's geometric study of curves, tangents, normals to a curve, etc.

7. The term "area" as it is understood in modern maths refers to real numbers. From Descartes the study of Geometry is reduced to the study of real numbers. Explain.

8. By means of Cartesian coordinates Gauss gave geometric interpretation of complex numbers. What does this phrase mean? Explain.

9. P. Fermat and R. Descartes came to develop Analytic Geometry almost simultaneously. Why was the time ripe for the creation of the synthesis of Geometry and Algebra in the mid of the seventeenth century? Explain.

10. Much terminology, like our classification of curves (and surfaces) into linear, quadratic, cubic and so forth, stems from our use of Cartesian coordinate systems. Some curves, however, such as many spirals, have intractable equations when referred to a Cartesian frame whereas they enjoy relatively simple equations when referred to some other skillfully designed coordinate system. What system? Explain.

VI. *Reproduce the topic "L. Euler", using the given sentences.*

1. Euler — a mathematician, physicist, astronomer, worker of mechanics, engineer, teacher concurrently — was a true 18th century scientist. 2. Euler was a great geometer though there are no drawings and constructions in his works. 3. Mathematicians like Euler are born, not made. 4. Euler calculated without apparent effort; he was never bored with lengthy computations which were his passion and hobby. 5. Euler loved beautiful formulas for their own sake. 6. Geometry was lost among countless formulas due to Euler. 7. Geometry became a precise science due to Euler. 8. Euler was never surpassed in devising algorithms for the solu-

tion of difficult problems. 9. Work at the St. Petersburg Academy comprised the most productive chapter in Euler's life. 10. While in Berlin (1741—1766) Euler carried out many commissions for the St. Petersburg Academy and gave expert advice. 11. Euler dealt with analytic functions only. 2. "A function of a variable is any analytic expression whatsoever composed of that variable quantity and numbers or constant quantities" (Euler). 13. "A curve is specified by analytic expression" (Euler). 14. Euler classified all the curves according to their **order**. 15. Euler's theory of all algebraic curves is distinguished by its generality and clarity of presentation. 16. Euler was the first to solve the equation in three variables corresponding to the surface of the second order (ellipsoid, hyperboloid, paraboloid). 17. Euler developed the notion of geometric and conformal transformations. 18. "The knowledge of Analytic Geometry and the theory of functions is necessary to master mathematical analysis" (Euler). 19. All modern topological theories stem from Euler's work. 20. Euler's notation for "e", "π", and "i" received universal acceptance. 21. Euler can be compared to Euclid. 22. "Euler's works are the best possible school for mathematicians" (Gauss). 23. One is still amazed by Euler's prolific writings. It seemed as if age had no effect.

VII. *What do we mean when we say.*

Eulerian a) analytic method, b) straight line, c) angles, d) integrals, e) variables, f) coordinates, g) spherical geometry, h) theorem of polyhedra, i) conformal transformations.

VIII. *Say it in English.*

Применение алгебры в геометрии имело к началу XVII в. долгую историю. Еще древние вавилоняне решили многие геометрические задачи, выражая искомые отрезки как корни численных квадратных уравнений. Важным средством геометрического исследования у греков служила геометрическая алгебра, в которой место вычислений занимало построение отрезков. Успехи символической и числовой алгебры в XVI в. явились основой гораздо более обширных приложений алгебры в геометрии, приведших к созданию новой аналитической геометрии. Алгебраическим решением геометрических задач занимались многие выдающиеся математики. Предметом аналитической геометрии является не только нахождение отдельных отрезков, выражаемых корнями уравнений с одним неизвестным, но изучение свойств различных геометрических образов (линий, поверхностей), выражаемых уравнениями с двумя или более неизвестными или координатами. Координаты появились еще в древности, в различных формах, между собой непосредственно не связанных (астрономические, географические). Координатные отрезки древнегреческой геометрии стали известны в Европе благодаря трудам Архимеда и особенно Аполлония. К разработке начал новой аналитической геометрии независимо друг от друга и одновременно приступили оба крупнейших французских математика XVII в. — Ферма и Декарт. Основные уравнения конических сечений представляют собой у Ферма непосредственное выражение в терминах алгебры их свойств, известных по «коническим сечениям» Аполлония. «Аналитическая геометрия» Ферма, долгое время остававшаяся в рукописи, не нашла того широкого распространения, какое получила «Геометрия» Декарта. Изложение аналитической геометрии у Декарта во многом отличается от данного Ферма. Декарт не только применял более развитую символику и его изложение было доступнее и богаче примерами, он выдвинул несколько общих идей и предложений, весьма существенных для последующего. Декарт вводит метод прямолинейных координат и понятие об уравнении кривой, а вместе с тем понятие о функции как аналитическом выражении, составленном из «неопределенных» отрезков x и y. Декарт объяснил, как описывать кривую или, вернее, строить любое число ее точек, вычисляя значения x по данным значениям y — первой координатой у него служила y. Декарт приводит первую общую классификацию алгебраических кривых в зависимости от степени их уравнений. Он формулирует фундаментальное предложение об инвариантности рода кривой при замене одной системы координат другой. Историки математики немало спорили о том, имелась ли у Аполлония аналитическая геометрия и было ли творчество Ферма и Декарта в этой области новаторским. Ответ зависит от определения термина «Аналитическая геометрия», который понимается по-разному. Несомненно, что оба ученые чрезвычайно многим были обязаны древним и что в теории конических сечений они не внесли каких-либо новых теорем, а также не построили ее в чисто аналитическом виде. И вместе с тем Ферма и Декарт заложили фундамент поистине новой геометрии, в которой впервые нашло явное выражение понятие о функции, заданной формулой. Благодаря им начинает развиваться чисто аналитический метод исследования геометрических образов, не нуждающихся в обращении к геометрическим построениям и опирающийся лишь на алгебраическое исчисление. Дальнейшая разработка аналитической геометрии связана с трудами **Г. Лейбница**, **И. Ньютона** и особенно **Л. Эйлера**. Средствами аналитической геометрии пользовался Ж. Лагранж при построении аналитической механики и Г. Монж в дифференциальной геометрии. **В аналитической геометрии на плоскости** подробно изучаются геометрические свойства **эллипса, гиперболы** и **параболы.** Кроме конических сечений систематически исследуются так называемые алгебраические линии первого и второго порядков — эти линии в декартовых прямоугольных координатах определяются соответственно алгебраическими уравнениями первой и второй степени. В **аналитической геометрии в пространстве** также пользуются методом координат. При этом декартовы прямоугольные координаты x, y, z (абсцисса, ордината и апликата) точки вводятся в полной аналогии с плоским случаем. Каждой поверхности S в пространстве можно сопоставить ее уравнение $F(x, y, z)=0$ относительно системы координат $Oxyz$, и свойства этой поверхности выясняются путем изучения аналитическими и алгебраическими средствами свойств уравнения этой поверхности. Подробно исследуются так называемые алгебраические поверхности первого порядка (плоскости) и второго порядка (**эллипсоиды**, однополостной и двуполостной **гиперболоиды**, эллиптический и гиперболический **параболоиды**). Ныне аналитическая геометрия не имеет самостоятельного значения как наука, а лишь как раздел и метод геометрии. Однако ее методы широко применяются в математике, физике и других науках, и значение этих методов трудно переоценить.

IX. *Read the text and dispute the problem involved. Sum up the discussion. Use the following phrases.*

My point is that...
It seems reasonable to say...
I can start by saying...
Summarizing the discussion ...
On the whole. In the long run...
In conclusion I must say...

Four-Dimensional Geometry

Until almost the beginning of the present century the attitude of mathematicians and laymen alike toward geometries of more than three dimensions — if notice was given to them at all — was one of scepticism. It was generally thought that reference to physical considerations alone was sufficient to preclude the existence of more than three dimensions.

Aristotle said that a solid has magnitude "in three ways, and beyond these there is no other magnitude because the three are all". Ptolemy, as quoted Simplcius, said, "It is possible to take only three lines that are mutually perpendicular; two by which the plane is defined and a third measuring depth". To the Greeks no further explanation was necessary.

In the fourteenth century **N. Oresme** sought a graphic representation of Aristotlian forms as heat, velocity, sweetness and so on by laying down a line as a basis, called **longitude**, and taking one of the forms to be represented by lines perpendicular to this as a **latitudo**. The form was then represented by a surface. Taking a surface as a basis, with latitudo perpendicular at each point, a solid was formed. He even went on to take a solid as a basis and at each point considered an increment. But he rejected a fourth-dimensional figure out of hand and considered instead the solid as consisting of infinitely many planes. Taking perpendiculars at each point of each plane, the result was an infinite set of intersecting solids. However, he did use the phrase "fourth dimension".

Girolamo Cardano in his Ars magna (Great art) of 1545 referred to powers of numbers in geometric terms, saying, "The first power of a number refers to a line, the square to a surface, the cube to a solid, and it would be fatuous indeed for us to progress beyond for the reason that it is contrary to nature".

René Descartes tried to find a graphic representation of the motion of freely falling bodies. He said that if a body is acted on by one accelerating force, motion is represented by a triangle. If it is affected by two forces, it is represented by an angular pyramid. But if acted on by three forces, it is represented "by other figures". What these figures are Descartes did not attempt to point out.

John Wallis's "Algebra" of 1685 includes the passage: "A line drawn into a line shall make a Plane or Surface; this drawn into a line shall make a Solid; but if this Solid be drawn into a line, or this Plane into a Plane, what shall it make? A Plane-Plane? That is a Monster in Nature, and less possible than a Centaure. For Length, Breadth and Thickness take up the whole of Space. Nor can we imagine how there should be a Fourth Local Dimension beyond those three".

Some progress is noted in the eighteenth century. A more modern note was struck by **Jean Le Rond D'Alembert** in 1754. "I stated above that it is impossible to conceive of more than three dimensions. A man of parts, of my acquaintance, holds that one may, however, look upon duration as a fourth dimension. This idea may be challenged but it seems to me to have some merit other than that of mere novelty".

Joseph Louis Lagrange, in 1787, said, "Since the position of a point in space depends upon three rectangular coordinates, these coordinates in the problems of mechanics are conceived as being functions of t. Thus we may regard mechanics as a geometry of four dimensions, and mechanical analysis as an extension of geometrical analysis".

Most scientists rejected four dimensions because in nature we do not know of any quantity which has more than three dimensions. There are, however, several references in the nineteenth century to four dimensions in occult, mystical and theological writing. With the advent of **Einstein**'s theory of relativity and subsequent discussions of time-space and the possible curvature of our three-dimensional in a four-dimensional space, four-dimensional concepts are spoken of calmly, and we now realize that the physical existence or nonexistence of a four-dimensional body has nothing to do with its existence as a mathematical entity.

What is a four-dimensional geometry? Approached pictorially the concept has no meaning. But we can think about four mutually perpendicular lines, i.e., four lines each perpendicular to the other three. A point in a four-dimensional space may also be regarded as represented by four numbers or coordinates (=the distance one must proceed along the four axes to reach that point). Thus, these coordinates are written (x, y, z, w).

How can one think of geometric figures in four-dimensional space? The most convenient way is through the language of Analytic Geometry, i.e., equations. The figures of four-dimensional geometry exist in the same sense as do the figures in two and three dimensions. The hyperplane is as "real" as the straight line and plane; and the hypersphere as "real" as the circle and sphere. The same applies to all other objects of higher-dimensional geometry. The difficulty most people experience in accepting a four-dimensional geometry and the corresponding equations is due to the fact that they confuse mental constructions and visualization. All of geometry deals with ideas that exist in the human mind only. Fortunately, people can visualize or picture two- and three-dimensianal ideas by means of drawings on paper. No one can visualize four-dimensional structures, unfortunately; one must rely on the mind alone. One can study four-dimensional figures in terms of two- and three-dimensional **sections** of these figures. As a matter of fact, it is possible to visualize **sections** of figures in four-dimensional space. One finds the equation of the section first and obtains its shape with the knowledge of ordinary two- or three-dimensional coordinate geometry. Thus the problem of studying a figure in four-dimensional space is reduced to that of studying figures in three- and two-dimensional space.

COMPOSITION

Read and translate the text. Write a technical description, i. e., *an o b j e c t i v e description of the structure of a thing or concept. Some type of coordinate systems in common use should be described (e. g., oblique, affine, cylindrical, etc.).*

Frames of Reference

Much terminology, like our classification of curves (and surfaces) into linear, quadratic, cubic and so forth, stems from our use of Cartesian-coordinate system. Some curves, however, such as many spirals, have intractable equations when referred to a Cartesian frame, whereas they enjoy relatively simple equations when referred to some other skilfully designed coordinate system. Particularly useful in the case of spirals is the polar-coordinate system.

Polar Coordinates. Points are most commonly described today by ordered pairs (x, y) in the Cartesian system, where x is the directed distance from the vertical axis and y is the directed distance from the horizontal axis. For certain kinds of curves, however, a more convenient and useful form of representation is that of polar coordinates. The polar ordered pair is (r, θ), where θ is the angle the vector makes with the reference line or polar axis and r is the length of the vector.

Isaac Newton was the first to think of using polar coordinates. In a treatise **Method of Fluxions** (written about 1671) which dealt with curves defined analytically, Newton showed ten types of coordinate systems that could be used; one of these ten was the system of polar coordinates. However, this work of Newton was not published until 1736; in 1661 **Jakob Bernoulli** derived and made public the concept of polar coordinates; his polar system used for reference a point on a line rather that two intersecting lines. The line was called the "polar axis" and the point on the line was called the "pole". The position of any point in a plane was then described first by the length of a vector from the pole to the point and second by the angle the vector made with the polar axis. After Bernoulli, **Jakob Hermann**, in a paper of 1729, asserted that polar coordinates were just as useful for studying geometric loci as were the Cartesian coordinates. However, Hermann's work was not well known, and it remained for **Euler**, about twenty years later, to make the polar coordinates system really popular.

Further coordinate systems were not investigated until toward the close of the nineteenth century, when geometers were led to break away from the Cartesian systems in situations where the peculiar necessities of a problem indicated that some other algebraic apparatus is more suitable. An interesting development in coordinate systems was inaugurated by **J. Plücker** in 1829, when he noted that our fundamental element need not be the point but can be any geometric entity. This, in turn, led Plücker to the concept of the dimension of a manifold of geometric entities as simply the essential number of coordinates needed to determine one of the entities in the manifold. The first nebulous and vague notions of a hyperspace which is n-dimensional ($n>3$) in points are lost in the dimness of the past and were confused by metaphysical considerations. Aristotle, for example, in his "Physics" talked of six dimensions, but these were up and down, before and behind and right and left.

Nowadays any collection of points, lines, curves or any geometric entities whatever that are used to set up a system of coordinates for the points of a plane is called a **frame of reference** for this plane. It is customary, that a pair of rectangular or oblique axes, their point of intersection (0,0) and the two so-called unit points on the axes, namely (0,1) and (1,0) consitute such a frame of reference for coordinates in the plane. One designates as the ordinary frame of reference in a plane the frame consisting of a pair of rectangular axes with the same-sized unit on each axis. The formulas for the distance between two points, for the angle between two lines, for the area of a triangle, the equations of conics and many other formulas and equations in Analytic Geometry presuppose the use of this common frame of reference.

Later on a still more general frame of reference for the plane called a **triangle of reference** can be introduced. It is obvious that any triangle ABC can be used as a triangle of reference and if we join any point P in the plane to the vertices A and B of this triangle, the lines AP and BP will cut the sides CB and CA respectively in points whose coordinates are assigned to P. Rectangular and oblique axes are only special cases of a triangle of reference. Just as in a plane, so on a line and in space whenever we assign coordinates to a point we shall use a frame of reference. For example, when we attach to the points P_1, P_2, P_3 ... of a line L the coordinates X_1, X_2, X_3 ... respectively, we shall employ a frame of reference composed of any two distinct points on L to which we give the coordinates 0 and 1.

COMPREHENSION EXERCISES

Questions

1. What is your favourite subject: algebra or geometry and why? 2. Can Analytic Geometry be described as the "royal road" in geometry that Euclid thought did not exist? 3. Why is Cartesian Geometry the essential base for modern applied sciences? 4. How can one set up a two-, three-, four-, n-dimensional coordinate system? 5. What is meant by Cartesian coordinates? 6. What do we call the point 0 in different frames of reference? 7. What is meant by the axis of abscissas? 8. What is meant by the axis of ordinates? 9. In what order are the four quadrants formed by the axes of coordinates designated? 10. What directions are considered positive (negative)? 11. How are points located in Cartesian coordinates? 12. What is meant by the distance? 13. What are polar coordinates? 14. What is the principal value of polar coordinates? 15. What other types of frames of reference are worthy of investigation? 16. Which is the most convenient coordinate system and why? 17. Why were both Descartes and Fermat dissatisfied with the limited methods of Euclidean Geometry? 18. Is algebra really a universal science for an analytic method? 19. What was Descartes's point of view concerning algebra? 20. Was Descartes alone in the history of mathematics to interprete algebra as a universal language? 21. What can you say about Leibnitz's objective to set up such a language? 22. Actually, both Descartes and P. Fermat were very much interested in optics, weren't they? 23. Descartes published an essay on the passage of light through lenses. Was he concerned with light rays, the structure of lenses or conic sections in this research? 24. P. Fermat contributed several fundamental laws to optics. What are they? 25. How can one approach a four-dimensional geometry? 26. If a curve of a four-dimensional space lies in a plane can it be visualized despite the fact that it is part of four-dimensional world? 27. Do mathematicians actually believe in the real existence of a world of four spatial dimensions and hope some day to train our visual apparatus to perceive this world?

Discussion

1. "As long as algebra and geometry proceeded along separate paths, their advance was slow and their applications limited. But when these sciences joined company, they drew from each other fresh vitality and thence forward marched on at a rapid pace toward perfection" (J. L. Lagrange). Prove it or disagree.

2. R. Descartes claimed that he ought to find the simple, clear and distinct truths that could play the same part in his philosophy that axioms play in mathematics proper. The results of his search are famous. From the one reliable source — his consciousness of self — he extracted the building blocks of his philosophy: a) I think, therefore I am; b) each phenomenon must have a cause; c) an effect cannot be greater than the cause, and d) the ideas of perfection, space, time and motion are innate to the mind and could be obtained only from the existence of a perfect being, who is God. Therefore God exists. With Descartes theology and philosophy parted company. Characterize R. Descartes's philosophy (my-

stical, metaphysical, theological, rational, mathematized, etc.). Is it tied only to a particular time of Descartes or is it up-to-date? What produced co-ordinate geometry: Descartes's philosophical interest in method or his intellectual delight in mathematical activity?

3. In contrast to Descartes's adventurous, romantic and purposive life P. Fermat's was dull, highly conventional, and matter-of-fact. He lived quietly, ignored problems involving God, man, and the nature of the Universe and devoted his spare time to mathematics. Whereas to Descartes mathematics served to solve philogophical and scientific problems and to master nature, to P. Fermat the subject offered beauty, harmony and the pleasures of contemplation. Characterize P. Fermat as a mathematician — the greatest "amateur" in the history of mathematics. His accomplishments and anticipations, his famous theorems, his method of proof.

4. **Scale, chart, drawing, picture, graph, diagram, nomogram, frame of reference**. Their characteristics, distinctions and applications.

5. **Conic Sections**. Their mathematical properties and applications.

6. Advantages and disadvantages of the algebraic language of Analytic Geometry. Consider the equation $x^2+y^2=25$, representing a circle. Where is the rounded figure, the path that knows no end, the beauty of the most perfect shape? Does this formula represent all the properties of the geometric circle? Algebra replaced geometry, the mind replaced the eye. Is it a convenient way, after all?

7. The word "focus" (F) means "a hearth" or "burning-place" in Latin. Why? Suppose that the parabola is a cross-section of a reflecting surface and it is held so that its axis points to a distant star. The light rays will come in practically parallel to the axis of the parabola, will strike the parabola, and will be reflected to the point F (focus). Hence there will be a great concentration of light at F enabling scientists to view the distant star more clearly. What will happen to an object placed at F if the sun is viewed instead of a star?

8. N-dimensional Geometry. Illustrate it by some problems.

LESSON SIX

INTRODUCTION TO MECHANICS

Grammar:

1. Continuous Tense-Aspect Forms.
2. Compound Word-Building Patterns.

LAB. PRACTICE

Repeat the sentences after the instructors.

1. Mechanics is a classical subject which deals with motion of bodies and construction of machines and various mechanical devices. 2. **Mechanics is closely identified with physics and engineering.** 3. There exist many extensive branches of General or Theoretical Mechanics which have their own principles and independent significance. 4. All the fields and subjects of Mechanics apply the methods and equations of Theoretical Mechanics. 5. The primitive elements of Mechanics are **Bodies, Forces, Motion.** These basic elements are governed by assumptions, principles or **Laws** which describe Mechanics as a whole. 6. Mechanical Laws abstract, codify and record the common features of all mechanical phenomena. 7. Scientists are seeking for Laws because Laws are theory and experimental knowledge combined. 8. Extracting and deducing Laws is one of the great activities of a scientist. 9. Most Laws in science state the relationships between measurements of two (three) quantities. 10. Almost all scientific Laws can be reworded with the word "constant" as their essential characteristic. 11. Most mechanical laws were derived inductively from experiments. 12. In some cases, however, scientists deduce Laws or rules from some theoretical scheme. 13. The general principles in Mechanics are illustrated by **constitutive** equations which abstract the differences among **Bodies.** 14. The fundamental entity in Mechanics is the **material particle** and Bodies are considered as aggregates of such particles. 15. The general Laws of Mechanics apply to all Bodies and all Motions. 16. The elements in terms of which **Motion** is described are **Position in Space** and **Time.** 17. **Space** that is used in Mechanics is Euclidean three-dimensional; **Time** is absolute. 18. The assumption of absolute time did not lead to contradictions when applied to facts known up to the nineteenth century. 19. With the advent of Einstein's theory of relativity Mechanics came to be regarded as a geometry of four-dimensions and space became finite but unbounded. 20. The concept of time was changed: there is no absolute time according to the relativistic viewpoint. 21. It was in the concept of time that the classical and relativistic developments diverged. 22. Mechanics is seeking the simplest possible description of **how** bodies actually move, it makes no pretence of explaining **why** bodies move. 23. The description of motion involves general

principles stated in mathematical terms embracing all particular motions as special cases. 24. The natural philosophers of ancient Greece liked to do experiments in their heads. 25. Centuries later Galileo developed the "thought experiment", intended for the imagination only, into a fruitful method of inquiry of free fall. 26. In our time "thought experiments" appealed strongly to such scientists as Einstein and Fermi. 27. Of all the Forces playing part in Mechanics (contact, i. e., pushes and pulls, frictional, electrical, nuclear, etc.) the most important is the **force of gravity**. 28. The force of gravity acts on any object vertically downward and it is proportional to its mass or inertia. 29. Under the influence of gravity all objects fall with the same acceleration. 30. Throughout recorded history the speculations of civilized men on the nature of gravitation ranged from the naive (Aristotle: objects fall to the earth, because that is their natural place) to the sophisticated (Einstein). 31. Newton was the first to recognize that the force of gravity is only a special case of a general attraction between any two masses. 32. This general attraction is responsible for keeping the earth and other planets on their courses around the sun. 33. Every scientific theory, though speculative in its character, is meaningful only if it can be tested and verified by **Experiment**. It dies if it fails such tests. 34. A genuine understanding of theory and its relationship with experiment is essential if one wishes to know science. 35. Einstein presented a new concept of gravitation. There is, he claimed, no absolute force of gravity pulling objects down. 36. On the contrary, every mass has within it a force in proportion to its mass which attracts objects to it. 37. This attraction force of masses is also responsible for the curvature of the universe and for variations in orbits of celestial bodies. 38. Today many scientist firmly believe that **Einstein's general theory of relativity** which explains gravity as a curvature or warping of space and time, is the correct theory of gravitation. 39. They praise its beauty and agreement with observation and experiment. 40. Other workers, however, are openly dubious of the general relativity and suggest that alternative theories provide a better description of gravity. 41. The general theory of relativity nowadays has many competing successors. 42. For such new theories to be viable they have to meet observational and theoretical criteria that are steadily becoming more rigorous. 43. The general theory of relativity predicts that accelerated masses radiate gravitational waves, i.e., gravitational fields propagating with the speed of light. 44. Such waves resemble electromagnetic waves as they carry energy, momentum and information. 45. Whereas electromagnetic waves interact only with electric charges and currents, however, gravitational waves interact with all forms of matter energy. 46. Experiments designed to detect gravitational waves record evidence that they are being emitted in bursts from the direction of the centre of all galaxy. 47. The origin of the observed gravitational radiation is not determined, only the direction of its arrival. 48. These findings are stimulating much theorizing (conjecturing) and a good deal of disagreement among astrophysicists and gravitationalists. 48. It is conjectured that the source might be an unusual object such as a pulsating neutron star very much closer than the galactic centre. 50. It is conceivable that the mass at the galactic centre is acting as a giant lens, focussing gravitational radiation from an earlier epoch of the universe. 51. Since gravitational radiation is not appreciably absorbed by matter, the authors of this hypothesis maintain, it should have been accumulating since, perhaps, the beginning of **Time**. 52. The relatively large radiation intensity apparently observed may be telling us when the **Time** began. 53. In the past few years several versions of Einstein's "thought experiments" were carried out with real apparatus to verify some of the new hypotheses. 54. The current existing mechanical theory is the **Quantum Theory.** Quantum Mechanics domain begins in the nucleus and extents as far as the solar system.

Key Grammar Patterns

Continuous Tense-Aspect Forms

Tense	*Active*		*Passive*	
Present	am is are	asking	am is are	being asked
Past	was were	asking	was were	being asked
Future	shall will	be asking	—	

Continuous-Aspect forms are used to express an action which is **going on (=is in progress)** at a definite moment of Present, Past or Future Tense. Time **indicators are** not always necessary and such adverbial modifiers as *now, at present, always, constantly, all day, all that year, the whole morning, at this time tomorrow*, etc. are found. As a rule, the precise time limits of the action are not specified.

It should be borne in mind that though most English verbs can be used in the Continuous Aspect, some of them, however, do not admit of it. This refers to verbs denoting actions of unlimited duration, of physical perception, of emotions, mental processes, etc. such as **to hear, to notice, to watch, to see, to hate, to like, to love, to desire, to want, to wish, to appreciate, to assume, to imagine, to know, to mind, to think, to recognize, to understand, to agree, to be, to feel, to find, to seem, to satisfy, to succeed, to suffice, to value,** etc. However, in spoken English some of the verbs listed may be occasionally used in the Coninuous Aspect; in this case the Continuous Aspect gives them emotional colouring. Thus, the division of verbs into those which admit of the Continuous form and those which do not as a rule admit of it, cannot be explained by any grammatical reasons but is purely semantically coloured and traditional.

Translate the following sentences and try to explain the use of the Continuous-Aspect forms of the predicates.

1. Forces are pushes and pulls, i.e., the things you feel (=you'**re feeling**) when they are acting on you; things that **are stretching** springs; things that **are making** moving bodies accelerate. 2. We can compare the results that **are being predicted** theoretically with those observed experimentally. 3. Since Dynamics is the science of motion, the question at once arises: what **is moving** (=moves) and what are the simplest elements in terms of which its motion may be described? 4. All forces **are always occurring** (=occur) in pairs which may be conveniently spoken of as action and reaction. 5. Large heavenly bodies **are moving** (=move) in regular orbits. 6. Something **is being interfered** (=interferes) with their straight-line motion. 7. They **are being attracted** (=are attracted) to some centre of force around which their circular motion occurs. 8. Galileo **was experimenting** and **thinking** and **teaching** new scientific knowledge of Mechanics. 9. The moon **must be revolving** around the earth because the earth **is attracting** it (=attracts it). 10. Whatever speed a space-ship of the future attains, it **will be going** in a straight line forever, unless it enters the gravitational field of another celestial body.

Compound Word-Building Patterns

Identify the parts of speech of the following compound words and give their Russian equivalents.

Sunshine, afternoon, beforehand, son-in-law, blackboard, letter-box, daybreak, nightfall, dark-grey, newly-discovered, the-Sun-at-centre-picture, a number-with-just-one-figure-in-front-of-the-decimal-point (2.3), single-handed-efforts, framework, straight-line motion, throughout the world, up-to-date methods, seventeenth-century-mathematicians, long-accepted idea, air-resistance formula, world-famous masterpiece, two-thousand-years-old-tradition, inverse-square-law-gravity, rate-of-gain-of velocity, steam-driven-engine„ action-at-a-distance-mysterious-force.

Great mathematicians once had difficulties with what today are **fairly-well-clarified concepts.**

THE INTRODUCTORY TEXT

THE SCIENTIFIC METHOD

Ancient scientists sought to learn **what** happens and **how** things happen and for many centuries wanted to explain **why** things happen. Greek thinkers founded new methods in science — they began to look for **general schemes**, which could account for and predict separate facts. About 2500 years ago there appeared scientists or natural philosophers trying to describe nature. They collected facts and combined them into distinct groups seeking to reveal properties common to the behaviour of the whole group and develop a theory. Thus, from the Greeks to Galileo science was being built by collectors, accurate observers, makers of schemes, authoritarian philosophers. The collectors gathered a lot of knowledge which by itself was too diverse to be called science. The scheme-makers organized this knowledge and extracted rules that were good working prescriptions, able to summarize the facts and often to make predictions. Rules and knowledge together with techniques for gaining more knowledge made the beginning of the genuine science.

The Greeks **deduced** their explanations and schemes for nature from a few general ideas which they just assumed, e. g., from "circles are perfect", they deduced epicycloids. In the course of the 17th century this kind of deductive reasoning fell into disfavour as it was really philosophical speculation flourishing with authority rather than science. By the middle of that century **Experiment** came to be regarded as the real source and test of science. Scientists were concerned and occupied with extracting rules or laws by **inductive reasoning from experiment.** In doing this, they too were making assumptions: that nature is simple and uniform, i. e., that in the same conditions the same behaviour occurs again and again. They still assumed that there are **causes** for things, but the meaning of causality remained as difficult a problem as ever. As general working rules emerged, e. g., Copernicus's Sun-in-centre scheme, Hooke's Law, Kepler's Laws, the sense of security and comfort increased and the early belief that nature is definite and reasonable gained ground as a basic belief in science.

Though this inductive method was an honest one leading to good rules, it lacked the general tying-together and mental satisfaction that a grand theory can give. **Newton**, with greater insight and judgement was the first to look at experiment, then reversed to theory and worked back **deductively** from theory, predicting results that could be tested and verified. This brought theory back into science as a framework of thought, but in a more respectable and responsible form. Theory was again considered valuable but this time as a servant to science rather than as master.

Later still, say in the last century, theory was subjected more and more to the test of productiveness. Scientists were asking: "Can this theory make (further) predictions?" If not, it was to be modified. In this century this seems too harsh a treatment for theory. Its use may lie not only in its ability to make predictions but also in the frame of thinking that it offers us. Nowadays scientists are following Galileo and Newton in one scheme of scientific method: scientists plan and frame their experimenting carefully and treat it by a formal system of inductive reasoning and testify, i. e., they collect information, extract rules, frame hypothesis, deduce consequences, test deductions, verify the results, etc.

Scientists are setting forth **ideal schemes** (=models) for science, but if you watch scientists at work you will see that there is **no one** scientific method. Science does not develop as a simple rigid chessgame; nor is the progress just a series of forward leaps. A first-round of thinking and experimenting may even lead back to the starting point, but, as with seeing a movie over again, we have a richer knowledge with which to pursue the second round. In the real development of science we approach our problems and build our knowledge by **many** methods: sometimes we start by guessing freely, sometimes we build a model for mathematical investigation, and then make experimental tests. Sometimes we just gather experimental information with an eye open for the unexpected; sometimes we plan and perform one great experiment and obtain an important result directly or by statistical sorting of a wealth of measurements. Sometimes a progressive series of experiments carries us from stage to stage of knowledge — the results of each experiment guiding both our reasoning and our planning of the next experiment. Sometimes we carry out a grand analysis thinking from stage to stage with a gorgeous mixture of information, rules, guesses and logic, with only an occasional experimental test.

Yet, **experiment** is the ultimate touchstone throughout good science, whether it comes at the beginning as a gathering of empirical facts or at the end in the final tests of a grand conceptional scheme. How far scientists' theoretical thinking will be developing at a given time depends on the state of knowledge and interest — on whether the time is ripe. When the time is ripe the same problem is often attacked by many scientists simultaneously and the same solution may be discovered by several. Yet, one scientist may get the credit for reaping the harvest — quite right if he is the only man with enough insight or skill to carry the innovation through. In Newton's day new interest in Motion, general thinking about the planets, Kepler's discoveries, new studies of magnitism and light phenomena, new attitudes towards experimenting and scientific knowledge — all made contributions of facts and approaches — the time was ripe for the great development. Hooke, Wren, Halley, Huygens and many others were all seeking to reach a unified theory for celestial and earthly motions. Each succeeded in grasping some parts of the solution, but it was Newton who gave the complete solution in one grand theory, making "not a leap but a flight".

ACTIVE VOCABULARY

1. to account (for)	13. to disturb	25. to predict
2. to annihilate	14. to diverge	26. to predominate
3. to attach	15. to exert	27. to prejudice
4. to behave	16. to experience	28. to propagate
5. to calcel	17. to guess	29. to repel
6. to clarify	18. to induce	30. to resist
7. to coincide	19. to insulate	31. to revise
8. to collide	20. to interfere	32. to scrutinize
9. to conduct	21. to modify	33. to speculate
10. to conserve	22. to obey	34. to spread
11. to discard	23. to occur	35. to transfer
12. to distribute	24. to penetrate	36. to transverse

Read and translate the text in class. Generalize the main ideas of the text. Reproduce it following the outline given below.

1. Scientific Law is not a legislation or a decree.
2. Alexandrian Mathematicians — the forerunners of Modern Science.
3. Archimedes's discoveries.
4. The usefulness of scientific Laws.

TEXT ONE

SCIENTIFIC LAWS

What is a scientific Law? Who makes it, who obeys? Who uses it, the great thinker or the engineer? The use of the word "Law" in scientific literature is not fortunate. We know of no legislation or decree that established the rules we describe; it is probably more accurate to think of Laws as invented rather than discovered. Historical records reveal the origin of some laws; others are concealed, because the man who first proposed them did not let us know how they had occurred to him first.

Scientific knowledge grew up with the early civilizations from simple noticing natural phenomena to systematic observing. The observations were not real science but they set the pattern of a speculative scheme to "explain" the facts. When Greek civilization formed, the wisest thinkers brought a new attitude to observations: their aim was to make a scheme that could account for facts. This was a grander business than either collecting facts or telling a new tale for each fact. This was an intellectual advance, the beginning of great scientific theory. The **scientists of Alexandria** made more accurate observations, devising new methods and new mechanical devices, producing better and more sophisticated mathematical and astronomical theories.

The man whose work best epitomizes the character of the Alexandrian age is **Archimedes** whose fame was based for many centuries not upon the immortal achievements explained in his own works, but upon the legends around his name. These legends had a core of truth: he did invent machines, such as compound pulleys, burning mirrors, etc., but these activities were secondary, he was primarily a mathematician, the greatest of antiquity and one of the very greatest of all times. Archimedes lacked the encyclopedic tendencies of Euclid who tried to cover the whole field of geometry; he was, on the contrary, a writer of a number of works limited in their scope, but his treatment of any subject was masterly in its order and clarity.

The ingenuity of the mechanical devices invented by the Alexandrians in response to the new interests is astonishing even by modern standards. Most spectacular of those was Archimedes's huge mirror which concentrated the sun's rays on Roman ships besieging his native city of Syracuse. The ships were burnt under the intense heat. Perhaps the most famous of Archimedes's scientific discoveries is the **hydrostatic principle** now named after him. A story preserved in history tells how Archimedes was led to make his great discovery. The king of Syracuse ordered a crown made of gold. When the crown was delivered, the king suspected that it contained some baser metals; so he sent it to Archimedes and asked him to devise a method of testing the contents without, of course, destroying the workmanship. Archimedes pondered the problem and one day while bathing, suddenly grasped the principle that enabled him to handle the problem. Archimedes discovered that **a body immersed in water is buoyed up by a force equal to the weight of the water displaced.** Since the weight of the displaced water as well as the weight of a body in air can be measured, the ratio of the weights is known. This ratio is **constant** for a given metal no matter what its shape and differs from metal to metal. Hence Archimedes had to determine this ratio for a piece of metal known as gold and compare it with the corresponding ratio for the crown. Unfortunately, history does not record his decision.

The principle that Archimedes discovered is one of the first universal **Laws** of science; he incorporated it among others in his book "On Floating Bodies". Two branches of Mechanics — **Statics** and **Hydrostatics** — were founded on mathematical bases by Archimedes who must be called **the first rational scientist of mechanics**. Archimedes's creation of those two branches of Theoretical Mechanics is perhaps even more remarkable than his mathematical investigations. Two of his mechanical treatises came down to us. They both begin with definitions and postulates on the bases of which a number of propositions are geometrically proved. There was no other scientist comparable to him until the time of Galileo who was born more than eighteen centuries later. Possessed of a lofty intellect, great breadth of interests both theoretical and practical, extraordinary mechanical skill, fertile imagination and the inquiring mind, Archimedes was greatly respected and admired by his contemporaries.

The history of science is not simply the history of great scientists. When one investigates carefully the genesis of any discovery of a universal Law, one finds that it was gradually prepared by a number of smaller ones and the deeper one's investigation the more intermediary stages are found. The word "Law" is misleading, in fact. It is used in science for a relationship or description of behaviour discovered and which seems very general and appeals to us as simple and important. Most scientific Laws are first derived inductively from experience; others are first deduced from some theoretical scheme. Sometimes a different title is awarded: "principle" or "rule" or even the honest word "relation", for example, "The Principle of Conservation of Energy", "The Quantum Rules", "The Mass-Energy Relation $E=mc^2$". Thus, the words Law — Principle — Rule at present you may regard them as all much the same, all are **summaries** of what we find or think does happen in nature. There is a tendency to use the word "Law" for great simple outcomes of experiment, "Principle" — for general beliefs which are built into theory and "Rule" — for more working statements. Nevertheless, one can't appreciate the value of the discovery by the label alone.

We take it for granted that there are simple Laws to be found and they are true descriptions of nature when we do find them. But the modern philosophy of science warns us that we are being overconfident. **It** reminds us that our whole behaviour in seeking laws is artificial. The nature we codify is just our idea of nature. Our laws are man-made, because we make assumptions to suit our hopes. The equations scientists write seem to contain in themselves the working of the world; by their manipulation, it seems, one can manipulate nature itself. The most remarkable aspect is psychological. When a relation such as the rule of vector addition was first suggested as the rule for the combination of forces, the suggestion was tentative, but when buildings and bridges were being constructed for years following the rule, it became a **Law** of nature. And we often hear "forces are vectors". Of course, forces are **not** vectors — they are forces; but the association of forces with vectors is so successful that the distinction becomes blurred and obliterated.

In codifying our knowledge of nature in simple Laws, scientists are looking first for **constancy**; the mass of a body remains constant; total electric charge remains constant; momentum is conserved; all electrons are the same, etc. Almost as simple and equally fruitful is **direct proportionality** when two measured quantities increase together in the same proportion: stretch of a spring with its load; force and acceleration; gas pressure and gas density, etc. **Extracting Laws** is one of the great activities of a scientist, but there is imaginative thinking, too, and above all much scheming to combine Laws together, hoping to find a common key or to reveal new predictions. The diverse motions may be linked together by a common characteristic. Without the help of combined Laws, the common behaviour may remain unknown and some of the motions never put to use or fully understood.

So it is unfortunate that scientists say, "...obey ...Law". Scientifinc Laws do not command nature like policemen. Nor should we use them to "explain" the observations that suggested them — though they can throw light on other experiments. **Laws are**, rather, **simple guiding threads** which we draw from the tangled web (=nature) we study, the main threads of experimental knowledge which we weave into the fabric of science. Science gets nowhere if knowledge is just a vast tangle of facts or random observations. Thus, scientists are trying to organize facts into groups and extract common pieces of behaviour. They call the extracted statements or relation a "rule", a "law", occasionally a "principle". Hence **"Law" is a generalized record of nature,** not a command that compels nature.

Read the text. Be ready to speak in greater detail about the scientists mentioned in the text, their discoveries and contribution to science.

TEXT TWO

CELESTIAL MECHANICS

Astronomy is almost as old as man himself, it is older than mechanics or physics. Astronomy is that part of science which provides a clear example of the growth and use of **Theory** in science. In fact it got science started by showing the beautiful simplicity of the motion of the planets and stars, the understanding of which was the beginning of science. It deals with the history of our knowledge of the solar system from early watching and observations, simple fables to the magnificient success of Newton's gravitational theory and Einstein's Relativity theory. Every scientist is aware of this important and historical continuity in science. Discoveries in science are made only when time is ripe for them. Discoveries are followed by the development of more sophisticated theories. The growth of Celestial Mechanics is largely cyclic in nature. The first period was that of the ancient Greek scientists.

All the Greek scientists were trying to build a picture and a theory of the Universe founded on observed facts and speculation. The earliest of the Greek "natural philosophers" (**Thales** — 600 B. C., **Pythagoras** — 530 B. C., **Eudoxus** — 370 B. C.) tried to start with a few simple assumptions or general principles and draw from them as logically as possible a complete "explanation" of the observed natural behaviour. This explanation served to coordinate the information available and to make predictions, but above all, to give a feeling that there is a **pattern** that holds diverse behaviour together, that nature makes sense. Although some of the search for a good scheme was prompted by practical needs such as e. g., calendar-making, this delight in a unified clear explanation went far beyond that. Driven by an urge to ask "why", the Greek philosophers were seeking and making scientific theory. Though our modern tradition of experimenting and our modern wealth of scientific tools made great changes, we still hold the Greek delight in a theory that can account for the natural phenomena.

Aristotle (340 B. C.), the great philosopher, teacher and encyclopedic scientist was the "last speculative philosopher" in ancient astronomy. The Pythagoreans began the great debate concerning the earth's place in the world which continued for over 2000 years. The earth was the centre of Aristotle's universe which was closed, bounded at its outer edge by the celestial sphere, populated with air, earth, fire, water and celestial substance or the staff out of which the planets and stars were made. Each planet was following its natural circular motion about the earth in contrast to material bodies whose natural motions were up and down, as light things rose and heavy things fall to the earth. In supporting the scheme of concentric spinning spheres Aristotle gave a dogmatic reason: the sphere is the perfect solid shape and it prejudiced astronomical thinking about orbits for centuries. For ages Aristotle's writings were the only attempt to systematize the whole of nature. They were translated from language to language, printed and studied and quoted as authority.

In the course of these thanslations and retranslations errors were made resulting in a great deal of confusion and in a new profession: that of a **scholar** who spent his time trying to decide what was actually contained in these ancient texts. Aristotle's conception of the Universe — his own research, criticism, his repetition of hearsay and the ideas of others — became part of christian religion and an attack thereafter on Aristotle was an attack on the church itself. In Mechanics proper Aristotle discovered the law of the **lever** for vertical forces and proposed incorrect theory on moving bodies. His importance in the history of science is due to the fact that he was investigating in over twenty five different fields of knowledge and that until the Renaissance no comparable systematic survey of knowledge was made. His works are an encyclopedia of the learning of the ancient world and in every field he contributed something of value. He was probably the first to conceive the idea of organized and systematized research. The science of Aristotle was purely speculative and dogmatic not compared with and controlled by observation and experiment.

The astronomers of Alexandrian school **Aristarchus** (240 B. C.), **Hipparchus** (140 B. C.) and **Ptolemy** (120 A. D.) made more accurate observations and produced better theories. **Cl. Ptolemy** compiled the summary of the Greek effort to order the motion of the stars and planets in his book **Almagest** meaning "The Greatest of Books", which survived until modern times, exerting a great influence on all astronomical thought. The Ptolemaic picture of the Universe dominated astronomy for the next fourteen centuries. Ptolemy set forth the general picture: the heaven of the stars is a sphere turning steadily round a fixed axis in 24 hours; the Earth is a sphere at rest at the centre of the heavens. The sun moves round the Earth. The Ptolemaic system was believed and hardly questioned. Corrected, amended, revised with circles added to circles, it passed from generation to generation until the fifteenth century, when it lost its ethics appeal it once claimed. Medieval world of dogmas and printed authorities created an unbearable pressure and there appeared scientists who were building that rational world which was to dominate science thereafter.

In **N. Copernicus** (1473—1543) the dissatisfaction with the Ptolemaic picture was so great that he was led finally to challenge the hypothesis of the ancient cosmology and to change the picture of the Universe. He suggested Sun-at-centre (heliocentric) picture which was a revolutionary advance in science. He wrote a great book setting forth the details of his system, showing calculations of its size and predicting tests. After his death this view spread, though it was not universally accepted for a long time.

G. Bruno (1548—1600) burst the starry immobile sphere, he made the Universe endless, space infinite, earth and sun lost among countless other planets and suns. He was burned for his heresy. So, the Universe, no longer, according to G. Bruno, centered about a unique fixed point, was no longer filled or finite; objects were moving through this space uniformly from point to point, because the space on all sides is the same. **R. Descartes** (1596—1650) rejected all philosophical doctrines and dogmas putting aside all authorities, Aristotle in particular, doubted everything. His new method was to analyze complex notions into their constituents until the irreducible elements are simple, clear, distinct ideas. He described a world system as mechanical, i. e., in terms of general principles of Mechanics. He opposed the idea of vacuum and filled space with whirling vortices to carry the planets. God, R. Descartes claimed, created matter and endowed it with motion; after that the world is evolving by the laws of Mechanics without interference.

Tycho Brahe (1546—1601) a Danish nobleman and astronomer, who, fired with curiosity about the planets, became a brilliant observer, a genius at devising and using precise instruments. He built the first great observatory. He did not challenge the simplicity of the Copernican system; but simplicity was not a sufficient reason for him to accept the notion that the earth is moving. This refusal did not prevent him in his great work of mapping the position of the stars and the planets over a long period of time. It was soon clear from his observations that the Copernican orbits of the planets were only roughly correct. He constructed far more accurate planetary tables than anybody before him and left his pupil Kepler to complete their publication.

J. Kepler (1571—1630) a German was a striking contrast to T. Brahe, his former teacher, to brilliant **observer** who gathered data and recorded what he saw. Kelper — the **theoretician** — was fascinated by the power of mathematics and was seeking for a numerical scheme underlying the planetary system. Using Tycho Brahe's observations and data he extracted three General Laws for the motion of the planets without finding any explanation of these laws. After Kepler ("Legislator of the Heavens") the main question became: "What theory will give Kepler's Laws?" And the planets — what of them? According to Kepler they were no longer moving uniformly, no longer in circles, no longer in harmonic proportion. Kepler used an imaginary spoke connecting the sun to the planet and driving the planets in their orbits. He found simple curves — ellipses — along which the planets are moving, sweeping out equal areas in equal times.

Galileo Galilei (1564—1642) advocated Copernicus's picture. With a telescope of his own design he gained evidence supporting Copernicus's theory. To the dismay of the classical philosophers, and at his own peril, he preached the need to abide by **Experiment**. He was experimenting and teaching new scientific knowledge of astronomy and mechanics. The scope of Galileo's interests and activities was unbelievably broad even for a great intellect of the Age of Genii (XVII c.). He was always keenly interested in mechanical devices. At home, he kept a workshop in which he spent a great deal of his time. There he produced so many new and ingenious devices that he can be called the father of modern invention. The most fruitful creation of the myriad-minded Galileo was a grand plan for reading the book of nature. The key of the success of modern science was the selection of a new goal for scientific activity set by Galileo and pursued by his successors. This new goal was that of obtaining **quantitative description** of scientific phenomena independently of any physical explanation instead of unsuccessful qualitative and causal inquiries into nature. Galileo was the first to formulate explicitly this new plan and put it into effect by establishing a number of **fundamental Laws** and the **Scientific theory**, the connective tissue and a body of mathematical Laws.

Mathematical deduction, Galileo proved, produces knowledge of the physical world. While studying the motion of projectiles (e. g., cannon balls) Galileo observed that an object's motion can result from two **independent simultaneous motions**. The meaning of this discovery can be clarified by an example. An object dropped from an airplane flying horizontally possesses two motions. In accordance with Galileo's third Law of motion — **if one body is carried by another, the first shares the motion of the second** — one motion is straight out in the same direction as the plane is going, the other motion is straight down. The combination of these two simultaneous motions causes the object to travel downward along a curve, which, as Galileo pointed out, is path of a parabola. However, the horizontal and vertical motions of the falling object are independent of each other. The now famous rule — **Parallelogram Rule** (also called **the triangle** for the composition of forces, velocities, etc.) — was introduced first by **Simon Stevin** (1548—1620) an engineer with an immediate practical need for this knowledge. Just as different things (apples, stones, and people) can be counted with numbers, different physical quantities (e. g., forces, velocities) can be associated with **vectors.** The parallelogram addition of forces, velocities, etc. essentially implies that one vector does not disturb another: they act independently and just add geometrically. All through his experiments Galileo insisted that motions (forces, etc.) are independent of each other. For example, a constant horizontal motion and a vertical accelerated motion simply add by vectors — one motion does not modify the other but each has its full effect. Galileo preached this independence of vectors again and again in his problems and "thought experiments" and showed that a steady motion of the laboratory does not affect experiments on statics, free fall, or projectiles. A laboratory steady motion cannot be detected by any mechanical experiments inside. This is **Galilean Relativity** — one of his greatest discoveries. The potential value of this relativistic principle of motion is obvious enough and it was **A. Einstein** — the creator of relativistic theory — who capitalized on it.

Isaac Newton (1642—1727) with the tools, the insights, the knowledge set by the seventeenth-century-scientists and his own investigations and experiments created the first great mechanical theory which "explains" the whole Copernicus-Kepler-system. He restated and generalized Galileo's discoveries concerning mass, motion and force into clearly-worded "Laws of Motion" and proposed further that bodies attract one another according to universal inverse-square-law of gravitation $F = gM_1M_2/d^2$. Newton achieved one of the major objectives in Galileo's plan by showing that 1) the Laws of motion and gravitation are fundamental; 2) they apply equally well to so many varied situations on heaven and Earth; 3) all three Kepler's laws follow from the basic Laws of Motion and the Law of Gravitation. Newton's theory was world-wide success, it impressed educated people not only as the brilliant ordering of celestial nature, but as a model for other grand explanations to come. During the next two centuries further perfection of Newton's theory was made and consequences were worked out by other mathematicians and astronomers including the French mathematicians **J. L. Lagrange** and **P. S. Laplace**. One remarkable deduction from the general astronomical theory of Lagrange and Laplace is especially worth mentioning. This was the purely theoretical prediction of the existence and the location of the planet Neptune by two astronomers: **J. C. Adams** in England and **U. J. Leverrier** in France by its minute gravitational effects on the known planet. This discovery of a new planet was regarded as a great triumph of theory and the universal application of Newton's law of gravitation.

Early in this century, **Albert Einstein** (1879—1955) suggested modifications and reinterpretation of the laws of Mechanics. These did not destroy Newton's work but enabled scientists to explain many phenomena then unaccounted for. In addition to much modification of the "working rules" of Mechanics, the great value of Relativity lies in the light it throws on the relation between experiment and theory, ruling out unobservable things from even the speculation of wise scientists. It makes the model of the physical world more susceptible to proof by experiments. The major difference in point of view between Newton's and Einstein's theory of gravitation (General theory of Relativity) lies in their convention concerning geometry of space and time. In the Newtonian theory (mechanical theory) Space is Euclidean and particles that move on curved paths do so because of forces. In General Relativity (field theory) space-time is not Euclidean and particles always move in such a way that they traverse the shortest distance between any two points, given the constraints of space. These two points of view (mechanical and field theory), although there are important differences, lead in most cases of practical experience to almost identical results.

Read the text. Which theory (hypothesis) do you favour? Why are you "for" or "against" a particular theory? Are the advanced hypotheses presented in the text building a bridge from the universe to the microcosm or vice versa?

TEXT THREE

GRAVITATION

Gravity is a force that holds together the hundred billion stars of the Milky Way. It makes the Earth revolve around the Sun and the Moon around the Earth. There are three great names in the history of man's understanding of Gravity: **Galileo** who was the first to study in detail the process of free and accelerated fall; **I. Newton**, the first to have the idea of Gravity as a universal force. However, Newton admitted that he did not know its ultimate cause ("I will not feign hypothesis") but he offered many a keen guess at the nature and mechanism of gravitation; and **A. Einstein**, who said that gravity is nothing but the curvature of the four-dimensional space-time continuum.

In specifying gravitation on the new geometrical view Einstein did not prove "Newton's Law of Gravitation" wrong but offered a refining modification — though this involved a radical change in viewpoint. We must not think of either law as right (or wrong) because it is suggested by a great man or because it is embodied by beautiful mathematics; we are offered it as a brilliant guess from the real universe. The changes from Newton's predictions to Einstein's, though fundamental in nature, are usually too small in effect to make any difference in laboratory experiments or even in most astronomical measurements. By reducing Gravity to geometrical properties of a space-time continuum, Einstein concluded that the electromagnetic field must also have some purely geometrical interpretation. The **Unified Field Theory**, which grew from this conclusion had rough going and Einstein died without completing it. Some scientists claim that it is very odd indeed that the theory of gravitation originated by Newton and developed further by Einstein should stand now in majestic isolation, having nothing to do with the rapid development of other branches of science. This is not the case, however. The progress in Quantum Mechanics, modern cosmology and astrophysics makes this claim unjustified.

The World of Hypotheses

The universal law of gravitation is claiming more and more attention from scientists. When Newton described gravitation for the first time, he gave science the law of universal gravitation. Einstein "exploded" traditional classical notions about it, linking gravitation with the curvature of space with his General Theory of Relativity. In both cases gravitation was seen as a phenomenon of a cosmic scale, since gravitational fields are "perceptible" only with the existence of huge masses. Now scientists hope that it may provide a key to understanding processes in a microcosm, at the quantum level. It is at the junction of quantum and gravitational ideas that science can expect to make the most sensational discoveries. This expectation characterized gravitionalists' discussions at the 6th All-Union Gravitational Conference and at symposiums in Moscow and Leningrad (1984). What ideas do scientists have who are trying to connect the gravitational processes of the universe with the world of elementary particles? Here are just three of them.

Was Einstein Right?

The general theory of relativity, which is being brilliantly confirmed by experiments, the equations of which are used by astrophysicists to compute the gravitational fields of space objects, possesses fundamental difficulties which are not clarified to this day. The chief one is the problem of determining the energy of the gravitational field. In the framework of Einstein's theory this question remains a veritable "headache" for scientists. Opinions on how exactly to compute this energy invariably differ. Recently Academician **A. Logunov** and Professor **M. Mestvirishvili** advanced a new theory of gravitation, in which the energy of the gravitational field can always be determined. Unlike Einstein, Logunov and Mestvirishvili maintain that our world is homogeneous, while gravitational attractions in it are conditioned not by the curvature of space but by some physical force field like the electromagnetic field. They draw on the methods used in the field theory of elementary particles.

What is to be done with the equations of the general theory of relativity which faithfully serve science? Are they not in antagonistic contradiction with Logunov and Mestvirishvili's theory? In fact, they are not. They are perfectly consistent with it if another four equations are added. Moreover, the curvature of space, which is the main element for Einstein, plays only a secondary part in the new theory. It is interesting to note that all the experiments, which hitherto corroborated the general theory of relativity, also confirm Logunov and Mestvirishvili's theory.

So, what does the universe look like according to the new theory? Einstein's theory allows for the existence of different models of the universe — "open", "closed", etc. — but Logunov and Mestvirishvili allow for only one model. Their universe can only be "flat". This, in turn, presupposes the existence in it of some concealed, unobservable mass. Surpassing the mass of all galaxies taken together many times over, this invisible mass ensures the evolution of the universe as a flat world. As often happens with new theories, Logunov and Mestvirishvili's theory is hostilely being received by many gravitationalists. However, it is mathematically correct, not open to doubt. Nor is it at variance with known experimental data. To solve the question of which is correct — Einstein's general theory of relativity or the theory advanced by Logunov and Mestvirishvili — there will have to be more experiments.

...The Incredible Kerr Disc!

Most people invariably tend to associate UFO's — one of the enigmas of our civilization — with antigravitation; it is mankind's cherished dream to master antigravitation. But however splended this dream may be, neither theories nor experiments of modern science provided any grounds for optimism. However, there is already a distant glimmer of hope. **Physicist A. Burinsky** of Moscow forwarded a hypothesis suggesting that some quantum phenomena indicate a path to achieving an antigravitational effect.

Scientists considered a vacuum not to be a void but an intricate structure filled as it were, with photon "gas" with the minimum amount of energy. What is known as the **Casimir effect** became highly popular in today's searches for a connection between quantum and gravitational theories. Long known to scientists, it makes it possible to tangibly "perceive" the existence of this "gas". This leads to a very unexpected conclusion: a certain body (conductor) placed in the vacuum, obstructs the penetration into it of the photon "gas", ousts it, with a resulting loss in energy, and mass (in exactly the same way as a body submerged in a liquid loses weight). This is nothing short of antigravitation.

However, for known bodies from ordinary materials this effect is so insignificant that it does not give us a chance to "feel" or see antigravitation. But isn't there a condition under which this effect will become palpable. According to Burinsky the antigravitational effect becomes more acute with the increase in density of objects. For materials where the particles are "packed" with extreme density, the antigravitational effect becomes so strong that its quantity can be compared with mass of the object itself.

"It may be supposed that in objects of extreme density such as the neutron stars, for instance, the antigravitational effect 'eats up' a large part of their mass," said Burinsky. "In the process of a star's compression (collapse) its density increases and reaches a state of extreme density. In this case a large part of the star's mass sort of converts to a concealed form while the collapsing star itself assumes the form of **a swiftly rotating disc.** This follows from the famous solution of Einstein's equations, discovered by **Roy Kerr** in 1963. Scientists found that the material of this 'Kerr disc' possesses highly unusual properties: it is superconductive and **weighs nothing!**"

The hypothetical Kerr disc is of interest to scientists not only for the purpose of describing the processes taking place in space. Burinsky, in particular, believes that the rotating Kerr disc (like the disc of the neutron star, but "made" not of neutrons but of densely packed "quarks") constitutes the basic structure of ... elementary particles! But then the masses "eaten up" by the antigravitational effect must be present in a concealed form in the elementary particles. If so, the physical picture of the world, as we know it today, must owe its completeness to the existence of both gravitation and antigravitation. To what extent does this conception (if Burinsky's hypothesis is correct) bring us nearer to the mastering of this force? "The main condition for this would be the creation, at least in laboratory conditions, of a material with the superdense packing of extreme density particles," says Burinsky. Evidently today this is an extremely complicated problem, even for omnipotent physics. But the history and logic of the development of science show, that the "impossible" simply requires a little more time. And sometimes — more chance.

Does the Universe Rotate?

Does the whole of the universe rotate? This is the central question today for scientists in relativist cosmology and gravitation theory. The supposition on the possible rotation of the meta-galaxy was engendered by the announcement of British astrophysicists in the global unevenness they discovered in the radiation of space radiosources. Moscow University Professor **D. Ivanenko** believes that the universe does have a general rotation at a slow speed. This fact can be explained and mathematically described from positions of Einstein's general theory of relativity. This is being done by Ivanenko's co-author **V. Krechet.** The solution obtained by Ivanenko and Krechet link the speed of the universe's rotation and its angular momentum (spin) with the average density of the matter contained in it. Thus, the rotation of the universe can serve as one more observable phenomenon corroborating the correctness of the general theory of relativity (along with the already known expansion of the universe, the deflection of star light near the sun, etc.).

The formula of the dependence of the spin and mass found by Ivanenko and Krechet for the universe coincides with the dependence known for elementary particles. This coincidence points to the existence of a profound analogy between the universe and the microcosm which is an additional argument in the so-called hierarchic concept of the structure of the physical world, developed by Ivanenko and Krechet. It regards the universe and microparticles as two elements of a single system possessing a number of common properties but differing in level: in exactly the same way as, say, a giant matryoshka doll differs from the tiniest one fitted into it. Will science ever be able to encompass the universe as a whole, to comprehend the single laws connecting the macro- and microcosm, to confirm the correctness of Academician Markov's hypothesis that our entire universe is nothing more than an elementary particle and that every elementary particle is a vast, infinite world like ours? Evidently, not. Knowledge is boundless. But it is the only path of Reason.

VOCABULARY EXERCISES

I. *Illustrate the following word combinations with sentences of your own.*

to deduce to derive to extract to infer	a law rule relation principle	law is	invented estimated discovered manufactured
to obey to observe to break to violate	law	law	holds is valid is obeyed is satisfied
to gather to collect to process to accumulate	facts data information	general common specific erratic	behaviour

(un)steady (non)uniform (ir)regular (in)violent	motion movement	to change to alter to modify to revolutionize	a viewpoint	dogmatic scholastic aristotelian speculative	philosopher
to solve to settle to tackle to handle	a problem	the distance	covered travelled traversed	applied exerted impressed transferred	force
the time	elapsed passed taken	the body	solid rigid heavenly	single regular random routine	observation

mean average	time speed velocity	to take into account to take into consideration to take for granted
mutual	action attraction repulsion	to be responsible (for) to be prejudiced (in) to be destined (to)

II. *Synonyms:*

speed	rate	order	clock	void
velocity	frequency	command	timepiece	vacuum

speculation	core	stop	ray	wave
meditation	kernel	halt	beam	billow
thought	nucleus	cease	bundle	surge

inquiry	shrinking	to become less	to speculate
research	reduction	to decrease	to ponder
quest	contraction	to diminish	to meditate

III. *Antonyms:*

to attract	to accept	to reveal	safe	light
to repel	to discard	to conceal	risky	heavy

wide	regular	natural	upwards	internal
narrow	random	artificial	downwards	external

real	fall	to emit (light)	susceptible
assumed	rise	to consume	uncapable of

IV. *Don't mix them up.*

low	peace	fast	random	particle	lawyer
law	piece	vast	routine	particular	legislator

vertices	futile	course	pressure	experiment
vortices	fertile	curse	precision	experience

technique	geocentric	physicist	refute	supernational
engineering	geliocentric	physician	refuse	supernatural

V. *Explain in your own words the difference between.*

to push	to emit	to shift	to bend	to account for
to pull	to radiate	to move	to curve	to explain

to destroy	to eliminate	to refute	to follow
to destruct	to annihilate	to prove wrong	to pursue

VI. *Give one Russian equivalent of the following groups of words.*

a) Law — act — bill — decree — legislation / pulse — impulse — impetus / order — command — writ / search — quest — inquiry / speed — velocity — rate — frequency / fault — blame — fallacy / resistance — opposition / surge — waveswing — billow / superstition — prejudice / quotation — citation — excerpt.

b) To turn about — to move around — to revolve — to rotate — to orbit — to whirl — to circle — to career — to spin / to consider carefully — to reflect — to meditate — to speculate — to cogitate / to draw — to pull — to attract / to repel — to push — to rebound / to meet — to come across — to encounter — to blunder — to collide / to take pace — to happen — to occur / to interfere — to disturb — to bother — to hinder / to predict — to foretell — to forecast / to ponder — to speculate — to reflect / to inhabit — to populate — to people / to unlock — to uncover — to discover.

c) Legal — lawful — legitimate — valid / suitable — appropriate / heavenly — celestial / thoughful — speculative / mistaken — erroneous — fallacious — misleading / strange — curious — problematic —

doubtful / void — empty / man-made — false — artificial / viable — vital — fruitful — productive — useful / fundamental — basic — essential.

VII. *Find the illustrative example of the following synonyms in the texts of the lesson.*

a) 1. to make smb. do smth.
2. to have —"—
3. to force smb. to do smth.
4. to cause —"—
5. to get —"—
6. to urge —"—
7. to compel —"—
8. to impel —"—
9. to challenge —"—
10. to bring (oneself) —"—

b) Somebody was led to do smth.
Somebody came to do smth.

c) to explain — to account for

d) to take into account — to take into consideration

e) on no account — by no means

f) a refutation — a disproof

g) experienced — skillful

h) continuity — succession

VIII. *Translate the following text at sight, giving synonyms, antonyms and definitions of the bold-faced words of the active vocabulary.*

Inertial Frame

One of the great contributions of mathematics to physics is Relativity which is both mathematics and physics; you need good knowledge of both mathematics and physics to understand it. The theory of Relativity, which **modifies** our mechanics and **clarified** scientific thinking, arose from a simple question: "How fast are we moving through space?" Attempts to answer that by experiment led to a conflict that forced scientists to think out their system of knowledge afresh. Out of that reappraisal came Relativity — a brilliant application of mathematics and philosophy to our treatment of space, time and motion. Since Relativity is a piece of mathematics, popular **accounts** that try to explain it without mathematics almost all fail. To understand Relativity you should either follow its algebra through in standard texts, or examine the origins and final results, taking the mathematical machine-work on trust.

What can we **find out** about space? Where is its fixed framework and how fast are we moving through it? Nowadays we find the Copernican view comfortable, and picture the **spinning** Earth moving around the Sun with an orbital speed of about 70,000 miles/hour. The whole Solar system is moving towards the constellation Hercules at some 100,000 miles/hour, while our whole galaxy...

We must be **careering** along a huge, epicycloid through space without knowing it. Without knowing it, because, as Galileo pointed out, the mechanics of motion — projectiles, collisions, etc. — is the same in a steadily moving laboratory as in a stationary one; as though the Earth's velocity changes around its orbit, we think of it as steady enough during any short experiment. Galileo **quoted** thought-experiments of men walking across the cabin of a sailing ship or dropping stones from the top of its mast. Review thought-experiments in moving trains. Suppose one train is passing another in constant velocity without bumps, and in a fog that **conceals** the countryside. Can the passengers really say which is moving? Can mechanical experiments in either train tell them? They can only observe their **relative** motion. In fact we **developed** the rules of vectors and laws of motion in earthly labs that are moving; yet those statements show no effect of that motion.

We give the name **inertial frame** to any frame of reference of laboratory in which Newton's Laws describe nature truly: objects left **alone** without force **pursue** straight lines with constant speed, or stay at rest; forces produce proportional accelerations. We find that any frame moving at constant velocity relative to an inertial frame is also an inertial frame. Newton's Laws **hold** here too. In the discussions that **concern** Galilean relativity and Einstein's Special Relativity, we **assume** that every laboratory we discuss is an inertial frame — as a laboratory at rest on Earth is, to a close **approximation**.

We are not **supplied** by nature with an obvious inertial frame. The **spinning** Earth is not a perfect inertial frame (because its spin imposes certain **accelerations**), but in case we ever find a perfect one then our relativity view of nature assures us we could find any number of other inertial frames. Every frame moving with constant velocity relative to our first inertial frame is an equally good inertial frame — Newton's Laws of motion, which **apply** by **definition** in the original frame, apply in all the others. When we do experiments on force and motion and find that Newton's Laws hold we are, from the point of view of Relativity, simply showing that our earthly lab, does **provide** a practically perfect inertial frame.

LAB. PRACTICE

Grammar Rules Patterns

I. *In the following statements use Present Continuous Tense forms.*

Models. 1. Large heavenly bodies (to move) in regular orbits. Large heavenly bodies **are moving** in regular orbits.

2. Something (to be interfered with) their straight-line motion.
Something **is being interfered** with their straight-line motion.

1. Astronomy (to provide) a clear example of the growth and use of **Theory.** 2. We (to honour) the great Greek scientists — the founders of Astronomy. 3. The earth (to spin) about its axis while it (to move) in an orbit about the sun. 4. The planets (to rotate) on elliptical orbits with the sun at one focus of the ellipse. 5. Celestial bodies (to be attracted) to some centre of force around which their circular motion takes place. 6. All the planets (to whirl) at tremendous speeds around the sun and (rotate) at the same time. 7. A system of orbits along which the planets (to move) (to be constructed). 8. The moon (to circle) around the earth because the earth (to attract) it. 9. All forces (to occur) always in pairs which are always referred to as action and reaction. 10. The planetary Laws (to be verified) in today's cosmic flights.

II. *In the following statements use Past Continuous Tense forms Active or Passive voice.*

Models. 1. Astronomical knowledge **grew** with early civilizations.
Astronomical knowledge **was growing** with early civilizations.

2. Ancient astronomers **were supplied** with the material for calendar-making by observations and collecting facts.
Ancient astronomers **were being supplied** with the material for calendar-making by observations and collecting facts.

1. Astronomical knowledge **originated** from simple noticing facts to systematic observing. 2. Ancient astronomers **sought** to learn what happens and how things occur. 3. With the accumulated knowledge **evolved** stories and myths describing the nature. 4. The stories and myths were not real science but they **set** the pattern of a scheme to "explain" the facts. 5. For many centuries ancient astronomers **tried** to explain "why" phenomena occur. 6. New attitude and methods **were brought and founded** by the Greek thinkers. 7. They **sought** general schemes and principles to account for and predict separate facts. 8. They **were awarded** the title "natural philosophers". 9. Greek speculative philosophers **gave** simple pictures of the Universe. 10. They **started** with a few general principles, **drew** logically a complete "explanation" of the observed behavior and **produced** theory.

III. *In the following statements use the Future Continuous Tense forms.*

Model. "Mr. Experiment" **enters** science as the only reliable test for a true theory.
"Mr. Experiment" **will be entering** science as the only reliable test for a true theory.

1. Scientists **object** to be ruled by dogmas and authorities. 2. We **deal with** unprejudiced scientists who **make** astronomy and mechanics inductive-deductive sciences. 3. They **devise** their theories with the help of guesses from experiments. 4. They **base** their theories on assumptions that are consistent with experiment. 5. From their theories they **draw** many predictions which in turn they **verify** again by experiments. 6. Scientists **devise** new astronomical instruments and methods of indirect measurements. 7. Scientists **distinguish** scientific sophisticated methods of reasoning from those used in our ordinary life. 8. They **order** facts, **deduce** laws and **predict** events to understand the world of our sense impressions. 9. Scientists of Mechanics **develop** Mechanics as a rational structure. 10. They **clarify** and **modify** scientific concepts.

IV. *Use the proper Indefinite or Continuous Tense forms.*

1. **He observes the motion of heavenly bodies.**
Now, all night through, to-morrow, at midnight, the day before yesterday, regularly.
2. **Great progress is made in mathematics.**
Last century, next year, at Cambridge University, at present, not long ago, at that time, soon, nowadays.
3. **We discuss this important law.**
The other day, at this time in a week, the whole lesson, next Monday, now, while they were away.

V. *Use the Present Continuous Tense forms to express actions in the near future due to a previous arrangement.*

Model. We **shall connect** mechanical world-view with the name of Newton.
We **are connecting** mechanical world-view with the name of Newton.

1. Similarly, we **shall connect** the field theory with the names of Maxwell and Einstein. 2. Nowadays scientists **will distinguish** between the mechanical and field views. 3. A mechanist **will characterize** the principal features of Newtonian view as particles and simple forces

acting between them. 4. A mechanist **will assume** the Universe forms a complicated machine that obeys Newtonian laws. 5. A mechanist **will claim** that mechanical view is most successful in mechanics and astronomy. 6. Maxwell's field theory **will deal with** the description of changes that spread continuously through space in time. 7. Maxwell's theory **will describe** electromagnetic waves and the laws of their propagation. 8. The field view **will prove** successful in the domain of electrical and optical phenomena. 9. According to the Newtonian theory of gravitation we **shall assume** that time and space are absolute and that there exists an inertial system. 10. General Relativity theory **will attack** the problem of gravitation in an entirely new way. 11. In relativity theory we **shall consider** space and time as a four-dimensional background of all events. 12. We **shall assume** further that field theory laws are valid in any system.

VI. *Use the Future Continuous Tense forms to express an action to take place in the future in the normal, natural course of events. Mind, that this is the main application of the Future Continuous in modern English.*

Model. The scientist (to comment) on the evolution of the problem of Gravity.
The scientist **will be commenting** on the evolution of the problem of Gravity.

1. To explain the phenomena of heat, light and flowing fluids the scientist of Mechanics (to invent) an appropriate mechanical picture or a model. 2. Mechanical waves (to spread) only in a material medium. 3. According to Galileo's relativity principle we (not to detect) uniform motion in two systems in uniform relative motion. 4. We (to describe) as an achievement Einstein's destroying once and for all the concept of ether. 5. We (to assume) that the speed of light is always the same irrespective of what system we (to measure) it in. 6. In relativity theory we (to change) the most basic concepts of space and time. 7. When we are considering bodies moving with speeds small compared to that of light, we (to employ) the principles of classical mechanics. 8. If the velocities of moving bodies are approaching that of light (electron motion), we (to use) the Einstein's relativistic mechanics. 9. Mechanists (to claim) that perhaps the greatest triumph of Newtonian mechanics was its solution of the gravitational problem. 10. Modern physicists (to object) that the Newtonian gravitational law is the law of a gravitational field changing in time and space; so this law is invalid.

VII. *Turn from Active into Passive.*

Models. Scientists **are seeking** for Laws describing nature.
Laws describing nature **are being sought for.**
Scientists **were codifying** the knowledge of nature in simple Laws.
The knowledge of nature **was being codified** in simple Laws.

1. Ancient scientists **were discovering** Laws by collecting facts and speculating. 2. Modern physical science **is deriving** most scientific laws inductively from experiment. 3. Natural scientists **are using** deduction to extract common behaviour from a few laws. 4. Physicists **are testing** the deductions by experiments. 5. While experimenting the scientists **are looking for** constancy. 6. Scientists **were** gradually **discovering** the conservation of Mass, Momentum and Charge. 7. Scientists **are considering**

Conservation Laws as a basis of Mechanics. 8. R. Hooke **was experimenting** with Springs and Loads. 9. He **was observing** direct proportionality between a stretch of a spring and the force applied. 10. Engineers are estimating Hooke's Law as accurate over a wide range of stretches. 11. Engineers **are encountering** similar Hooke's Law behaviour in all varieties of elastic deformations. 12. Scientists **are claiming** Constancy as the most essential characteristic of all scientific Laws.

VIII. *Compose sentences using Continuous Tenses forms according to the models.*

Model. This is the research ... (I, to carry on now).
This is the research **I am carrying** on now.

This is the
- theory of classical mechanics ... (we, to try to appreciate)
- mechanical picture ... (mechanicsts, to apply at that time)
- light phenomenon ... (physicists, to explain in a new way then)
- hypothesis of absolute time and space (Einstein, to discard early in this century)
- General Relativity theory ... (Einstein, to develop to revise the gravitation problems)
- field viewpoint ... (physicists, to spread and deepen)

Model. **What are you going to do with** this speculative theory?
(to put it to experimental verification)
We'll **be putting** it to experimental verification.

What are you going to do with
1) the results of the experiments performed? (to check them up once more)
2) the data collected? (to devise a general scheme or principle)
3) the model of inertial system? (to apply Einstein's principle)
4) the small disagreements between two theories? (to confirm that they lie beyond experimental error)
5) the time worlds of observers moving relative to each other? (to show that they are different)
6) the light ray? (to show that the light ray is curved in a gravitational field)

IX. *Explain the use of Indefinite and Continuous-Aspect Tense forms.*

1. The modern existing mechanical theory **is** the quantum theory. 2. Its domain **begins** in the nucleus and **extends** to the solar system. 3. Quantum theory **is becoming** the classical theory. 4. There **is** a transition between the classical and quantum domains. 4. Scientists **are tracing and locating** subtle penetration of quantum effects into a completely classical domain. 6. Quantum Mechanics **was developing** the undeterministic idea in the specification of position and velocity of a particle. 7. Quantum Mechanics **is the** mathematical theory of particles. 8. Quantum Rules **obey** in any system. 9. Gravitation **is extending** over enormous distances. 10. All bodies **are interacting** gravitationally. 11. The

gravitational field **is being created and annihilated** in small regions by the process of tranformation. 12. To solve the problem of gravitation scientists **are considering** the time-space geometry in a new way.

CONVERSATIONAL EXERCISES

I. *Express your views on the following statements trying to prove your point. The given phrases may come in handy.*

What is missing (lacking) in the statement is that ... In view of the idea ... I have reason to believe that ...

1. It is characteristic of the scientific Laws that they have abstract character. 2. If Law does not work even in one place where it ought to, it must be wrong. 3. One can claim for scientific Laws a universal validity. 4. Any great discovery of a new Law is useful only if we can take more out than we put in. 5. The fundamental hypothesis of science is: the sole test of the validity of any idea is **Experiment.** 6. Experiments help produce Laws; they can give one hints and clues. 7. Scientists need curiosity, imagination, insight, persistence to guess and create from the hints and clues the great and broad generalizations. 8. This imagining process is so difficult that there is a division of labour in science. 9. There are **treoreticians,** who imagine, deduce and guess at new Laws, but do not experiment. 10. There are **experimentators,** who experiment, imagine, deduce and guess. 11. Observations, reasoning and experiment make up what we call the scientific method. 12. Fundamental scientific Ideas arose from the application of the scientific method. 13. The basic theoretical problem is to find Laws behind experiment that can amalgamate different phenomena. 14. We make **Conservation Laws** of energy, mass and momentum a great basis of Mechanics. 15. Important conclusions are drawn from simple idealized "thought experiments". 16. There are no absolute Laws in the sense of laws independent of observers. 17. A Law must be framed in terms of the measurements of a particular observer. Laws are tied to observers. 18. One should not speak about things one cannot measure.

II. *The fundamental fields of Mechanics. Characterize the problems they deal with. If you are the Mechanics student what field are you going to specialize in? Give your reasons.*

Model. It is easy to show that forces are vectors, i. e., that they obey geometrical addition. The vector treatment of ballanced forces is called **Statics.** The geometrical properties of the motion of rigid bodies constitute the subject matter of **Kinematics.** The study of motion subject to external forces is **Dynamics.** There are two basic problems in Dynamics: to find the **forces** exerted that cause the motion and determine the **motion** of the body when the forces applied are specified.

1. **Theoretical Mechanics.** It is as old as Mechanics itself. It makes research in various trends. The theories of stable motion, of automatic and optimal operations, the dynamics of flight, etc. are treated. The problems concerning the behaviour of mass-points, motion, density, liquids, gases, plasma and their states are dealt with. Ideal physical and mechanical models are introduced to make formulation of Mechanics possible with the aid of differential and integral calculus.

2. **Analytical Mechanics** formulated at least 200 years ago studies systems of mass-points or rigid bodies, finite in number. Analytic methods and the methods of differential equations are used.

3. **Applied Mechanics.** The theory of machines and mechanical devices, the theories of vibrations, regulations, gyroscope, automatic control, etc. are developed. Most problems are of engineering and manufacturing aspect.

4. **Electromechanics** commonly implies the interaction of currents with fluids and the construction of practical electromechanical energy-converting devices. The theory covers topics regarding the nature of the mechanical and electrical properties of the interacting medium. It makes a great difference whether the fluid is a gas, a liquid, or a plasma to say nothing of the diversity of properties associated with each of these media.

5. **Hydrodynamics. Aerodynamics. Magnetohydrodynamics. Electrohydrodynamics.** They are all based on classical Newton's Mechanics and the model of material continuum. The theories of perfect fluid and viscous fluid, the perfectly flexible line, the membrane, gas and wave dynamics, the perfectly elastic solid, the infinitessimally visco-elastic material and plastic material, the phenomena in the earth's interior and in its atmosphere are created.

6. **Celestial Mechanics. Stellar Astronomy. Astrophysics.** The investigations of the gravitational fields, celestial bodies with various configurations, the evolution of planetary and satellite systems and their stability, the motion of cosmic dust, topological peculiarities of the rotation of celestial bodies, stellar atmosphere, cosmic gas dynamics, the structure and evolution of stars, etc. are being performed in all these fields.

7. **Continuum Mechanics.** The theories of elasticity, plasticity, creep and relaxation in solids, the theory of motion of plasma constitute the subject matter of the field. The problems of the elastic limit, the origin of a residual plastic deformation, the motion of highly compressed liquids and gases, or conversely, rarefied gases, mechanical models for polymeric plastic materials, etc. are dealt with.

III. *Say it in English.*

Механика — одна из древнейших наук. Ее возникновение и развитие связаны с развитием общества и нуждами практики. Механика — наука о **механическом движении** материальных тел и происходящих при этом **взаимодействиях** между телами. Механика тесно связана и является одной из научных основ многих областей современной физики и техники. Под механическим движением понимают изменения с течением времени взаимного положения тел или их частиц в пространстве. Примерами таких **движений** являются: в природе — движение небесных тел, колебания земной коры, воздушные и морские течения, тепловое движение молекул. и т. п.; в технике — движения различных летательных аппаратов и транспортных средств, частей двигателей, машин и механизмов, деформации элементов различных конструкций, движения жидкостей и газов и т. п. Примерами **взаимодействий,** рассматриваемых в механике, могут быть притяжения тел по закону всемирного тяготения, взаимные давления соприкасающихся тел, воздействия частиц жидкости или газа друг на друга и др. Обычно различают классическую механику, в основе которой лежат законы механики Ньютона, изучающей движения любых материальных тел (кроме элементарных частиц), совершаемые со скоростями меньше скорости света. Движение тел со скоростями порядка скорости света рассматривается в теории относительности, а внутриатомные явления и движение элементарных частиц изучаются в квантовой механике. При изучении движения в механике вводят ряд абстрактных понятий, отражающих свойства реальных тел: материальная точка, абсолютно твердое тело, сплошная изменяемая среда. Изучение основных законов и принципов, которым подчиняется механическое движение тел, и вытекающих из этих законов общих теорем и определяющих уравнений, составляет содержание общей и теоретической механики. Разделами механики, имеющими важное самостоятельное значение, являются теория колебаний, теория устойчивости равновесия и устойчивости движения, теория гироскопа, механика тел переменной массы, теория удара. При решении ряда задач газовой динамики, теории взрыва, теплообмена в движущихся жидкостях и газах, аэродинамики разряженных газов, магнитной гидродинамики, небесной механики и др. используются методы и уравнения теоретической механики. Во всех областях механики все большее значение приобретают задачи, в которых вместо «детерминированных», т. е. заранее известных величин, рассматривают «вероятностные» величины, т. е. те, для которых известна лишь вероятность того, что они могут иметь те или иные значения.

IV. *Suppose that the given statement seems to you insufficient and you want to add. Repeat the statement and add your own comments thus extending and developing the idea. Use the opening phrases.*

Model. **Archiemedes was the greatest scientist of Mechanics in Antiquity.**
In fact, Archimedes was by common consent the **founder of Mechanics** in Antiquity. Moreover, Archimedes was a natural philosopher, an engineer and a physicist. His works in both Mathematics and Mechanics are masterpieces. Archimedes is, indeed, one of the greatest scientists of all times.

1. Archimedes was interested in and concerned with both pure and applied science. 2. He originated two branches of Theoretical Mechanics, statics and hydrodynamics. 3. Archimedes was thoroughly schooled in the Euclidean tradition. 4. He employed the mathematical method in his works on Theoretical Mechanics. 5. In his treatise "**On Plane Equilibrium**" Archimedes established 25 theorems on Mechanics on the basis of three simple postulates. 6. In his treatise, "**On Floating Bodies**" he establishes 19 propositions on the two fundamental postulates. 7. This treatise is the first recorded application of mathematics to hydrostatics. 8. It begins with the developing of the Laws which are the base of modern hydrostatics. 9. It culminates with a remarkable investigation of paraboloid of revolution floating in a fluid. 10. In his work "**On the Sphere and Cylinder**" Archimedes explicitly stated the postulate introducing the concept of continuity that bears his name. 11. Archimedes was renowened for his mechanical inventions, some of them were put to use in the defense of Syracuse against the Romans. 12. The ingenuity of the mechanical devices invented by Archimedes are astonishing even by modern standards. 13. He designed waterclocks, sun dials, pumps, pulleys, wedges, tackles, geared devices, mileage measuring devices. 14. The principles underlying water- and steam-driven devices were explained in his treatises on pneumatics and hydrostatics. 15. Mathematical prescriptions for the construction of vaults, catapults and digging tunnels in a mountain were explained by him in detail. 16. Distingui-

shed in the classical pursuits of mathematics Archimedes also displayed profound genius in Mechanics. 17. There was a gap of eighteen hundred years between Archimedes and **Simon Stevin** (1548—1620), the next major contributor to the knowledge of hydrostatics and the statics of solids. 18. The work of S. Stevin appreciably advanced Statics and Hydrodynamics beyond the points reached by Archimedes. 19. Stevin was developing the principles of Statics while G. Galileo was working on Dynamics. 20. Stevin and Galileo laid the foundations of applied Mechanics.

V. *Listen to the tape or read the text. Reproduce it orally and speak of some other important Laws of Mechanics based on observavion of mechanical behaviour and discovered through experiment.*

Hooke's Law

In 1676 Robert Hooke announced his discovery concerning Springs. He discovered that when a spring is stretched by an increasing force **the stretch varies directly as the force.** It was a simple law, accurate over a wide range, destined to play an important part in science and engineering. As you know from your own work, this relation holds for a steel spring with remarkable accuracy over a wide range of stretches. It holds for springs of other materials, perhaps best of all for a spiral of quartz (pure melted sand). The law is surprising and useful for the relation holds until the spring's stretch is several times its original length. The law is remarkable not just for its simplicity but for its wide range.

We meet similar Hooke's Law-behaviour in many cases of stretching, compression, twisting, bending — all varieties of elastic deformations. The general form of Hooke's Law "stress / strain is constant" or "deformation varies directly as the deforming force" applies to all materials (within limits) and to many types of distortion. A wooden beam may be bent, or a hair-spring coiled up, through a large angle and still fit with Hooke's Law. Even a simple metal wire when stretched fits Hooke's Law over a surprising range of stretches — far beyond the tiny expansion caused by heating. Its atoms dragged apart against electrical attractions experience individual Hooke's-Law-forces. This general rule is called "Hooke's Law" in honour of R. Hooke's discovery.

Once extracted, may scientific laws be discredited by the discovery of exceptions or limitations? Some scientists idealize Laws. They take each Law as simple and exact and award them much more permanent privilege. They take the view that the law is there, a clear statement of possible simple behaviour, with no question of its being wrong or untrue, it just states what it states. When we are trying to extract a law we usually restrict our attention to particular aspects of nature. When we are finding Hooke's Law our spring may be twisting, the loads may be painted different colours, the loads may even be evaporating, but ignore those distractions. Or our spring may be growing hotter in an overheated laboratory; and then we find the stretch changing less simply.

In discussing Hooke's Law, "stretch varies as load", we should not ask: "Is that statement true?" but rather, "How closely do the facts fit the statement? Do many substances in many shapes "obey" it? Does it apply over a small range of stretch or a large one?" When we find that most springs and wires obey it over a large range of stretching, we consider it a useful law, worth naming. We may picture the law itself as going on for ever, right out towards infinite stretches and back into compressions, but we have no illusion that real materials obey it over such a range. Instead we pride ourselves on a cunning knowledge (drawn of course from experience) of its limitations. We consider we know within what range of stretches it applies to, say, a steel wire, and in that range how closely experimental measurements fit it. And we keep track of special substances, such as glass and clay, that we suspect of serious deviations from the Law.

VI. *Summarize orally the topic: "Laws of Motion".*

1. Aristotle approached nature in terms of concepts such as **origins, essences, form, quality, causality** and **ends** that do not lend themselves to quantization. 2. By the end of the sixteenth century there were available alternatives to the Universe as conceived by Aristotle. 3. Rather than closed the Universe was open; rather than filled, it was empty. 4. Space, rather than having a unique point was the same in all directions. Space was peopled with particles that do not fall or rise but remain **in uniform motion** unless they collide. 5. Galileo, unlike Aristotle, approached the problem as a mathematician and emphasized and fixed on **matter moving in space and time** as the fundamental phenomenon of nature. 6. Galileo concentrated his study on such concepts as **space, time, weight, velocity, accelaration, inertia, force** and **momentum.** 7. Galileo offered a totally new concept of scientific goals and the **role of mathematics** in achieving them. He claimed that science was to be patterned on the mathematical model. 8. Galileo sought mathematical formulas that can describe the **Motion of Bodies** and nature's behaviour. 9. Bare mathematical formulas explain nothing; they simply describe in precise language. Yet, such formulas are the most valuable knowledge man can acquire about nature. 10. Science is not a series of experiments regardless of how skillfully they are executed. The value lies in the **Theory** that unifies experiments and facts deduced from them. 11. Galileo created a structure of bodies in motion very much like Euclid developed a structure of relations of objects in space. 12. Newton, born the year that Galileo died, created the first great **Mechanical Theory** which dominated scientific thought for two centuries. 13. Newton restated and generalized Galileo's findings in the form of two **Laws of Motion** and added a third. 14. Newton tested and verified, in a way, the Laws, while investigating motions of planets and the moon. 15. Newton's successors extended them to molecules, atoms and even parts of atoms. 16. Newton's Laws of Motion are clear, powerful **working rules** based on experiment, the clarification of terms and deduction. 17. Right down to this day scientists disagree over the status of Newton's Laws of Motion. 18. Some scientists claim that the Laws are wholly **definitions** and **conventions** and contain no experimental ties to the natural world. 19. Some arguers see **Law One** as chiefly a **description** of force, and **Law Two** as a **definition of force-measurement.** 20. Such appraisals are misleading. Newton's Laws of Motion and Gravitation are fundamental and apply equally well on heaven and Earth. 21. Like the axioms of Euclid, Newton's formulas serve as a logical basis for other valuable laws in mechanics and physics.

VII. *Starting from general assumptions, Newton tied together in a single scheme many diverse and disconnecting things. Explain what enabled him to make his great guess, concerning each and all taken together.*

Moon's circular motion.
Disturbances of Moon's simple motion.
Planetary motions (Kepler's Laws).
Planetary perturbations.
Motion of Comets. Tides.
Bulge of the Earth.
Differences of gravity.
Precision of equinoxes.
The motion of a gyroscope.

} All related by inverse-square-law of Gravitation and a spinning Earth.

VIII. *What Law can be deduced from the following statements?*

1. A spinning top has the same weight as a still one. 2. Mass is constant, independent of speed. 3. Mass increases with velocity, but appreciable increases require velocities near that of light. 4. If an object moves with a speed of less than one hundred miles a second the mass is constant to within one part of a million.

IX. *Speak on the topic: "How much of Science became Mathematized in the form of Geometry" by extending each statement into a paragraph, adding some illustrations, proofs, evidence, your own viewpoint etc. (You must do it in writing first).*

Models. 1. **The classical Greeks believed that reality could be best understood in terms of geometrical properties.**
I think, they were right, after all. They sought knowledge of what is universal and eternal, rather that individual and fleeting. Geometry, the study of forms, was the special concern of the Greeks and their greatest accomplishment. Their astronomical inquiries too led the Greeks to favour geometry. The Pythagoreans observed the fact that natural phenomena which are physically most diverse exhibit identical geometrical properties, e. g., size, shape, volume, dimensions, ets., and they claimed that mathematical relations underlie diversity and must be essence of phenomena.

2. **One should distinguish between geometry as a branch of maths and geometry as a branch of physics.**
Quite right. As a branch of maths geometry is an abstract body of theorems deduced from a set of postulates. Postulates are perfectly arbitary subject to the requirements that they should be consistent and mutually independent. Postulates and theorems of geometry have nothing to do with observations, experiments and verifications. **As a branch of physics** geometry is the description of the results of a vast body of observations and experiments. It tells us what will happen if you do certain things. It is the main concern and objective of the physicist to verify the validity of postulates and theorems. Thus, we postulate the existence of certain idealized objects (point, line, circle, sphere or manifold of these elements — plane of points, ordinary space of points, pencil of circles, etc.). The theory (geometry) of these idealized objects with the assigned and explicitly stated properties is a branch of maths. Geometry as the body of experiments and observations in which this maths is used is a branch of physics.

1. From the days of Euclid the laws of physical space were no more than theorems of geometry. 2. Hipparchus, Ptolemy, Copernicus and Kepler summarized the motion of the heavenly bodies in geometrical terms. 3. With his telescope Galileo extended the application of geometry to infinite space and to many millions of heavenly bodies. 4. Descartes' doctrine is that the phenomena of matter and motion can be explained in terms of the geometry of space. 5. Lobachevsky, Bolyai, and Riemann showed how to construct different geometrical worlds. 6. The study of the properties of the points of a plane which remain invariant under the group of transformations is known as a (special) Lorentz geometry. 7. Minkowski insisted that the universe was naturally a four-dimensional-space-time unity. 8. Einstein seized Minkowski's idea to fit our physical world into a four-dimensional mathematical world. 9. The Lorentz transformation is always valid as it recognizes the relative character not only of space geometry but also of time. 10. Gravity, time and matter became, along with space, merely part of the structure of geometry. 11. The twentieth-century science is being mathematized in the form of geometry. 12. The science of the vast universe and the infinitesimal realms was both geometrized by quantum theory.

X. *Disagree with the following negative statements. Give your reasons or possible justification.*

1. The basis of science is not built through scientific laws. 2. The Laws of nature must not be valid in any system. 3. Our inability to detect absolute motion is not a result of experiments. 3. The classical concepts of **absolute space, light path** (distance) and **time interval** were not to be revised. 5. The idea of absolüte frame of reference must not be abandoned. 6. We don't give mass the relativistic property of increasing with motion. 7. Riemannian and Minkowski's geometries found no application in Relativity theory. 8. Time, distance and simultaneity are not different for the people in the Universe. 9. Relativity theory did not enlarge the framework of classical mechanics. 10. General Relativity theory did not clarify the connection between Mechanics and Geometry. 11. Einstein did not revise the problem of gravitation. 12. Gravitation does not go out inversely as the square of the distance. 13. Gravity does not exist at bigger dimensions. 14. Einstein did not predict the deflection (bending) of a lightray in the gravitational field of the sun. 15. The gravitational field does not annihilate by the process of transformation. 16. The work done in going around any path in a gravitational field is never zero. 17. Einstein did not discover the existence of gravitational waves. 18. General Relativity theory does not explain the abnormal behaviour of Mercury's orbit. 19. There exist no experimental verification and confirmation of General Relativity theory predictions. 20. Relativity theory did not clarify scientific thinking and modify Laws of Classical Mechanics.

XI. *Discuss the significance of the great discoveries in Mechanics and Physics.*

(1807) **J. Fourier** presented to the French Academy a **theorem** of unprecedented importance for the progress of science, which advanced the mathematical mastery of the motion of waves: Any wave, whatever its form, can be treated as a sum of a set of simple harmonic waves.

(1864) **J. Maxwell** published a paper synthesizing notions about electricity, magnetism and light. According to Maxwell's theory of electromagnetic radiation visible light, ultraviolet light and any possible raditions of still higher frequencies must be emitted by oscillating electric charges within atoms. Accelerated charges produce the electromagnetic waves that we observe as light when they strike our eyes.

(1900) **M. Planck** introduced rather startling hypothesis that light is emitted in bundles in the black body enclosure and that the amount of energy in each bundle is related to the frequency of light by $E=hy^2$. In this way h, **Planck's constant** ($h \approx 6{,}6 \times 10^{-27}$ *erg/sec*) — the quantum of action was first introduced.

(1905) **A. Einstein** picking up the theme introduced by Planck proposed that light is not only emitted in units of energy $E=hy^2$, but it is also absorbed in such bundles which he called **photons.** The photon theory did not abandon the wave concept completely but stated that the energy of light is not distributed over the whole wave front (Maxwell), but rather is concentrated or localized in tiny bundles "photons".

One must not think that the photon theory is a revision of corpuscular theory. Corpuscules were thought of as actual particles of matter, whereas photons represent bundles of energy that have no rest mass. This means once the photon stops, it ceases to exist and its energy is transferred to whatever stopped it.

(1909) **Loretz**'s investigations showed that if the material bodies are made up of charged particles and if the forces between them behave as Maxwell's equations indicate, then, because of the change in forces between charged particles when they are in motion, one can conclude that a body should **contract** (shrink) in the direction of its motion, e. g., the mass of the electron changes when it is in motion. In modern atomic physics, where nature provides us with velocities close to that of light Galileo's transformation fails and it has to be replaced by the **Loretz** transformation — a means of finding the space and time coordinates of events in one system if they are known in the other, and if the relative speed of these two systems is known. The structure of Maxwell's equations does not change under the Loretz transformation.

(1911) **A. Einstein.** General Relativity. The light is curved in the gravitational field. (1915) Accelerated mass should radiate energy in the form of gravitational waves.

(1924) **De Broglie Duality.** Any moving particle (electron, atom, neutron, a quantum of light) is an extensive **wave** in some of its behaviour, and a compact **particle** in some of its behaviour. All objects should have with them a wavelength related to their momentum, e. g., an electron must have a wavelength associated with it.

(1926) **E. Schrödinger's equation:** the law of motion for quantum system. Shortly after de Broglie introduced the idea of the associated wave of an electron, E. Schrödinger gave the answer to the question of what happens to the associated wave if a force acts on it. Schrödinger equation — the heart of Quantum Mechanics — gives the possible waves associated with a particle designated by the wave function $\psi(x, y, z, t)$, i. e., given a particle and given the force system that acts, it yields the wave function solutions for all possible energies. The wave function satisfies the most fundamental properties of waves — e. g., the property of **superposition**, i. e., that a trough and a crest can be added to cancel one another.

(1927) **Heisenberg's Uncertainty Principle.** There is some uncertainty in the specification of position and velocity of a quantum particle. We can say, at least, that there is certain probability that any particle will have a position near some coordinate x. The most precise description of nature must be in terms of probabilities.

XII. *Dispute the problem concerned. The phrases may be helpful.*

I am confirmed in my opinion that...	**I deny that this is the case.**
The statement may be confirmed by...	**I deny that the statement is true.**
One can confirm it by further...	**It can(not) be denied that...**
Although no confirmation is available...	**There is no denying the fact...**

Are there gravitational waves?

The possible existence of gravitational waves, similar to electromagnetic ones but possessing unusual properties, was first predicted by Einstein's general theory of relativity at the beginning of the century. But their rather fractional energy and very weak absorption by substances account for the fact that the existence of gravitational waves is up to this day a subject of very heated discussion. Nevertheless, an active search for them is on in the USSR, the USA, Italy, Japan, West Germany and recently in China. It is not coincidental that questions pertaining to the study, irradiation and reception of gravitational waves were high on the agenda of the 6th all-Union conference of gravitationists among the problems of the general theory of relativity and gravitation. The problem is so important that it is not easy even to assess the revolution in physics, if the waves are to be discovered. Soviet gravitationists have disigned several types of antennae which are currently being used in conjunction with their foreign colleagues in probing for gravitational waves that are supposed to come from outer space.

There are two elements in Einstein's theory — one, that light from a distant star will be bent by the gravitational pull of the sun and the other, that the sun's gravitational pull will have a distinctly measurable effect on the way the innermost planet Mercury revolves around the sun. Einstein's 1916 extraordinary prediction **that the accelerated mass should radiate energy in the form of gravitational waves** is supported nowadays by evidence that a pulsar's orbit around a companion star is slowly shrinking. Yet, the waves are so weak and their interaction with matter is so feeble that Einstein himself wondered whether they would ever be detected. In 1974 nevertheless, an object (a pulsar) suitable for testing the prediction was found.

The numerous experiments conducted in the last years (since 1970) to see if starlight is bent by the sun's pull all verified Einstein. Two experiments showed that light from distant quasars was bent by the sun's gravity in just the way Einstein predicted, another that pulsar light did the same thing. A fourth experiment showed that radio signals from the Viking spacecraft that landed on Mars in 1976 were bent in the same way by the sun's gravity when Mars was on the other side of the sun from Earth. More recent experiments bouncing radar signals off the planet Mercury back to radio antennae in California, Massachusetts and Puerto Rico also verified that Mercury moves around the sun in just the way that Einstein said it would. Is Einstein's general theory of relativity being challenged? It is. Three astronomers from the University of Arizona recently found that the sun is not a perfect sphere as Einstein assumed it was when he developed his theory in 1916. There are fluctuations in the way the sun's edge darkens at the equator that strongly suggest the sun's equator is bulging and its north and south poles are flat. If true, this means the sun is more oblate than it is spherical. They found that the sun's interior spins once every 3.5 earth days, a brand new discovery that means the sun is spinning seven times faster in its interior than it is on the surface. The solar exterior's spin rate is once every 25.4 earth days, a fact known already for some time.

There is still enough uncertainty with planetary orbits that nobody could measure Mercury's orbit with enough precision to say what it really is. If the interior of the sun is rotating as rapidly as the three astronomers of Arizona say it is, it makes an important contribution to the way Mercury is orbiting the sun. Einstein's theory of how Mercury orbits the sun is based on the assumption that the sun is a perfect sphere, which they do not believe it is. They claim that there is a 95 percent chance that there is a problem with Einstein's theory. It is a true and fresh challenge to Einstein's theory of relativity. What will happer if Einstein is disproved? Not much. Our atomic clocks might be off by an infinitesimal fraction of a second.

XIII. *Agree or disagree with the following statements. Use the introductory phrases and develop the idea further.*

That's right. It's O. K.
I quite agree to it.
Exactly. Quite so.
I doubt it.
This is not the case.
Quite the reverse.
Far from that.
It's hardly likely that...

Models. 1. **The precise form of the wave function for all future time is determined by the Schrödinger equation.**
That's right. Given the wave function for a system at some time and the forces to which the system is subjected the precise form of the wave function for all future time is determined by the Schrödinger equation.

2. **We can't understand what is going on inside a star.**
This is not the case, because what is going on inside a star is well-understood since we can calculate what the atoms in the star should most likely do in various circumstances.

3. **There is no probability that a "quantum" particle will have a position near some coordinate x.**
But there is. According to Heinsenberg Uncertainty Principle there is certain probability that a "quantum" particle will have a position near some coordinate x.

1. A wave is a disturbance that propagates through a medium. 2. Interference is not characteristic wave phenomenon. 3. Neither the wave viewpoint nor the particle viewpoint is correct. 4. Both theories were approximate and both will change. 5. The wave function from the quantum viewpoint contains all the information. 6. All energies, all momenta, all wave lengths are allowed for a quantum particle by the Schrödinger equation. 7. Light or electromagnetic waves are abstract entities propagating through nothing. 8. A photon is never moving. 9. Any large number of photons can occupy the same place at the same time and can be described by identical wave function. 10. The precise quantum description of nature must not be in terms of probabilities. 11. Logically consistent formulation of Mechanics should be explainable on a purely relativistic basis. 12. Science is able to locate unequivocally absolute inertial frame. 13. The centre of the system of the world is immovable. 14. We understand the distribution of matter in the interior of the sun

far better than that of the Earth. 15. The momentum of quantum particle is always zero. 16. Outside nucleus scientists know all; inside it — Quantum Mechanics is valid. 17. The principles of Quantum Mechanics never fail. 18. Nuclear reactions must be going on in the stars to make them shine. 19. The quantum view is not consistent as it fails to specify precisely the position of the quantum particle. 20. The rules of Quantum Mechanics can be employed to calculate the results of the experiments.

XIV. *What do we mean when we say:*

1. Four-dimensional geometry gained validity and theoretical justification. 2. The classical concept of the "state of rest" is being modified into the concept of the "zero oscillations state". 3. In Mechanics a "field" is a continuous set of points. 4. There exists the **unity** of all natural forces interactions. 5. Quarks (never visualized) always appear in pairs. 6. Vacuum is full of particles and antiparticles; gravitation and antigravitation. 7. A wave function is not a physically existing entity, it is an **information containing** wave. 8. A wave function changes by jumps depending upon the conditions specified. 9. Conditional probability is determinative in Quantum Mechanics. 10. The significance and scientific contributions of 1910-1930 years to Mechanics and Physics cannot be overassessed. 11. A scientific bloodless revolution occurs whenever lots enough facts and experimentally verified proofs are accumulated to produce a new theory. 12. A scientific revolution is a big breakthrough and advance in an understanding of nature.

XV. *Translate the text and give your definition and assessment of Quantum Mechanics as a science.*

The Quantum Theory and Reality

One expects that any successful theory in the physical sciences makes accurate predictions. Given some well-defined experiment, this theory should correctly specify the outcome or should at least assign the correct probabilities to all possible outcomes. From this point of view Quantum Mechanics must be judged highly successful. As the fundamental modern theory of atoms, of molecules, of elementary particles, of electromagnetic radiation and of the solid state, it supplies methods for calculating the results of experiments in all these realms.

Apart from experimental confirmation, however, something more is generally demanded of a theory. It is expected that it not only determines the results of an experiment but also provides some understanding of the physical events that underlie the observed results. In other words, the theory should not only give the position of a pointer on a dial but also explains why the pointer takes up that position. When one seeks information of this kind in the Quantum theory, certain conceptual difficulties arise. For example, in Quantum Mechanics an elementary particle such as an electron is represented by the mathematical expression called a **wave function**, which often describes the electron as if it were smeared out over a large region of space.

This representation is not in conflict with experiment; on the contrary, the wave function yields an accurate estimate of the probability that the electron will be found in any given place. When the electron is actually detected, however, it is never smeared out but always has a definite position. Hence, it is not entirely clear what physical interpretation should be given to the wave function or what picture of the electron one should keep in mind. Because of the ambiguities such as this many scientists find it most sensible to regard Quantum Mechanics as merely **a set of rules** that prescribe the outcome of experiments. According to this view Quantum Mechanics is concerned only with observable phenomena (the observed position of the pointer) and not with any underlying physical state (the real position of the electron).

It now turns out that even this renumeration is not entirely satisfactory. Even if Quantum Mechanics is no more than a set of rules, it is still in conflict with a view of the world that many people consider obvious or natural. This world view is based on three assumptions, or premises that must be accepted without proof. One is **realism**, the doctrine that regularities in observed phenomena are caused by some physical reality whose existence is independent of human observers. The second premise holds that **inductive inference is a valid mode of reasoning** and can be applied freely, so that legitimate conclusions can be drawn from consistent observations. The third premise is called **Einstein separability** or Einstein locality, and it states that no influence of any kind can proparate faster than the speed of light. Of the three premises realism is the most fundamental. The three premises, which are often have the status of well-established truths, form the basis of what is called **local realistic theories of nature.**

XVI. *Discuss the Laws of Quantum Mechanics.*

1. A wave transfers energy. 2. Wavelike objects show particle properties. 3. No two electrons can be found in exactly the same state (including spin). 4. The relations between the fields and the charges retain the same form in the moving system as they had in the fixed system. 5. The energy of a body always equals mc^2. 6. Light is always propagates in empty space with a constant speed c, independent of the state of motion of the emitting body.

XVII. *Say it in English.*

В начале XX в. выяснилось, что классическая механика Ньютона имеет ограниченную область применения и нуждается в обобщении. Во-первых, она неприменима при больших скоростях движения тел — скоростях, сравнимых со скоростью света. Здесь ее заменила **релятивистская механика,** построенная на основе специальной теории относительности А. Эйнштейна. Идеи Эйнштейна изменили господствовавшие в науке со времен Ньютона механистические взгляды на пространство и время и привели к новым законам движения и новой, материалистической картине мира, основанной на органической связи этих понятий с материей и ее движением. Одним из проявлений этой связи оказалось тяготение. Идеи Эйнштейна стали составной частью современной теории динамической, непрерывно расширяющейся Вселенной. Релятивистская механика включает в себя Ньютонову механику как частный случай. Для классической механики в целом характерно описание частиц путем задания их положения в пространстве (координат) и скоростей и зависимости этих величин от времени. Такому описанию соответствует движение частиц по вполне определенным траекториям. Однако опыт показывает, что это описание не всегда справедливо, особенно для частиц с очень малой массой (микрочастиц). В этом состоит второе ограничение применимости механики Ньютона. Более общее описание движения дает **квантовая механика,** которая включает в себя как частный случай классическую механику.

Квантовая механика, как и классическая, делится на нерелятивистскую, справедливую в случае малых скоростей, и релятивистскую, удовлетворяющую требованиям специальной теории относительности. Квантовая (волновая) механика — это теория описания законов движения микрочастиц (элементарных частиц, атомов, молекул, атомных ядер) и их систем (напр. кристаллов). Законы квантовой механики составляют фундамент изучения строения вещества. Они позволили выяснить строение атома, установить природу химической связи, объяснить периодическую систему элементов, понять природу таких астрофизических объектов, как белые карлики, нейтронные звезды, выяснить механизм протекания термоядерных реакций в Солнце и звездах и др. Ряд крупнейших технических достижений XX в. основан по существу на законах квантовой механики — например, работа ядерных реакторов. Квантовая механика становится в значительной мере «инженерной наукой».

COMPOSITION

I. *Write an abstract of Text Three "Gravitation". Your abstract must not exceed 12-15 sentences.*

II. *Write a composition: "Theoretical and engineering aspects of modern Mechanics".*

COMPREHENSION EXERCISES

Questions

1. Where do Laws which are to be tested and verified come from? 2. What is the source of scientific knowledge? 3. Why was Pythagoras awarded the title "philosopher"? 4. Why should philosophers be concerned with science? 5. What do we mean by "understanding" and "explanation" in science? 6. What is the criterion for choosing and employing one theory or the other? (classical — relativistic — quantum?) 7. Did Galileo guess his famous times-squared (odd-number) Law of Free Fall by pure reasoning? 8. What common features do different motions have? 9. Does the inertia of a body depend on its energy? 10. What causes a body to accelerate uniformely near the surface of the Earth? 11. What is acceleration? Does a greater speed imply a greater acceleration? What is the relation between the force and the acceleration? 12. Why do planets remain in their orbits? 13. How can one calculate the motion of planets? 14. Does light travel at a finite or infinite speed? 15. Can there be velocity greater than that of light? 16. How can the two theories of light (corpuscular and wave) both be true? 17. Newton's Laws of Motion do not mention the size, the shape or colour of the body. Can you give an example of a motion that depends on the size of the body? Will Newton's Laws be saved in such a case? 18. What is the speed of the Earth in its orbit? 19. What is the force that the sun exerts on the Earth to keep it in a circular orbit? 20. Does Newton's Law of Gravitation imply that there is only one kind of force acting between two bodies — gravity? 21. Do Newton's Laws give any information about the cause of the gravitational force and its origin? 22. Is the theory of Gravitation now a completed theory? 23. What is the magnitude of the gravitational force that the Earth (the sun, the moon) exerts on the body? 24. What happens to mass, velocity and momentum if a constant force is exerted on a particle for a long time? 25. What happens to the relativistic modifications of Newton's equations if the

speed of light becomes infinite? 26. From Newton's viewpoint can one understand why light bends as it passes near the sun? 27. Do we have an idea about the overall curvature of our universe on a large scale? 28. Where is it most convenient to stand if we want to describe the motion of the solar system? 29. What is the picture of an atom? What is the machinery of interaction between atoms? Is it gravitational or electrical? 30. How much stronger electricity than gravitation? 31. What happens while the electrons in the wire are being accelerated first positively, and then negatively? 32. How can we force particles to move with a speed only slightly smaller than that of light? 33. How can one explain interference and difraction with the Quantum theory? 34. What Laws of Quantum Mechanics are worth discussing and why? 35. What Russian scientists contributed to the developments of Mechanics?

Discussion

1. "The scientist must order. One makes science with facts as a house with stones; but an accumulation of facts is no more science than a pile of stones is a house" (Poincare). Prove it or disagree.

2. "If science is more than an accumulation of facts; if it is not simply positive knowledge, but systematized positive knowledge; if it is not simply unguided analysis and haphazard empiricism, but synthesis; if it is not simply a passive recording, but constructive activity then, undoubtedly, ancient Greece was its cradle" (G. Sarton). Extract from the above quotation the main features of **ancient Greek science,** add some more comments of your own and characterize it.

3. **Aristotle's Mechanics** — the only significant system of Mechanics which Renaissance World possessed. Describe its merits and fallasies.

4. **Copernicus** put the sun at the centre of the Universe, leaving for the Earth the reduced status of one among the other planets. Compare this system with that of Ptolemy. Dispute Copernicus's contribution to science.

5. **G. Bruno** — apostle of infinite space — burst the starry sphere and made the Universe endless, the Earth and Sun lost among countless other planets and suns. Why was the church so violently opposed to his theory? G. Bruno: "Thus let this surface be what it will, I must always put a question, what is **beyond?** If space ends, what is **beyond?**" Explain in your own words the meaning of the quotation. Wasn't G. Bruno's question challenging?

6. "It is **Descartes** who gave us the new method of reasoning much more admirable than his philosophy itself, in which a large part is false, or very doubtful according to the very rules that he is teaching us" (Fontenelle). Characterize Descartes's philosophy and his Mechanics.

7. **G. Galileo** was teaching new science of Mechanics, advocating Copernicus's picture of the world. How did the church manage to stop him? Did he yield earnestly? One of Galileo's early discoveries was the remarkable property of pendulums: that (for small amplitude) the time of swing is independent of amplitude. Discuss the significance of this discovery and its application.

8. **J. Kepler** affirmed: "The reality of the world consists of its mathematical relations. Mathematical Laws are the true cause of phenomena". What made Kepler believe that way? Did his belief help him discover his famous Laws of planetary motions? What do mathematical models represent and reflect? Why must the ideal mathematical universe in your mind be the same as in mine?

9. **I. Newton** said, "I don't know what I may appear to the world; but to myself I seem to have been only like a boy playing on the seashore, and diverting myself now and then by finding a smoother pebble or a pretier shell than ordinary; whilst the great ocean of truth lay all undiscovered before me. **If I saw a little farther than others it is because I stood on the shoulders of giants"**. Who, to your mind, of his predessors did Newton call "giants" and why? How did Newton estimate his creations?

10. "Newton was not only the greatest but the most fortunate among scientists, because the science of the world can be created only once, and it was Newton who created it" (Lagrange). Your viewpoint.

11. Newton's scientific thought rested ultimately on methaphysical assumption involving God, absolute space, absolute time, absolute Laws of Motion and Gravity, that rule the Universe. Are Newton's Laws absolutely right in our solar system? Do they extend beyond the relatively small distances of the nearest planets? Do stars attract each other as well as planets? The two stars are going around each other. Do they rotate according to Newton's Laws?

12. "No one must think that Newton's great creation can be overthrown by Relativity or any other theory. His clear and wide ideas will forever retain their significance as the foundation on which our modern conceptions of physics were built" (Einstein). What is the modern view of Newton's Mechanics?

13. The new relativistic frame became one of the most important guides in modern Mechanics and Physics. Why? Dispute **Einstein's theory** as a rational structure: in part philosophical and speculative, yet capable of experimental verification.

14. Einstein showed that the assumptions of absolute length, time and simultaneity were unjustified. A true scientist refuses to accept an empirical formula until he (or someone else) developed a theory to justify it, e. g., Planck had to find some theoretical justification for his radiation formula. Give some more examples.

15. The general theory of relativity has numerous successors and competitors. Enumerate some alternative theories. How are they tested?

16. There are a few wise scientists who decline to favour one theory of gravitation over another. They attempt to study all the theories as a class, hoping thereby to unlock some of the secrets of gravitation in an unbiased manner independent of any one particular theory. Are they right? Your viewpoint.

17. "What goes up must come down" is a classical saying which is not true any longer. Some of the rockets launched in recent times from the surface of the Earth became artificial satellites of the Earth with indefinitely long lifetimes, while others are lost in the vast expanse of interplanetary space. Explain the reason and possible justification.

18. **The Quantum Mechanics founders:** Einstein, Dirack, Ruzerford, Bohr, Planck, De Broile, Schrödinger, Heinserberg, Swinger, Fermi, Fok, Tamm, Landaw.

19. According to **Quantum Mechanics** — the mathematical theory of particles — our most precise description of nature must be in terms of probabilities. For the description of the small-scale phenomena of atomic and nuclear physics, Classical Mechanics was superseded by Quantum Mechanics. For phenomena involving speeds approaching that of light — by Relativity. Characterize modern Mechanics as a Science. The interrelation and interconnection of all branches of modern Mechanics.

LESSON SEVEN

INTRODUCTION TO ALGEBRA

Grammar:
1. Perfect Tense-Aspect Forms. Non-Continuous and Continuous Aspect.
2. Modal Compound Predicate.
3. Adjectives, Adverbs and their Russian Equivalents.

LAB. PRACTICE

Repeat the sentences after the instructors.

1. The origin of the title "Algebra" is rather exotic. 2. We owe the word "Algebra" to the Arab mathematician al-Khowarismi. 3. Although originally Algebra referred only to equations and their solution the word today has acquired a new connotation. 4. Algebra in its development passed successively through three stages: the **rhetorical** (or verbal), the **syncopated,** and the **symbolic.** 5. Rhetorical algebra is characterized by the complete absence of any symbols and the words were used in their symbolic sense. 6. In syncopated algebra certain words of common and frequent use were gradually abbreviated. 7. Eventually these abbreviations have become symbols. Modern algebra is symbolic. 8. One of the most interesting problems of algebra is that of the algebraic solution of equations. 9. Elementary algebra (from 1700 B. C. until abut 1700 A. D.) dealt exclusively with the general properties of numbers and the solution of algebraic equations. 10. Nearly all mathematicians of distinguished rank have treated this subject. 11. They arrived (XVI c.) at the general expression of the roots of equations of the first four degrees. 12. However, these solutions were achieved by ingenious devices rather than advances in insight and theory. 13. It was believed that a uniform method of solving equations should be applicable to an equation of any degree. 14. The mathematician's failure to reach the objective led to the presumption that the solution of general equations was impossible algebraically. 15. Early in the XIX c. a new view of mathematics began to emerge. 16. Mathematics came not to restrict itself to numbers and shapes. 17. Algebra nowadays deals effectively with anything (although "anything" often continues to be related in some way to numbers). 18. The mainstream in the development of algebra followed a parallel and concurrent stream in the development of the complex number system. 19. The introduction and acceptance of negative, imaginary, complex and hypercomplex numbers contributed to the development of modern algebraic notation. 20. Modern higher algebra can deal effectively with anything and occasionally it is pursued without reference to anything in particular. 21. The word "Algebra" was gradually being expan-

ded to include any system of handling symbols according to prescibed rules. Any one is free now to invent his own algebra. 23. It is known that there are only three algebras over the real field: a) the real number system, b) the complex number system, c) the system of quaternions. 24. Modern abstract algebra is the study of mathematical structures such as **groups, rings, fields, integral domains** etc. 25. It has many distinct departments but each separate branch cannot be treated in isolation: all of them constitute the subject-matter of Algebra.

Key Grammar Patterns

Perfect Non-Continuous Tense-Aspect Forms

	Active	*Passive*
	to have + done	**to have been + done**
Present	I **have** just **done** my work.	The work **has** just **been done.**
Past	I **had done** my work before he came.	The work **had been done** before he came.
Future	I **shall have done** my work by (at) 5 o'clock tomorrow.	The work **will have been done** by 5 o'clock tomorrow.

Perfect Continuous Tense-Aspect Forms

	Active	*Time Indicators*
	to have been + doing	
Present	I **have been doing** my work since early morning.	**For** a month (week, term, etc.) Lately. **For** quite a while
Past	I **had been doing my** work for two hours when he came.	**Since** morning (5 o'clock, etc.) **How long?**
Future	I **shall have been doing** my work for two hours before the lecture begins tomorrow.	**Since when? Up to now So far**

Read and translate the following text. Comment on the Perfect Tense-Aspect forms used.

Mathematics and Modern Civilization

Mathematics is the queen of natural knowledge.
C. F. Gauss

There are two ways in which mathematics **has become** so effective in our age. The first is through its relationships with science, the second is through its connection with human reasoning. Mathematical method is reasoning of the highest level known to man, and every field of investigation — be it law, politics, psychology, medicine or anthropology — **has felt** its influence and **has modelled** itself on mathematics to some extent ever since its creation. In order to gain a more comprehensive view of the relation of mathematics to the sciences, let us analyze the various ways in which mathematics **has been serving** scientific investigations.

I. Mathematics **has been supplying** a language for the treatment of the quantitative problems of the physical and social sciences. Much of this language **has taken** the form of mathematical symbols. Symbols also permit concise, clear (unambiguous) representation of ideas which are sometimes very complex. Scientists **have learned** to use mathematical symbols whenever possible.

2. Mathematics **has been supplying** science with numerous methods and conclusions. Among the important conclusions are its formulas, which scientists **have accepted** and **used** in solving problems. The use of such formulas is so common that the contribution of mathematics in this direction **has not been** fully **appreciated.**

3. Mathematics **has been enabling** the sciences to make predictions. This is perhaps the most valuable contribution of mathematics to the sciences. The ability to make predictions by mathematical means was exemplified in the most remarkable way in 1846 by the two astronomers Leverrier and Adams. As a result of calculations, they predicted, working independently, that there must exist another planet beyond those known at the time. A search for it in the sky at the predicted place and time revealed the planet Neptune. Prediction **has played** a part in every mathematical solution of a quantitative problem arising in the physical and social sciences.

4. Mathematics **has been furnishing** science with ideas to describe phenomena. Among such ideas may be mentioned the idea for functional relation; the graphical representation of functional relations by means of coordinate geometry; the notion of a limit; the notion of infinite classes which helps us to understand motion. Of special importance are the statistical methods and theories which **have led** to the idea of a statistical law. The description is not complete without mentioning the fact that for many physical phenomena no exact concepts exist other than mathematical ones.

Mathematics **has been** of use to science in preparing men's mind for new ways of thinking. The concepts of importance in science **had been coming** to men with great difficulty. The concepts of gravity, of energy and of limitless space took years to develop and men of genius were required to express them precisely. Great as is the genius of Einstein, it is almost certain that he was able to achieve some of his results only because the mathematics of preceding decades **had suggested** new ways of thinking about space and time.

To summarize: Mathematics **has been supplying** a language, methods and conclusions for science; **enabling** scientists to predict results; **furnishing** science with ideas to describe phenomena and **preparing** the minds of scientists for new ways of thinking.

It would be quite wrong to think that mathematics **had been giving** so much to the sciences and **receiving** nothing in return. Physical objects and observed facts **had** often **served** as a source of the elements and postulates of mathematics. Actually, the fundamental concepts of many branches of mathematics are the ones that **had been suggested** by physical experiences. Scientific theories **have** frequently **suggested** directions for pursuing mathematical investigations, thus furnishing a starting point for mathematical discoveries. For example, Copernican astronomy **had suggested** many new problems involving the effects of gravitational attraction between heavenly bodies in motion. These problems **had stimulated** the further activities of many scientists in the field of differential equations.

Modal Compound Predicate

A modal verb in combination with any form of the infinitive (Active or Passive) forms a **modal compound predicate.**

must

He **must solve** the problem himself.	...должен...
He **must be solving** the problem at present. He **must have solved** the problem. He **must have been solving** the problem for a long time.	...**должно быть**...

can, may **could, might**→more uncertainty, doubt, improbability

1. He **can (may) make** the research all alone.	...может...
2. He **can (may) be making** the research all alone. 3. He **can (may) have made** the research all alone. 4. He **can (may) have been making** the research all alone.	...**возможно**...

Translate the following sentences.

1. Algebraic formulas for finding the volumes of cylinders and spheres **may have been used** in Ancient Egypt to compute the amount of grain contained in them. 2. Babylonians **must have been** the first to solve the cubic equations by substitution. 3. The discovery of the theorem of Pythagoras **can** hardly **have been made** by Pythagoras himself; but it was certainly made in his school. 4. The Pythagoreans **may have been** the first to give a rigorous proof to the famous theorem. 5. Regardless of what mystical reasons **may have motivated** the early Pythagorean investigators, they discovered many curious and fascinating number properties. 6. Before Archimedes there **might have been no** systematic way of expressing large numbers. 7. The oath **must have been galling** to Cardano. 8. Viète's inability to accept negative numbers (not to mention imaginary numbers) **must have prevented** him from attaining the generality he sought and partly comprehended in giving, for example, relations between the roots and the coefficients of a polynomial equation. 9. R. Descartes's geometric representation of negative numbers **could have been helping** mathematicians to make negative numbers more acceptable. 10. Newton's earliest manifestations of the higher mathematical talent **may** well **have passed unnoticed.** 11. Imaginary numbers **must have been looking like** higher magic to many eighteenth-century mathematicians. 12. L. Euler **can have been using** imaginary numbers quite successfully as it is to him that we owe the formula $e^{2\pi i}=1$. 13. Gauss **must have been** the first to introduce the special sign $\equiv$ for the concept of congruence — although, of course, the concept itself was not original with Gauss. 14. In ordinary algebra, in which the letters represent real numbers, the field axioms **must be assumed.** 15. One **must be arriving at** nothing significant by adding telephone numbers. 16. The symbol $\sqrt{\ }$ **may have been used** in the XVI c. and it resembled a manuscript form of the small r (radix), or it **might have been invented** arbitrarily. By the XVII c. the use of symbol $\sqrt{\ }$ for square root had become quite standard.

Adjectives, Adverbs and their Russian Equivalents

Mind the difference:

Adjective	Adverb I	Adverb II	
bad	bad(ly)	badly	— очень, весьма
close	close	closely	— внимательно
fair	—	fairly	— весьма, совершенно, справедливо
hard	hard	hardly	— едва, почти не
high	high	highly	— очень, весьма
late	late	lately	— недавно, за последнее время
like	—	likely	— вероятно
large	—	largely	— главным образом
near	near	nearly	— почти
present	—	presently	— сейчас, скоро
ready	—	readily	— легко, охотно
short	—	shortly	— вскоре
sure	—	surely	— конечно
most	most	mostly	— главным образом, обычно
mere	—	merely	— только, просто, единственно

I. *Translate the sentences into Russian.*

1. The word "Al-jabr" may be **likely** translated as "the transposition of subtracted terms to the other side of an equation" and "the cancellation of like (equal) terms on opposite sides of the equation". 2. **Nearly** all mathematicians of distinguished rank have treated the subject of the algebraic solution of equations. 3. The Arabs acquired most **readily** the Greek and Hindus scientific writings which they translated into Arabic and preserved through the Dark Ages of Europe. 4. Powerful commercial cities arose first in Italy and **surely** it was here that the algebraic renaissance in Europe began. 5. Descartes's **La géométrie** consists **mostly** of what we now call the "theory of equations" and it contains his famous rule of signs for determining the number of positive and negative roots of an equation. 6. In the symbolic stage algebraic notation went through many modifications and changes until it became **fairly** stable by the time of Newton. 7. A great many persons have been involved in developing the foundations of modern algebra, **largely** the members of the British school of algebraists. 8. Considering **closely** the algebraic aspects of the real numbers mathematicians came to broad generalizations such as those of rings and fields, which not only contributed to an expansion of algebra but gave a penetrating perspective to elementary arithmetic. 9. More than two hundred algebraic structures have been studied **lately.** 10. In 1826 the twenty-four-year-old Abel, poverty-stricken and suffering from tuberculosis, wanting **badly** some recognition, published the first general proof of the binomial theorem for arbitrary complex exponents. 11. Abel died when he was only 27 leaving behind a wealth of **highly** original work which stimulated research for many years after. 12. **Shortly** after Galois was killed in a duel in 1832 the development of group theory was substantially advanced by Cauchy. 13. The contributions of Abel and Galois to modern algebra can **hardly** be overestimated. 14. In collaboration with Sylvester, Cayley (1846) began the work on the theory of algebraic invariants, which had been **merely** in the air for some time and which, like matrices, received some of its motivation from determinants.

II. *Give the Russian equivalents of:*

Specific — specifically, comprehensive — comprehensively, unambiguous — unambiguously, customary — customarily, usual — usually, essential — essentially, similar — similarly, principal — principally, respective — respectively, particular — particularly, simultaneous — simultaneously, successive — successively, eventual — eventually, evident — evidently, obvious — obviously, literal — literally, occasional — occasionally, rapid — rapidly, sensitive — sensitively, consequent — consequently, natural — naturally, curious — curiously, unique — uniquely, (in)consistent — (in)consistently, arbitrary — arbitrarily, rigorous — rigorously, sufficient — sufficiently, subsequent — subsequently, careful — carefully, implicit — implicitly, significant — significantly, apparent — apparently, strict — strictly, accurate — accurately, fair — fairly.

III. *Give the corresponding adverbs and their Russian equivalents.*

Adequate, congruent, considerable, conventional, different, effective, equal, exceeding, exclusive, frequent, general, gradual, immediate, main, normal, ordinary, previous, qualitative, quantitative, rare, reasonable, recent, repeated, reverse, satisfactory, scientific, suitable, thorough, unfortunate.

THE INTRODUCTORY TEXT

THE HISTORY OF ALGEBRA

Exotic and intriguing is the origin of the word "algebra". It does not submit to a neat etymology as does, for example, the word "arithmetic", which is derived from the Greek **arithmos** ("number"). **Algebra** is a Latin variant of the Arabic word **al-jabr** (sometimes translated aljebr) as employed in the title of a book, "**Hisab al-jabr w'al mugabalah",** written in Baghdad about A. D. 825 by the Arab mathematician **Mohammed ibn-Musa al-Khowarismi.** This treatise on algebra is commonly referred to, in shortened form, as **Al-jabr.** A literal translation of the book's full title is "science of restoration (or reunion) and opposition", but a more mathematical phrasing is "science of transposition and cancellation". Perhaps the best translation is simply "the science of equations".

Although originally "algebra" referred to equations, the word today has a much broader meaning, and a satisfactory definition requires a two-phase approach: 1. Early (elementary) algebra is the study of equations and methods of solving them. 2. Modern (abstract) algebra is the study of mathematical structures such as groups, rings, and fields — to mention only a few. Indeed, it is convenient to trace the development of algebra in terms of these two phases, since the division is both chronological and conceptual.

The Early Algebra

Babylonian Algebra — Rhetorical Style

Since algebra might have probably originated in Babylonia, it seems appropriate to credit the country with the origin of the rhetorical style of algebra, illustrated by the problems found in clay tablets dating back to c. 1700 B. C. The problems show the relatively sophisticated level of their algebra. Nowadays such problems are solved by **the met-**

hod of elimination. The Babylonians also knew how to solve systems by elimination but preferred often to use their **parametric method.** The Babylonians were able to solve a rather surprising variety of equations, including certain special types of cubics and quartics — all with numerical coefficients, of course.

Algebra in Egypt

Algebra in Egypt must have appeared almost as soon as in Babylonia; but Egyptian algebra lacked the sophistication in method shown by Babylonian algebra, as well as its variety in types of equations solved. For linear equations the Egyptians used a method of solution consisting of an initial estimate followed by a final correction, a method now known as the **"rule of false position".** The algebra of Egypt, like that of Babylonia, was rhetorical.

The numeration system of the Egyptians, relatively primitive in comparison with that of the Babylonians, helps to explain the lack of sophistication in Egyptian algebra. European mathematicians of the sixteenth century had to extend the Hindu-Arabic notion of number before they could progress significantly beyond the Babylonian results in solving equations.

Early Greek Algebra

The algebra of the early Greeks (of the Pythagoreans and Euclid, Archimedes, and Apollonius, 500—200 B. C.) was geometric because of their **logical** difficulties with irrational and even fractional numbers and their **practical** difficulties with Greek numerals, which were somewhat similar to Roman numerals and just as clumsy. It was natural for the Greek mathematicians of this period to use a geometric style for which they had both taste and skill.

The Greeks of Euclid's day thought of the product ab (as we write it nowadays) as a rectangle of base b and height a and they referred to it as "a rectangle contained by CD and DE". Some centuries later, another Greek, Diophantus, made a start toward modern symbolism in his work "Diophantine equations" by introducing abbreviated words and avoiding the rather cumbersome style of geometric algebra. Diophantus introduced the **syncopated** style of writing equations.

Hindu and Arabic Algebra

Little is known about Hindu mathematics before the fourth or fifth century A. D. because few records of the ancient period have been found. India was subjected to numerous invasions, which facilitated the exchange of ideas. Babylonian and Greek accomplishments, in particular, were apparently known to Hindu mathematicians. The Hindus solved quadratic equations by **"completing the square"** and they accepted negative and irrational roots; they also realized that a quadratic equation (with real roots) has two roots. Hindu work on indeterminate equations was superior to that of Diophantus; the Hindus attempted to find **all possible integral solutions** and were perhaps the first to give general methods of solution. One of their most outstanding achievements was the system of Hindu (often called Arabic) numerals.

Algebra in Europe

In the eleventh century many Greek and Arabic texts on mathematics were translated into Latin and became available in Europe. However, even more important for Europe, especially Italy was the "Liber abaci" (1202) of **Fibonacci** (Leonardo of Pisa) in which he solved equations in the rhetorical and general style and strongly advocated the use of Hindu-Arabic numerals, which he discovered on his journeys to many lands as a merchant and tradesman. It is not surprising that at first the local chambers of commerce (in Pisa and neighbouring city-states of Italy) resisted the adoption of the "new" Hindu-Arabic numerals and in fact viewed them with suspicion; but they were gradually adopted, and the old abacus was stored in the attic.

The algebra that entered Europe (via Fibonacci's "Liber abaci" and translations) had retrogressed both in style and in content. The semi-symbolism of Diophantus and relatively advanced accomplishments of the Hindus were not destined to contribute to the eventual breakthrough in European algebra.

Symbolic Algebraic Notation

Modern symbolism began to emerge around 1500. A banner year was 1545: in that year **G. Cardano,** an Italian scholar, published his "Ars Magna" ("Great Art") containing the solution of the cubic and the quartic. These solutions represented the first really new material since antiquity, even though these essentially general solutions were achieved by "ingenious devices" rather than advances in insight and theory. The watershed of algebraic thought (separating the early shallow flow of "manipulative solution of equations" from the deeper modern stream which began with the theoretical properties of equations) is personified in the Frenchman **Fr. Viète,** who in 1573 was the first to introduce letters as general (positive) coefficients and to put some other finishing touches to symbolism. Later **R. Descartes** systematically used the first letters of the alphabet for the given quantities, and the last letters for the unknowns and made algebraic notation finally up-to-date by the time of I. Newton. Just as the discovery of zero created the arithmetic of today, so did the literal notation ushered in a new era in the history of algebra. Wherein lies the power of this symbolizm? First of all, the letter liberated algebra from the slavery of the word. This is important enough; but what is still more important is that the letter is free from the taboos which have been attached to words through centuries of use. In the second place, the letter is susceptible of operations which enable one to transform literal expressions and thus to paraphrase any statement into a number of equivalent forms. But the most important contribution of symbolism is the role it has been playing in the formation of the generalized number concept.

Algebra is not only a part of mathematics; it also plays within mathematics the role which mathematics itself had been playing for a long time with respect to physics. What does the algebraist have to offer to other mathematicians? Occasionally, the solution of a specific problem; but mostly a language in which to express mathematical facts and a variety of patterns of reasoning, put in a standard form. Algebra is not an end in itself; it has to listen to outside demands issued from various parts of mathematics. This situation is of great benefit to algebra; for, a science, or a part of science, which exists to solve its own problems only, is always in danger of falling into peaceful slumber and from there into a quiet death. But in order to take full advantage of this state of affairs, the algebraist must have the ability to derive profit from what he perceives is going on outside his own domain. Algebra, like every other modern branch of mathematics and science, continues to proliferate with the vitality and expansiveness of a tropical forest and every particular part of algebra has much new mathematical knowledge that is being discovered, so that the algebraist should keep his eyes open for the small piece that may be of great value to him. Mathematics is changing constantly, and algebra must reflect these changes if it wants to stay alive. This explains the fact that algebra is one of the most rapidly changing areas of mathematics; it is sensitive not only to what happens inside its own boundaries; but also to the trends which originated in all other branches of mathematics. The most important new demands on algebra come from topology, analysis and algebraic geometry.

The mainstream in the development of algebraic structure followed a parallel and concurrent stream in the development of the complex-number system. The introduction and acceptance of negative, imaginary and complex numbers contributed to the development of modern algebraic notation. The foundations begun by Viète for the modern structural formulation of algebra had to wait some two hundred years before **Niels Henrik Abel** (1824) and especially **Évariste Galois** (1831) introduced **the idea of a group,** in their independent proofs that **a polynomial equation of degree greater than four has no general algebraic solution.** During the two hundred years from Viète to Abel and Galois, mathematicians were not idle; the group concept, of course, did not emerge suddenly with Abel and Galois. In the works of the best mathematicians of the time an implicit grasp of the group concept was already to be found.

ACTIVE VOCABULARY

1. to adopt
2. to appoint
3. to appropriate
4. to approve
5. to baffle
6. to benefit
7. to betray
8. to comperehend
9. to confer
10. to dawn
11. to divulge
12. to eliminate
13. to encounter
14. to endeavo(u)r
15. to expand
16. to evoke
17. to forbid
18. to fordoom
19. to govern
20. to haunt
21. to permute
22. to pervade
23. to presume
24. to prevail
25. to proliferate
26. to release
27. to remedy
28. to revive
29. to revoke
30. to submit
31. to surrender
32. to surround
33. to survive
34. to suspect
35. to swear
36. to unfold
37. to unravel
38. to vanish
39. to vindicate

Read the text. Give some more details and your own comments concerning all the algebraists mentioned in the text. Practise questions and answers.

TEXT ONE

SOLUTION OF POLYNOMIAL EQUATIONS OF THIRD AND HIGHER DEGREE

The first records of man's interest in **cubic equations** date from the time of the old Babylonian civilisation, about 1800-1600 B. C. Among the mathematical materials that survive, are tables of cubes and cube roots, as well as tables of values of n^2+n^3. Such tables could have been used to solve cubics of special types. For example, to solve the equation $2x^3+3x^2=540$, the Babylonians might have first multiplied by 4 and made the substitution $y=2x$, giving $y^3+3y^2=2{,}160$. Letting $y=3z$, this becomes $z^3+z^2=80$. From the tables, one solution is $z=4$, and hence 6 is a root of the original equation.

In the Greek period concern with volumes of geometrical solids led easily to problems that in modern form involve cubic equations. The well-known problem of duplicating the cube is essentially one of solving the equation $x^3=2$. This problem, impossible of solution by ruler and compasses alone, was solved in an ingenious manner by **Archytas of Tarentum** (c. 400 B. C.), using the intersections of a cone, a cylinder, and a degenerate torus (obtained by revolving a circle about its tangent).

The well-known Persian poet and mathematician **Omar Khayyam** (A. D. 1100) advanced the study of the cubic by essentially Greek methods. He found solutions through the use of conics. It is typical of the state of algebra in his day that he distinguished thirteen special types of cubics that have positive roots. For example, he solved equations of the type $x^3+b^2x=b^2c$ (where b and c are positive numbers) by finding intersections of the parabola $x^2=by$ and the circle $y^2=x(c-x)$, where the circle is tangent to the axis of the parabola at its vertex. The positive root of Omar Khayyam's equation is represented by the distance from the axis of the parabola to a point of intersection of the curves.

The next major advance was the algebraic solution of the cubic. This discovery, a product of the Italian Renaissance, is surrounded by an atmosphere of mystery; the story is still not entirely clear. The method appeared in print in 1545 in the "**Ars magna**" of Girolamo **Cardano** of Milan, a physician, astrologer, mathematician, prolific writer, and suspected heretic, altogether one of the most colourful figures of his time. The method gained currency as "Cardan's formula"; (Cardan is the English form of his name). According to Cardano himself, however, the credit is due to **Scipione del Ferro,** a professor of mathematics at the University of Bologna, who in 1515 discovered how to solve cubics of the type $x^3+bx=c$. As was customary among mathematicians of that time, he kept his methods secret in order to use them for personal advantage in mathematical duels and tournaments. When he died in 1526, the only persons familiar with his work were a son-in-law and one of his students, **Antonio Maria Fior** of Venice.

In 1535 Fior challenged the prominent mathematician **Niccolo Tartaglia** of Brescia (then teaching in Venice) to a contest because Fior did not believe Tartaglia's claim of having found a solution for cubics of the type $x^3+bx^2=c$. A few days before the contest Tartaglia managed to discover also how to solve cubics of the type $x^3+ax=c$, a discovery (so he relates) that came to him in a flash during the night of February 12/13, 1535. Needless to say, since Tartaglia could solve two types of cubics whereas Fior could solve only one type, Tartaglia won the contest. Cardano, hearing of Tartaglia's victory, was eager to learn his method. Tartaglia kept putting him off, however, and it was not until four years later that a meeting was arranged between them. At this meeting Tartaglia divulged his methods, swearing Cardano to secrecy and particularly forbidding him to publish it. This oath must have been galling to Cardano. On a visit to Bologna several years later he met Ferro's son-in-law and learned of Ferro's prior solution. Feeling, perhaps, that this knowledge released him from his oath to Tartaglia, Cardano published a version of the method in **Ars Magna.** This action evoked bitter attack from Tartaglia, who claimed that he had been betrayed.

Although couched in geometrical language the method itself is algebraic and the style syncopated. Cardano gives as an example the equation $x^3+6x=20$ and seeks two unknown quantities, p and q, whose difference is the constant term 20 and whose product is the cube of 1/3 the coefficient of x, 8. A solution is then furnished by the difference of the cube roots of p and q. For this example the solution is

$$\sqrt[3]{\sqrt{108}+10}-\sqrt[3]{\sqrt{108}-10}.$$

The procedure easily applies to the general cubic after being transformed to remove the term in x^2. This discovery left unanswered such questions as these: What should be done with negative and imaginary roots, and (a related question) do three roots always exist? What should be done (in the so-called irreducible case) when Cardano's method produced apparently imaginary expression like

$$\sqrt[3]{81+30\sqrt{-3}}+\sqrt[3]{81-30\sqrt{-3}}$$

for the real root, —6, of the cubic $x^3-63x-162=0$? These questions were not fully settled until 1732, when **Leonard Euler** found a solution.

The general **quartic equation** yielded to methods of similar character; and its solution, also, appeared in **Ars Magna.** Cardano's pupil **Ludovico Ferrari** was responsible for this result. Ferrari, while still in his teens (1540), solved a challenging problem that his teacher could not solve. His solution can be described as follows: First reduce the general quartic to one in which the x^3 term is missing, then rearrange the terms and add a suitable quantity (with undertermined coefficient) to both sides so that the left-hand member is a perfect square. The undetermined coefficients are then determined so that the right-hand member is also a square, by requiring that its determinant be zero. This condition leads to a cubic, which can now be solved — the quartic can then be easily handled.

Later efforts to solve the **quintic** and other equations were foredoomed to failure, but not until the nineteenth century was this finally recognized. **Carl Fridrich Gauss** proved in 1799 that every algebraic equation of degree n over the real field has a root (and hence n roots) in the complex field. The problem was to express these roots in terms of the coefficients by radicals. **Paolo Ruffini,** an Italian teacher of mathematics and medicine at Modena, gave (in 1813) an essentially satisfactory proof of the impossibility of doing this for equations of degree higher than four, but this proof was not well-known at the time and produced practically no effect.

Read the text. Generalize the main ideas of the text and reproduce them in class.

TEXT TWO

THE THEORY OF EQUATIONS

History shows the necessity for the invention of new numbers in the orderly progress of civilisation and in the evolution of mathematics. We must review briefly the growth of the number system in the light of the theory of equations and see why **the complex number system** need not be enlarged further. Suppose we decide that we want all polynomial equations to have roots. Now let us imagine that we have no numbers in our possession except **the natural numbers.** Then a simple linear equation like $2x=3$ has no root. In order to remedy this condition, we invent **fractions.** But a simple linear equation, like $x+5=2$ has no root even among the fractions. Hence we invent **negative numbers.** A simple quadratic equation like $x^2=2$ has no root among all the (positive and negative) **rational numbers,** therefore we invent the **irrational numbers** which together with the rational numbers complete the system of **real numbers.**

However, a simple quadratic equation like $x^2=-1$ has no root among all the real numbers, hence, we invent **the pure imaginary numbers.** But a simple quadratic equation like $x^2+2x+4=0$ has no roots among either the real or pure imaginary numbers; therefore we invent **the complex numbers.** The story of $\sqrt{-1}$ the imaginary unit, and of $x+yi$, the complex number, originated in the logical development of algebraic theory. The word "imaginary" reflects the elusive nature of the concept for distinguished mathematicians who lived centuries ago. Early consideration of the square root of a negative number brought unvarying rejection. It seemd obvious that a negative number is not a square, and hence it was concluded that such square roots had no meaning. This attitude prevailed for a long time.

G. Cardano (1545) is credited with some progress in introducing complex numbers in his solution of the cubic equation, even though he regarded them as "fictitious". He is credited also with the first use of the square root of a negative number in solving the now-famous problem, "Divide 10 into two parts such that the product ... is 40", which Cardano first says is "manifestly impossible"; but then he goes on to say, in a properly advanturous spirit, "Nevertheless, we will operate". Thus he found $5+\sqrt{15}$ and $5-\sqrt{-15}$ and showed that they did indeed have the sum of 10 and a product of 40. Cardano concludes by saying that these quantities are "truly sophisticated" and that to continue working with them is "as subtle as it is useless". Cardano did not use the symbol $\sqrt{-15}$, his designation was "$R_x{\cdot}m$", that is, "radix minus", for the square root of a negative number. R. Descartes (1637) contributed the terms "real" and "imaginary". L. Euler (1748) used "i" for $\sqrt{-1}$ and C. F. Gauss (1832) introduced the term "complex number". He made significant contributions to the understanding of complex numbers through graphical representation and defined complex numbers as ordered pairs of real numbers for which $(a,\ b)\cdot(c,\ d)=(ac-bd,\ ad+bc)$, and so forth.

Now, we may well expect that there may be some equation of degree 3 or higher which has no roots, even in the entire system of complex numbers. That this is not the case was known to **C. F. Gauss,** who proved in 1799 the following theorem, the truth of which had long been expected: **Every algebraic equation of degree *n* with coefficients in the complex number system has a root (and hence *n* roots) among the com-**

plex numbers. Later Gauss published three more proofs of the theorem. It was he who called it "**Fundamental Theorem of Algebra**". Much of the work on complex number theory is Gauss's. He was one of the first to represent complex numbers as points in a plane. Actually, Gauss gave four proofs for the theorem, the last when he was seventy; in the first three proofs he assumes the coefficients of the polynomial equation are real, but in the fourth proof the coefficients are any complex numbers. We can be sure now that for the purpose of solving polynomial equations we do not need to extend the number system any further.

Algebraic Formulas for the Roots

The general linear equation can be written in the form $ax+b=0$ $(a\neq 0)$, hence the formula for its roots is $x=\frac{-b}{a}$ The mathematician's desire for several results makes it natural to ask the following question: Can we get similar formulas giving the roots as algebraic expressions in terms of the coefficients for the general equation of any degree? For the general quadratic and cubic equations and equation of degree four such formulas, as we have already seen, were obtained in the XVI c. The next task was naturally to obtain similar formulas for the general equation of degree five: $ax^5+bx^4+cx^3+dx^2+ex+f=0$. Attempts to find such formulas were made from the XVI c. until early in the XIX c. without success. The reason for this failure became evident (in 1824) when **N. H. Abel** and **E. Galois** (in 1831) proved that it is not possible to write the roots of the general equation of degree higher than **four** as algebraic expressions in terms of the coefficients. You may be tempted to ask: "How can you boldly assert that it is impossible to find such formulas? Perhaps some day some genius will discover them. All things are possible. Are you sure you don't mean simply that no one has found them yet?" The answer is that we do not merely mean that no one has found them yet; we mean that no one will ever find them because it is impossible for such formulas to exist. Notice that we have not said that the general equation of degree **five** cannot be solved. In fact, it can be solved by other means, but its roots cannot be given as algebraic expressions in the coefficients. However, the roots of some **particular** equations of degree five or more can be obtained. For example, if in the fifth degree equation above, we restrict ourselves to the particular case where $b=c=d=e=0$ $a\neq 0$, that is, to equations of the form $ax^5+f=0$, then we can clearly express one root as $x=\sqrt[5]{-f/a}$ which is an algebraic expression. Therefore a natural question to raise is: **Given a definite polynomial equation of degree five or more, how can we tell whether or not its roots are expressible as algebraic expressions in its coefficients?** This question was settled by E. Galois.

Before describing the momentous work of Abel and Galois, we must note some of the events immediately preceding and directly influencing the remarkable achievements of these gifted young mathematicians both of whom died in their twenties. In 1770 **Euler** devised a new method for solving the quartic equation but his optimistic hope that some similar method could solve the general polynomial equation was ill-fated. In the same year **Lagrange** considered the problem of solving the general polynomial equation by comparing the known solutions of quadratic, cubic and quartic equations and noting that in each of these three cases a certain reduction transformed the equation to one of the lower degree; but, unhappily, when Lagrange tried this "reduction" on a quintic equation, the degree of the resulting equation was increased rather than decreased. Although Lagrange did not succeed in his main objective, his attack on the problem made use of **permutations of the roots of the equations;** and he discovered the key to the theory of permutation groups, including the property mentioned earlier and now called Lagrange's theorem.

Both Abel and Galois built on Lagrange's work. It is not surprising that Abel approached the general problem of trying to solve the polynomial equation of degree n by trying to solve the general quintic equation. In fact, he thought he had succeeded and the "solution" was sent to a leading mathematician, but while waiting for a reply, Abel fortunately discovered his mistake and this misadvanture caused him to wonder **whether a general algebraic solution was indeed possible.** Although Abel succeeded in showing that for n greater than four the general polynomial equation could not be solved algebraically, he did not claim to have completely achieved the objective he set for himself: 1) to find all the equations of any degree which are solvable algebraically; 2) to determine whether a given equation is or is not solvable algebraically.

It was fortunate that Abel's proof, in which he used permutation groups to some extent, received early publication. This proof caught the imagination of **Galois** who gave complete answers to the questions proposed by Abel. Galois showed that every equation could be associated with a characteristic group and that the properties of this group could be used to determine whether the equation could be solved by radicals. In 1831 Galois stated his criterion: **A polynomial equation is solvable if and only if its group, over the coefficient field, is solvable.** The concepts associated with this result was usually characterized as Galois's theory. In his work he used the idea of isomorphic groups, and was the first to demonstrate the importance of invariant (or normal) subgroups and factor groups. **The term "group" is due to Galois.** The work of Galois was quite original in character and was not well understood at the time because of the sketchy expositions which he presented. Galois's mathematical abilities were not appreciated by his teachers, and in fact he received no recognition for his work while he lived. Although Galois's accomplishments were mathematical landmarks of the greatest significance and originality, they did not immediately make their full impact on his contemporaries because these men were slow to understand, appreciate, and publish Galois's work. However, what is now called **the Galois theory of equations** is studied everywhere by advanced students of mathematics.

Abel was not yet 27 when he died leaving behind a wealth of highly original work which stimulated mathematical research for many years after. Galois was killed in a duel at the age of less than 21. Abel and Galois proved in entirely different ways that **there cannot be any general formulas for solving polynomial equations of degree higher than four.** At least there can be no formulas which give the solutions in terms of the coefficients and which involve only addition, subtraction, multiplication, division and the extraction of roots.

Read the text. Reproduce orally the historical development of algebraic structures and explain why many different branches of modern mathematics are all interrelated by virtue of the "group" structures.

TEXT THREE

FIELDS, RINGS, GROUPS

The concept of a "field" was used by both Abel and Galois at an intuitive, subformal level in their work on polynomial equations. In algebra the word " field" is used to describe a structure that closely resembles ordinary arithmetic. The operations of addition, subtraction, multiplication and division occur in a field and are much like the corresponding operations in arithmetic. **The set of real numbers, under ordinary addition and multiplication, is the most familiar example of a field.** There exists a large variety of fields in algebra. In ordinary algebra in which the letters represent real numbers, the field axioms are assumed. One of the most interesting field properties usually assumed in ordinary algebra (actually it is not an axiom but a theorem) is the "nonexistence of zero divisors". This is used in solving quadratic equation by the factoring method and guarantees that if a product like $(x-2)(x-3)$ is zero, at least one of the factors must be zero. In 1871 **R. Dedekind** gave a concrete formulation and the earliest expositions of the theory of fields. One of the greatest accomplishments of the XIX c. in mathematics is expressed in the statement that the real number system is a **"complete ordered field"**.

More formally the word "field" means a mathematical system in which addition and multiplication can be carried out in a way that satisfies the familiar rules, namely (1) the commutative law of addition and multiplication, (2) the associative law of addition and multiplication, (3) the distributive law. Furthermore, a field must contain a zero element 0, characterized by the property, that $x+0=x$ for any element x. It contains a unit element, 1, that has the property that $1\times x=x$. For any given element x there exists another element $-x$ such that $-x+x=0$. Finally, for any elements x $(x\neq 0)$ a field must contain an element $1/x$ such that $x(1/x)=1$. **Thus, a field is a structure (exemplified by e. g., the rational numbers) whose elements can be added, subtracted, multiplied and divided under the familiar rules of arithmetic.**

Considering now the second word, a field is **"ordered"** if the sizes of its elements can be compared. The shorthand symbol used to denote this property is the sign $>$, meaning "greater than". This symbol must obey its own set of rules, (1) the trichotomy law: for any two elements x and y, exactly one of the following three relations is true, $x>y$, $x=y$, or $y>x$; (2) the transitivity law: if $x>y$ and $y>z$ then $x>z$; (3) the law of addition: if $x>y$, then $x+z>y+z$; (4) the law of multiplication: if $x>y$ and $z>0$, then $xz>yz$.

Finally, what do we mean by the word "complete" in describing the system of real numbers as a "complete ordered field"? This has to do with the problem raised by a number such as $\sqrt{2}$. Practically speaking, $\sqrt{2}$ is given by a sequence of rational numbers such as 1, 1.4, 1.41, 1.414 ... that provide better and better approximation to it. Squaring these numbers yields a sequence of numbers that are getting closer and closer to 2. So, we think of $\sqrt{2}$ as a "limiting value" of such a sequence of approximation. An ordered field is called "complete" if, corresponding to any regular sequence of elements, there is an element of the field that the sequence approaches as a limiting value. This is "the law of completenss", the final axiomatic requirement for the real-number system.

In a field as we have just seen, we can add, subtract, multiply and divide (except that division by 0 is barred). Not all algebraic structures have as comprehensive a list of operations. In a **Ring**, for example, we can add, subtract and multiply but not necessarily divide. A familiar example of a **Ring is the whole numbers, both positive and negative.** Even more restricted than a ring is the concept of a **group**, with the existence in it of only one operation, which can be thought of as a kind of **generalized multiplication.** The idea of a group is one which pervades the whole of modern mathematics both pure and applied. The theory of groups, a central concern of contemporary mathematics, has evolved through a progression of abstractions. **A group is one of the simplest and the most important algebraic structures of consequence.** Group theory traces its origin back to a problem that has fascinated mathematicians since the Middle Ages: the solution of algebraic equations of degree greater than two by algebraic processes. In the particular form of the study of symmetry, group theory can claim to have its origin in prehistoric times. Nowadays, group theory is developed in an abstract way so that it can be applied in many different circumstances but many of those applications still concern symmetry.

Some of the components of the group concept (i. e., those essential properties that were later abstracted and formulated as axioms) and also of the field concept, were recognized as early as 1650 B. C. when the Egyptians showed a curious awareness that something was involved in assuming that $ab=ba$. The Egyptians also freely used the distributive law, namely, $a(b+c)=ab+ac$, but without any comment. The Babylonians (c. 1700 B. C.) also used the commutative and distributive laws. These laws were tacitly assumed in their rhetorical algebra when, in effect they used such formulas as $(a+b)^2=a^2+2ab+b^2$. Looking at Greek mathematics, we see that Euclid was more aware of the explicit nature of the distributive law, declaring in his Proposition 1: $a(b+c+d)=ab+ac+ad$. Somewhat later Diophantus exhibited interesting insights regarding multiplicative inverses and the unity element. One may perhaps claim that the concept of a **cyclic group** is prehistoric in the sense that the Ancients measured a circle by using equal divisions of its circumference, or that the 24-hour clocks of the Babylonians and Egyptians were (implicitly) examples of finite additive groups with 24 used as a zero element and Euclid's work contains at the implicit level what is classified now as algebraic number theory and group theory.

The group concept was not recognized as explicitly as were some of its axioms, but even so it was implicitly sensed and used before Abel and Galois brought it into focus and before **Cayley (1854)** defined a general abstract group. During the two hundred years from Viète to Abel and Galois, in the work of some great mathematicians an implicit grasp of the group concept was already to be found. During **the seventeenth century** it was clear to those working with the nth roots of unity that these n elements formed a multiplicative cyclic group and that the primitive nth roots could be used as generators of the group. The use of group theory at the subformal level — and a striking one — is found in **Euler's** proof (1760—1761) of a generalization of Fermat's "little theorem". Euler was actually using an idea later formulated by **Lagrange** (1770) and now known as Lagrange's theorem, which says that the number of elements in the first-column subgroup divides the number of elements in the whole table. Lagrange gave the idea an explicit and general formulation and showed that the number of elements in a symmetric group is divisible by the number of elements in any subgroup (which is, of course, a permutation group). Hence his result was valid for non-Abelian permutation groups as well as for Abelian groups.

In 1779 **Paolo Ruffini** showed that the converse of Lagrange's theorem is not true; that is, a group of order n does not necessarily have a subgroup of order s just because s divides n. Just preceding Abel and Galois, **Cauchy** (1815) published his first article on group theory dealing with the permutation groups and **Gauss** formalized the modular system (which are additive cyclic groups) and in his work on the theory of the composition of quadratic forms, derived a complete set of properties, which taken axiomatically, define an Abelian group. In 1854 **Cayley** published an article entitled **"On the Theory of Groups as Depending on the Symbolic Equation $\theta^n = 1$"**, which is noteworthy because it gives what is probably the earliest definition of a finite abstract group. It also gives the result now known as Cayley's theorem, that "every finite group is isomorphic to a regular permutation group". In 1870 B. C. **Jordan** published his **"Traité des Substitutions"** — a masterly presentation of permutation groups — covering the results of Lagrange, Abel, Galois, Cauchy as well as his own contributions to the subject. In the same year **L. Kronecker** gave a set of axioms defining finite Abelian groups. Out of this complete axiomatic system for finite Abelian groups, Kronecker, working with a completely arbitrary abstract set of elements, derived the customary group properties.

It will be convenient at this point to give a formal definition of a group, and we shall illustrate the definition by giving an example of a permutation group, since permutation groups were historically important in developing the group concept. Consider the set $G = \{1, a, b, c, d, e\}$ where the six elements of G are, respectively, the six permutations 1) 123, a) 132, b) 213, c) 231, d) 312, e) 321. The "product" ab is defined to mean that the first permutation a is performed and then permutation b is performed on the result of a. Thus, a produces (1, 3, 2) and, applying b to (1, 3, 2), we get the result (2, 3, 1), which is c. So we say that $ab = c$. Notice that $ba = d$, and so our multiplication in this example is not commutative. All possible products are given in the figure below. (To find the product ab from the table, use the left and top margins, in their order, for a and b; the entry is c, indicated by the dotted lines.) By referring to the table the following defining properties (axioms) of a group are easily checked:

	I	a	b	c	d	e
I	I	a	b	c	d	e
a	a	I	c	b	d	e
b	b	d	I	e	a	c
c	c	e	a	d	I	b
d	d	b	e	I	c	a
e	a	c	d	a	b	I

1. The set G is closed with respect to the defined multiplication; that is, the product of any two elements of G is again an element of G. Thus, $be = e$, and e is an element of G.
2. The associative law holds for any three elements of G. Thus, $(ab)c = a(bc)$; $(c)c = a(e)$; $d = d$.
3. The set G has an identity, element (in this example, I) with the property that $a\mathrm{I} = \mathrm{I}a = a$, $b\mathrm{I} = \mathrm{I}b = b$, etc.
4. Each element of G has an **inverse**; for example, we can find

An x so that $ax = \mathrm{I}$ (in fact, $x = a$, since $aa = \mathrm{I}$)

A y so that $by = \mathrm{I}$ (in fact, $y = b$, since $bb = \mathrm{I}$)

A z so that $cz = \mathrm{I}$ (in fact, $z = d$, since $cd = \mathrm{I}$).

The set G is called a **group** if and only if the above four properties hold. Some groups have the additional property of commutativity, and they are called **Abelian (or commutative) groups.** Part of the fascination that group theory exercises on many mathematicians lies in the fact that the whole structure is created on the logical foundation of these four simple axioms. The relation of the group concept to that of a field may be briefly indicated by noting that the set F of real numbers is an example of a field because the elements of F satisfy the five axioms for a commutative group with respect to addition (where the identity element is represented by 0); the nonzero elements of F satisfy five more axioms for a commutative group with respect to multiplication (where the identity element is represented by 1); and the elements of F satisfy a final axiom called the distributive law, namely, $a\cdot(b+c)=a\cdot b+a\cdot c$, which "distributes" the multiplication "over" the addition.

Pure group theory is not usually the most fruitful part of the theory. The reason for this is that it is not the symmetry as such which is of prime importance but rather the way in which the symmetry governs and simplifies the discussion of the quantitative properties of the system. This quantitative aspect is known as **Group Representation Theory** since the abstract groups or symmetry groups are represented by groups of matrices.

Group Algebra

In order to be able to investigate the structure of a group more profoundly it is necessary to introduce addition as a second operation between group elements. This addition has to be considered as a formal operation in the sense that the coefficients of the various group elements are added separately just as algebraic expressions in different unknowns x, y, z, etc., are added. By introducing addition as a second operation into the group, a more elaborate algebraic entity has been created whose elements are the linear combinations of group elements. This entity in the group algebra is one example of the more general concept of a **linear associative Algebra.**

The Significance of Groups in Algebra and Geometry

Groups and semigroups have considerable significance in the foundations of mathematics. The importance of groups and semigroups in algebra lies largely in the fact that many algebraic systems are actually groups or semigroups with respect to one or more of the binary operations of the systems. In other words, **many algebraic structures contain the group structure or the semigroup structure within them as a substructure.** Groups and semigroups are like algebraic atoms from which many algebraic system can be constructed. These ideas are illustrated by the following alternative definitions of **ring,** commutative ring, ring with identity, integral domain, division ring and field.

1. **A ring** is a set S of elements for which two binary operations, addition and multiplication, are defined such that (1) S is an Abelian group under addition with identity element called zero, (2) S is a semigroup under multiplication, (3) the two distributive laws of multiplication over addition hold.

2. **A commutative ring** is a ring in which the multiplicative semigroup is Abelian.

3. **A ring with identity** is a ring in which the multiplicative semigroup has an identity element.

4. **An integral domain** is a ring in which the nonzero elements constitute a subsemigroup of the multiplicative semigroup.

5. **A division ring** (or a **field**) is a ring of more than one element in which the nonzero elements constitute a subgroup of the multiplicative semigroup.

6. **A field** is a division ring in which the multiplicative semigroup is Abelian. Two very important examples of ordered fields are the set of all rational numbers and the set of all real numbers, with the operations of addition and multiplication performed on these numbers.

Read the text. Speak about the extensions of the number concept and its generalizations.

TEXT FOUR

INTRODUCTION AND INFLUENCE OF QUATERNIONS

After I. Newton, the greatest mathematician of the English-speaking peoples is **W. R. Hamilton,** who was born in Dublin in 1805 and died in 1864. His fame has had curious and regular changes. During his lifetime he was celebrated but not understood; after his death his reputation declined and he came to be counted in the second rank; in the twentieth century he has become the subject of an extraordinary revival of interest and appreciation. Hamilton's scientific career is astonishing. When he was only 21 years old he submitted to the Royal Irish Academy a paper entitled "A Theory of Systems of Rays" which in effect made a new science of mathematical optics. The communication of the paper was soon followed by a great change in Hamilton's circumstances. The chair of Professor of Astronomy at Trinity College was vacated in 1826 and Hamilton was appointed to the chair which conferred to its occupant the title of Royal Astronomer of Ireland. The election of an undergraduate to a professorial chair was an astonishing event.

In 1832 Hamilton announced to the Royal Irish Academy a remarkable discovery in optics which followed up his theory of systems of rays. In 1835 Hamilton received the honour of knighthood, and two years after he was elected president of the Royal Irish Academy. In 1837, six years after Gauss invented his treatment of complex numbers, Hamilton arrived at his own independent discovery of the same ideas, which he applied to rotations and vectors in the plane, as others had done. In the second paper on this subject he generalized from ordered pairs to n-tuples with emphasis on quadruples (or "quaternions"), which extended the algebra of vectors in the plane to vectors in space. Thus, the concept of a complex number, $a+bi$ was extended to the form $a+bi+cj+dk$ (a, b, c, d real) where $i^2=j^2=k^2=-1=ijk$. Thus, in 1843 Hamilton made his greatest scientific discovery — **the calculus of quaternions.**

Hamilton was led to this discovery by long thought of the problem of finding a general rule for computing the fourth proportional to three straight-line segments when the directions of those lines were taken into account. The roots of **vector algebra** go back to the geometric concept of directed line segments in space. The composition of forces by the parallelogram law led to the idea of addition of vectors. Their representation as ordered sets of real numbers could occur only after the extension of number systems beyond the complex numbers. It had taken fifteen years to work before it dawned on Hamilton that it was possible to create a consistent and useful mathematical system that contradicted the time-honoured law that $AB=BA$. This flash of insight occurred one October day when he was out strolling with his wife along the Royal Canal in Dublin, and he carved the basic formulas on a stone in Brougham Bridge. Hamilton suddenly realized the answer: the geometrical operation of three-dimensional spaces required for their description not triplets but quadruplets. To specify the operation needed to convert one vector into another in space, one had to know four numbers: 1) the ratio of the length of one vector to the other; 2) the angle between them; 3) the node; 4) the inclinations of the plane in which they lie. Hamilton named the set of four numbers a **quaternion,** and found that he could multiply quaternions as if they were single numbers. But he discovered that the algebra of quaternions differed from ordinary algebra in a crucial respect: it was noncommutative. The surrender of the commutative law was a tremendous break with tradition. It marked the beginning of a new era.

From this time (1843) until his death, Hamilton's chief interest (for 22 years!) was to develop the new calculus. Hamilton's discovery was quickly followed by other new algebras, such as the **theory of matrices,** which is likewise noncommutative. Thus Hamilton started a glorious school of mathematics, though it was not to come into full flower for another half century. The discovery of Special Relativity brought quaternions to the fore because quaternions could be applied to the representation of rotations in four-dimensional space. Hamilton's work stirred up considerable disputation throughout the Western world on the question whether quaternions should replace vectors as an everyday tool in physics and mathematics and it resulted in the formulation of an international association to study the question. As it turned out quaternions were not as practical as Hamilton had believed, and they were soon eclipsed by later inventions that were easier to apply; but they began to do for algebra what non-Euclidean concepts were doing for geometry. Once it was realized that $BA=AB$ was not an irrevocable axiom, mathematicians began to experiment with new systems in which other axioms were also changed.

As a standard device for every day use in physics, quaternions disappeared entirely. They are, however, very much alive now with a different "raison d'être". Today mathematicians are interested in **studying number systems in their entirety,** in learning their properties and in learning how to construct new ones. One prominent type is called an associative division algebra over a field. It is known that there are only three such algebras over the real field: 1) the real number system, 2) the complex number system, and 3) the system of quaternions. Thus the system of quaternions may be designated as the only noncommutative associative division algebra over the real field.

It is only fair to mention in passing that in 1844 **Hermann Grassmann** simultaneously and independently created an even more general theory of n-tuples than Hamilton. Instead of considering ordered sets of quadruples of real numbers, Grassmann dealt with ordered sets of n real numbers. To establish such set $(x_1, x_2, \ldots, x_n)$ Grassmann associated a hypercomplex number of the form $x_1e+x_2e_2+\ldots x_ne_n$, where $e_1, e_2, \ldots, e_n$ are the fundamental units of his algebra. Two such hypercomplex numbers are added and multiplied like polynomials in $e_1, e_2, \ldots,$ $\ldots, e_n$. The addition of two such numbers yields, then, a number of the same kind. To make the product of two such numbers a number of the same kind requires the construction of a **multiplication table** for the units $e_1, \ldots, e_n$ similar to Hamilton's multiplication table for his quaternions. Here one has considerable freedom and different algebras can be created by making different multiplication tables. Hamilton's quaternions and to some extent Grassmann's calculus of extension were devised by their creators as mathematical tools for the exploration of physical space. These tools proved to be too complicated for quick mastery and easy application, but from them emerged the much more easily learned and applied the subject of **vector analysis.** This work was due to principally the American physicist **J. W. Gibbs** (1838—1903) and is encountered by every student of mathematics and physics.

Read the text. Generalize its main ideas and illustrate them with some problems in Linear Algebra.

TEXT FIVE

LINEAR ALGEBRA

Linear Algebra like several other mathematical disciplines may be considered from two different points of view: 1) as a branch of mathematics of independent interest with a development and with problems of its own; 2) as a tool for other mathematical disciplines and for mathematical physics. A large class of mathematical problems is generally called "linear", the simplest problem is the following: Let a and b be two given (real or complex) numbers — to find a number x that satisfies the equation $ax=b$. The problem has a **unique** solution x, if, and only if, $a \neq 0$; **no** solution at all if $a=0$, $b \neq 0$; an **infinity** of solutions, viz., all real (or complex) x if $a=0$ and $b=0$. This statement comprises the whole theory of the problem.

One of the main technical devices of linear algebra is the **theory of determinants** and for a long time it has been the only part of linear algebra which was studied systematically. This is the more surprising as the notion of **matrix** — the main object of modern linear algebra — is evidently more fundamental than that of a determinant because a **determinant is only a certain number associated with a given square matrix.** Whilst the notion of determinant was discovered by **Leibnitz** (c. 1690) the notion of matrix appeared only much later in 1854 in a paper by **Cayley** and independently in 1867 in a paper by **Laguerre.** Since then linear algebra and matrix calculus have developed into a vast domain of mathematics closely connected with a good many other mathematical branches, such as the theory of groups, the theory of invariants, tensor calculus, the theory of systems of differential equations, etc. Linear algebra provides the methods of proof as well as the adequate algebraic formalism for a considerable part of analytic geometry. It has served as a model in recent developments of Analysis (theory of integral equations and of linear transformation in infinite-dimensional spaces) which have considerably advanced this branch of mathematics and proved important in modern physics.

Discriminant, Determinants and Matrices

As a result of the historical development of ideas leading to the term "**discriminant**", there is today a slight inconsistency in the use of the word. Texts dealing with the equation $Ax^2+Bx+C=0$ call B^2-4AC **the discriminant of the equation.** Other texts discussing the binary quadratic form $Q(x, y)=Ax^2+2Bxy+Cy^2$ call $AC-B^2$ **the discriminant of Q.** Similar though these expressions are, it is not immediately obvious that we are justified in using the same name — that we have the same mathematical entity. By the middle of the eighteenth century it was well known that a necessary and sufficient condition for the equation $Ax^2+Bx+C=0$ to have two identical roots was $B^2-4AC=0$. The expression was known; mathematicians knew what it signified and how to work with it; but it was not yet recognized as a mathematical entity.

During the next hundred years mathematicians studied several expressions related to the quadratic form. In 1748 **L. Euler** used conditions involving expressions like those above to determine whether a quadratic surface is contained in finite space; but Euler did not give a name to these expressions. The expression that was not yet an entity reappeared in 1773. **J. L. Lagrange** was studying the binary quadratic form given above. He proved that if $x+\lambda y$ were substituted for x, leading to a new form $A(x+\lambda y)^2+2B(x+\lambda y)y+Cy^2$, then if the new expression is simplified to $A'x^2+2B'xy+C'y^2$, we must have $A'C'-B'^2=$ $=AC-B^2$. Other mathematicians turned to the study of such invariants, and similar expressions kept reappearing. **C. F. Gauss** called such an expression a "**determinant**" of the function. It remains for the **J. J. Sylvester,** who called himself the "mathematical Adam" because of his habit of giving names to mathematical creatures, to name this one. In 1851 he was studying invariants in reducing certain sixth-degree functions of two variables to simpler forms. What he found was what he called (and what we now recognize as) the "discriminant of a cube". **The discriminant is a combination of constants which vanishes if at least two factors of a function are the same.**

The Japanese mathematician **Seki Kowa** (1683) systematized an old Chinese method of solving simultaneous linear equations whose coefficients were represented by calculating sticks — bamboo rods placed in squares on a table, with the positions of the different squares corresponding to the coefficients. In the process of working out his system, Kowa rearranged the rods in a way similar to that used in our simplification of determinants; thus, it is thought that he had the idea of a **determinant.** Ten years later **Leibnitz** formally originated determinants and gave a written notation for them. In a letter to M. de L'Hospital Leibnitz gave a discussion of a system of three linear equations in two unknowns: "I suppose that*

$$\begin{aligned} a_{10}+a_{11}x+a_{12}y&=0\\ a_{20}+a_{21}x+a_{22}y&=0\\ a_{30}+a_{31}x+a_{32}y&=0 \end{aligned} \tag{1}$$

...Eliminating y first from the first and the second equations and then from the first and third and eliminating the letter x... as a result we shall have

$$\begin{vmatrix} a_{10} & a_{11} & a_{12}\\ a_{20} & a_{21} & a_{22}\\ a_{30} & a_{31} & a_{32} \end{vmatrix}=0 \tag{2}$$

The reader may recall, or easily verify, that (2) is the condition for the three straight lines represented by (1) to pass through a common point. The now-standard "vertical line notation" used in (2) right hand column above was given by **A. Cayley.** Determinants were invented in-

* in modern notation

dependently by **G. Gramer,** whose now well-known rule for solving linear systems was published in 1750, although not in present-day notation. Many other mathematicians also made contributions to determinant theory — among them **A. T. Vandermonde, P. S. Laplace, J. M. Wronski,** and **A. Z. Cauchy.** It is **Cauchy** who applied the word "**determinant**" to the subject; in 1812 he introduced the multiplication theorem. The fundamental importance of determinants as working tools in mathematics has come to be widely recognized.

Suppose there are two homogeneous linear equations in three variables x, y, z $\quad a_1x+b_1y+c_1z=0.$

$$a_2x+b_2y+c_2z=0.$$

Then in general they have a solution

$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{z}{a_1b_2-a_2b_1}$$

These denominators are called **determinants of the second order;** they can be written in various ways, all of which have a great value.

A more familiar notation for the determinant $b_1c_2—b_2c_1$ is the well-known square array $\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$ introduced by Cayley in 1841 long after determinants were first invented. Determinants provide an efficient computational tool for various purposes, notably to determine when vectors are linearly independent and it is consequently useful to have them for such computations. In spite of the great intrinsic interest of the subject, and the wonderful flexibility of determinants as practical working tools in many branches of pure and applied mathematics, there is still a considerable absence of systematic knowledge of even the main results in the theory.

Although the idea of a **matrix** was implicit in the quaternions (4-tuples) of W. Hamilton and also in the "extended magnitudes" (n-tuples) of H. Grassmann, the credit for inventing matrices is usually given to **Cayley** with a date of 1857, even though Hamilton obtained one or two isolated results in 1852. Cayley says that he got the idea of a matrix "either directly from that of a determinant; or as a convenient mode of expression of the equations $x'=ax+by$, $y'=cx+dy$". He represented this transformation and developed an algebra of matrices by observing properties of transformations on linear equations.

$$\begin{matrix} x^1=ax+by \\ y^1=cx+dy \end{matrix} \rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Cayley also showed (1885) that a quaternion could be represented in matrix form as shown above where a, b, c, d are suitable complex numbers. For example, if we let the quaternion units 1, i, j, k be represented by

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$$

the quaternion $4+5i+6j+7k$ can be written as shown below:

$$\begin{bmatrix} 4+5i & 6+7i \\ -6+7i & 4-5i \end{bmatrix}.$$

This led P. G. Tait, a disciple of Hamilton, to conclude erroneously that Cayley had used quaternions as his motivation for matrices. It was shown by Hamilton in his theory of quaternions that one could have a logical system in which the multiplication is not commutative. This result was undoubtedly of great help to Cayley in working out his matrix calculus because matrix multiplication, also is noncommutative. In 1925 A. Heisenberg discovered that the aglebra of matrices is just right for the noncommutative mathematics describing phenomena in Quantum Mechanics.

Cayley's theory of matrices grew out of his interest in linear transformations and algebraic invariants, an interest he shared with J. J. Sylvester. In collaboration with J. J. Sylvester, Cayley (c. 1846) began the work on the theory of algebraic invariants which had been in the air for some time and which, like matrices, received some of its motivation from determinants. They investigated algebraic expressions that remained invariant (unchanged except, possibly, for a constant factor) when the variables were transformed by substitutions representing translations, rotations, dilatations ("stretchings" from the origin), reflections about an axis, and so forth.

There are three fundamental operations in matrix algebra: addition, multiplication and transposition, the last does not occur in ordinary algebra. The law of multiplication of matrices which Cayley invented and his successors have approved, takes its rise in the theory of linear transformations. Linear combinations of matrices with scalar coefficients obey the rules of ordinary algebra. A **transposition** is a permutation which interchanges two numbers and leaves the others fixed, or in other words: the formal operation leading from x to x' and also that leading from x' to x is called transposition. A matrix of m rows and n columns has rank r, when not all its minor determinants of order r vanish, while of order $r+1$ do so. A matrix and its transposed have the same rank. The rank of a square matrix is the greatest number of its rows or columns which are linearly independent.

Today, matrix theory is usually considered as the main subject of linear algebra, and it is a mathematical tool of the social scientist, geneticist, statistician, engineer, and physical scientist.

VOCABULARY EXERCISES

I. *Check up the meaning of the following words and word combinations in the dictionary.*

Current — undercurrent / current of air / the current of events / alternative current / the current year / current prices / current opinions / in current use / to gain (to obtain) currency / words in common currency / paper currency / foreign currency.

Reciprocal correspondence / reciprocal differences / reciprocal equation / reciprocal matrix / reciprocal ratio / reciprocal theorems / reciprocity law.

Homogeneous coordinates / homogeneous equation / homogeneous function / homogeneous intergal equation / homogeneous process / homogeneous space / homogeneous substance.

Ill-fated / ill-advised / ill-affected / ill-favoured / ill-judged / ill-timed / ill-used.

Misfortunate — misadvantage — misunderstanding — misusage — misinformation — miscalculation — misconception.

Falsehood — likelihood — neighbourhood — knighthood — brotherhood — childhood — motherhood — manhood.

Irrational — irreconcilable — irreducible — irrefutable — irrevacoble — irregular — irrelevant — irresistible — irrespective — irresponsible.

II. *Give Russian equivalents of the following phrases.*

To put smb. off / to use smth for personal advantage / efforts were foredoomed to failure / to catch the imagination / to receive no recognition / to receive the honour of knighthood / to bring smth to the fore / to delete postulates / to show a curious awareness.

III. *Give one Russian equivalent of the following groups of words.*

a) Name — title — heading — headline / field — sphere — domain — province / landmark — watershed — boundary line symbol — turning point / contest — competition — tournament — duel / oath — swear word — solumn statement — solemn promise / assertion — declaration — claim / permutation — combination — arrangement in orders / answer — reply — response / connection — relation — tie / impact — collision / effect — results — influence — impression / presumption — presupposition / cancellation — reduction / proliferation — abundancy / disciple — follower / array — table — map / row — line / promulgation — announcement — proclamation.

b) Current — modern — today's — contemporary / ill-fated — unlucky — fatal — misfortunate — unsuccessful / galling — annoying / remarkable — out of ordinary — notable — worthy of notice / irrevocable — final — unalterable / appropriate — fit — suitable / understandable — comprehensible / erroneous — mistaken / infallible — unmistaken / inverse — reciprocal / tempestuous — violent — stormy / simultaneous — happening at the same time / rectangular — square.

c) To forbid — to prohibit — to ban — to outlaw / to delete — to bar / to word — to put (express) in words — to phrase — to couch / to make a secret known — to reveal a secret — to divulge a secret / to be eager — to long — to be anxious / to cause — to produce — to bring out — to evoke / to be false — to be unfaithful — to give away — to betray / to recall — to remember — to call back to mind / to have — to have got — to possess / to remedy — to cure — to put right / to reject — to refuse / to assert — to declare — to claim / to stem from — to originate / to surrender — to give up — to abandon — to sacrifice / to eclipse — to surpass — to outschine — to darken / to pervade — to penetrate — to spread through / to preclude — to prevent / to embrace — to include — to involve / to signify — to mean — to make known / to approve — to be satisfied with / to overcome — to vanquish / in search of — in quest of / to perfect — to improve — to refine — to purify.

IV. *Don't mix them up!*

contest	letter	literal	number	affair	surround
context	latter	literary	numeral	offer	surrender
currant	arrow	raw	oath	effect	wonder
current	array	row	aors	affect	wander

V. *Read and translate the text. Give synonyms, antonyms, definitions of all the bold-faced words.*

Descartes's Rule of Signs

In 1637 Rene Descartes published a book with a **lengthy title** "Discourse on the Method of Rightly Conducting One's Reason and Seeking Truth in the Sciences". Three **appendixes** were **included:** "Optics", "Meteorology", "Geometry". The third part of the third appendix is entitled in translation "On the construction of Solid and Supersolid Problems". It **deals with** many basic ideas for solving equations that arise **in connection with** geometric problems (**primarily** the study of conic sections by algebrais methods).

After **posing some problems** on **mean proportions,** Descartes **proseeds** to construct a fourth-degree polynomial equation by multiplying together the linear factors $(x-2)$, $(x-3)$, $(x-4)$ and $(x+5)$ to obtain $x^4-4x^3-19x^2+106x-120=0$. He **remarks** that the polynomial is divisible by no other binomial factors and that the equation has "only the four roots 2, 3, 4 and 5". The fact that the fourth root is -5 rather that 5 is **recognized** by speaking of 5 as a "false" root, **in contrast to** the positive numbers which are called "true" roots. (The minus sign is not used by Descartes to **designate** negative numbers.) Then comes the statement of the **celebrated** rule of signs:

"We can **determine** also the number of true and false roots that any equation can have, as follows: An equation can have as many true roots as it contains changes of sign, from + to — or from — to +; and as many false roots as the number of times two "+" signs or two "—" signs are found in succession".

Following this general comment, Descartes points out the three changes of sign and the one succession (**permanence**) of sign in his example and **concludes.** "We know there are three true roots and one false root".

As is often the case with the **promulgation** of a **significant** mathematical result, this first statement of the relation between changes in signs of the **successive** terms of the **polynomial** and the nature of the roots was not complete. Neither was any attempt made at proof, other than the illustrative example that **accompanied** it.

The process of **refining** the rule of signs **continued** over a period of two centuries. In this process two points, **specifically,** were **clarified:** (1) the fact that variations in sign determine only **upper bounds** for the number of positive roots because of the possibility of imaginary roots and (2) the fact that the permanence of sign determine bounds for the number of negative roots only for a **complete** polynomial — that is, one with no coefficients equal to zero.

Isaac Newton in his work "Arithmetica Universalis" (published in 1707 but written some thirty years earlier), gave an **accurate** statement of the rule of signs and presented without proof a procedure for determining the number of imaginary roots. At about the same time. G. Leibnitz pointed out a line of proof, although he did not give it in detail. Several proofs were given in the period from 1745 to 1828, some of them quite **insufficient.** K. Gauss added the significant contribution to the statement of the rule that if the number of positive roots **falls short** of the number of variations, it does so by an even integer. The complete statement of Descartes's rule of signs is as follows:

Let $P_n(x)=a_0x^n+a_1x^{n-1}+\ldots+a_n$, **where the coefficients** $a_0, a_1, \ldots, \ldots, a_n$ **are real number,** $a_0\neq 0$. **Then the number of positive real roots of the equation** $P_n(x)=0$ **[a root of multiplicity** m **being counted** m **ti-**

mes] is either equal to the number of variations in signs or less than that number by a positive even integer".

The matter of negative roots of $P_n(x)=0$ **is handled** simply by considering the positive roots of $P_n(-x)=0$. Thus the matter of permanence of sign is avoided. The **crux** of the proof **stems from** the work of Gua de Malves and Segner. It consists in showing that if $P_n(x) = (x-r)P_{n-1}(x)$, where $P_{n-1}(x)$ has real coefficients and r is positive, then $P_n(x)$ has at least one more variation in sign than does $P_{n-1}(x)$ — for the general case, an **odd number more.**

LAB. PRACTICE

Grammar Rules Patterns

I. *Compare Russian and English Tense forms.*

Я решаю задачи.	I **solve** problems. (Pr. Ind.) I **am solving** problems. (Pr. Cont.) I **have been solving** problems for an hour. (Pr. Perf. Cont.)
Я решал (решил) задачи.	I **solved** the problems yesterday. (Past Ind.) I **was solving** the problems when he came. (Past Cont.) I **have solved** the problems. (Pr. Perf.) I **had solved** the problems before he came. (Past Perf.) I **had been solving** the problems for an hour when he came. (Past Perf. Cont.)
Я буду решать (решу) задачи.	I **shall solve** the problems tomorrow. (Fut. Ind.) I **shall be solving** the problems when he comes tomorrow. (Fut. Cont.) I **shall have been solving** the problems for an hour when he comes tomorrow. (Fut. Perf. Cont.) I **shall have solved** the problems when he comes. (Fut. Perf.)

II. *Write all the Tense-Aspect forms of the predicate in the following sentences. Consult the above given patterns, if necessary.*

1. The knowledge of algebra **does not grow** steadily. 2. We **talk** about the way in which vectors are added. 3. Mathematicians **encounter** imaginary numbers as roots of polynomial equations. 4. In elementary algebra symbols **stand for** number. 5. Thanks to the Fundamental theorem of algebra the solution of polynomial equations **requires** no new kinds of numbers.

III. *Use the proper Tense-Aspect form (Indefinite, Continuous, Perfect and Perfect-Continuous) in the sentences.*

1. **The scientists express this number as a terminating decimal.**
(by the end of the last century, in future, since the birth of modern civilisation, usually, nowadays, for a long time, if necessary, in this particular problem)
2. **Axiomatic inquiry brings forth new concepts in algebra.**
(still, from the outset, in the 1873, eventually, this year, already, next decade)

IV. *Compare Russian and English Tense-Aspect forms of the Passive Voice.*

Такие задачи **решаются** в алгебре.	Such problems **are usually solved** in algebra. Such problems **are being solved** in algebra nowadays.
Такие задачи **решались** (решены) в алгебре.	Such problems **were usually solved** in algebra. Such problems **were being solved** in algebra at that time. Such problems **have already been solved** in algebra. Such problems **had been solved** by the end of the past century.
Такие задачи **будут** решаться (решены) в алгебре.	Such problems **will be solved** in algebra. Such problems **will have been solved** in algebra when the necessity comes.

V. *Give all the English Tense-Aspect forms of the Passive Voice.*

1. Contemporary algebra **is considered** as a mixture of much that is very old and still important, e. g., couting and newer concepts such as structures. 2. The beginnings of the development of numbers **are lost** in prehistory. 3. Complex numbers **are expressed** in the form of a number-couple. 4. The sign "+" **is used** in modern algebra only for commutative systems.

VI. *Translate the following sentences paying special attention to the way the predicate should be rendered in Russian.*

1. We **are so accustomed** to expressing relations by means of compact symbols that it **is tempting** to identify the symbols with algebra itself. 2. However, for a long time the subject matter of algebra **was written out** in common language. 3. The major task of historian seeking to understand ancient algebra **has** always **been** to express it, if possible, in modern symbols and thus to disclose its abstract form. 4. Any development toward a more compact way of displaying complex relationships **has been regarded** as a fundamental advance. 5. Such an advance in the field of algebra **was made** by the French mathematicians R. Descartes and F. Viète. 6. The concern for unambiguous terminology and symbolism **had preceded** by two centuries the concern with structures. 7. It was not until the end of the fifteenth century that algebra **was assuming** a somewhat modern form. 8. A general theory of structure (beginning with the solution of polynomial equations and the relations between their roots and coefficients) **has led** to the "completion" of the complex-number system. 9. Due to the extension of the complex numbers by hypercomplex numbers, new algebraic structures **have been created.** 10. The domain of algebra **has been** profoundly **expanded** thanks to the change of the whole nature of algebra. 11. Early in the nineteenth century algebra **has become** a science that could deal effectively with anything. 12. Modern algebra **is being made** to apply to situations, which at first sight in no way related to algebra.

VII. *Translate the sentences with the Prepositional Passive Constructions.*

1. Algebraic symbolism **is** frequently **thought of** as the hallmark of algebra itself. 2. Abel's first important paper on the quintic equation

was sneered at by his contemporary mathematicians. 3. Galois **was looked upon** as a lazy and queer boy by his school teachers. 4. The work of genii of mathematics **is** much **commented on** and **accounted for** by the historians of mathematics. 5. Their great discoveries and contributions **have been** a good deal **written about.** 6. Grassmann's work **was disapproved of** by his contemporaries so its true significance had to wait for the passage of time. 7. The general expressions of the roots of equations of the first four degrees **are arrived at** without difficulty. 8. Discriminants and matrices **are dealt with** in Linear Algebra. 9. This proof **cannot be relied on** — a more rigorous one **is** still **being looked for** by the mathematicians. 10. Such results **must not be wondered at** — they **are approved of** by practice.

VIII. *Turn from Active into Passive.*

Model. Descartes **introduced** (c. 1637) the use of Hindu-Arabic numerals as exponents on a given base.
The use of Hindu-Arabic numerals as exponents on a given base **was introduced** in c. 1637.

1. The Greeks **were expressing** the numbers only in sexagesimal system, not decimally. 2. Diophantus **developed** "syncopated algebra", the use of the abbreviated words (c. 250 A. D.). 3. At least three thousand years ago people **were employing** implicitly the notion of a function. 4. The Italian Luca Pacioli **applied** (1494) the rule of false position. 5. Natural scientists **had often used** the parallelogram as a means of addition. 6. A. Girard (1629) **approached** both negative and imaginary numbers with great boldness. 7. All the distinguished nineteenth-century mathematicians **have treated** the algebraic solution of equations. 8. People generally **associate** adding in arithmetic with the idea of "putting together". 9. Algebraists **refer** to this class of equations as "differential equations". 10. Modern mathematicians **are attaching** several different meanings to the expression $A+B$.

IX. *Express probability, certainty and strong likelihood in the following statements using may or must with the proper form of the Infinitive (Indefinite, Continuous, Perfect, Perfect — Continuous).*

Models. Algebra probably **originated** in Babylonia.
Algebra **may have originated** in Babylonia.
The historians claim that the rhetorical style of algebra **is** an accomplishment of that country.
The historians claim that the rhetorical style of algebra **must be** an accomplishment of that country.

1. Scientists do not have enough evidence to fix the date when the epochal discovery of cardinal number was made: the concept **was equally present** in Egypt, China, India, Mesopotamia. 2. Certainly, algebra **appeared** in Egypt as soon as in Babylonia. 3. In both the Babylonian and the Egyptian civilizations computations **were handled** mostly by the priests. 4. Babylonian and Greek accomplishments in mathematics **were** apparently **known** to Hindu mathematicians. 5. One of the Hindu famous achievements **is** obviously the system of Hindu (often called Arabic) numerals. 6. Arabic algebra probably **came** from both the Greeks and the Hindus. 7. Throughout the Dark ages the Arabs **preserved** the Greek and Hindu works in algebra for posterity — evidently without their translations most of the prior work could have been lost. 8. Of course, the ease and facility in handling Hindu-Arabic numerals **contri-**

buted to the growth of algebra in Europe. 9. It is likely that the invention of printing and the expansion of trade and travel **facilitated** the exchange of ideas. 10. The symbols "+" and "—" (as it seems) **occurred** in the fifteenth century in problems solved by false position to indicate excess and deficiency. 11. In modern algebra the sign "+" **does not** always **signify** addition in any real sense. 12. Surely, the $\sqrt{}$ sign is a modification of *r* for radix or root. 13. At present different branches of mathematics **are** evidently **interacting** to a surprising degree through algebra.

CONVERSATIONAL EXERCISES

I. *Repeat the given statements adding your own comments, examples, characteristics, viewpoints, historic evidence, etc., thus developing the idea further.*

Model. **The early (elementary) phase of algebra spanned the period from about 1700 B. C. to 1700 A. D.**
There is a lot of historical evidence that as long ago as Babylonian times elementary algebra and number theory were being originally developed. The first **general** treatment of polynomial equations was given by F. Viète in 1603. The statement may be thus justified, to my mind.

1. The elementary algebra was characterized by the gradual invention of symbolism and the solving of equations. 2. The development of algebraic notation progressed through three stages: the rhetorical (verval), the syncopated and the symbolic. 3. In the symbolic stage, algebraic notation went through many modifications and changes. 4. Algebraic notation became fairly stable by the time of Newton. 5. Egyptian algebra lacked the sophistication in method shown by Babylonian algebra. 6. Most of the standard Babylonian problems were phrased in geometric terminology by the early Greeks. 7. They solved e. g., quadratic equation by giving a "proof", i. e., a construction of the positive root of the equation, followed by a verification. 8. The early Greeks gave their algebra geometric formulation because of their conceptual difficulties with irrational numbers. 9. Perhaps it is not entirely joking to say that their linear continuum was literally linear. 10. **Diophantus** made a fresh start in algebra by introducting the syncopated style of writing equations. 11. Diophantus gave an ingenious treatment of indeterminate equations often called Diophantine equations. 12. These are usually two or more equations in several variables that have an infinite number of rational solutions.

II. *Read the sentences and characterize orally the algebraic activity during the Dark Ages of Europe.*

1. It was the Arabs who preserved the Greek and Hindu scientific writings through the Dark Ages of Europe. 2. Our main interest during the arabic period centers on **al-Khowarismi** and **Omar Khayyam.** 3. Al-Khowarismi's books — the **Al-jabr** and the **Liber algorismi** — greatly influenced European mathematics. 4. Al-Khowarismi's aim was to write a practical textbook on solving equations, his algebra seems prosaic. 5. Omar Khayyam's greatest contribution to algebra was a geometric solution of cubic equations. 6. **Brahmangupta** (c. 628) and **Bhaskara** (c. 1150) were the most prominent of the Hindu algebraists. 7. Hindu work on indeterminate equations was superior to that of Diophantus. 8. The Hindus solved quadratic equations by completing the square and

they accepted negative and irrational roots. 9. One of their greatest accomplishments was the system of Hindu (often called Arabic) numerals. 10. It was Italy that produced the greatest algebraists during 1200—1620 period. 11. **Fibonacci** (1202) did a great deal to popularize Hindu-Arabic numerals in his book on arithmetic and algebra. 12. **Tartaglia** (1535) solved two types of cubic equations. 13. **Cardano** ("gambling scholar") published (1545) the complete solution of all varieties of the cubic equations. 14. Cardano's Ars Magma contains **S. del Ferro's** solution of the cubic and **L. Ferrari's** solution of the quartic. 15. Later efforts to solve the quintic and other equations were foredoomed to failure up to the XIX c.

III. *Agree with the statements below and try to give some proof or justification. Use the introductory phrases.*

That's right.	**Quite so.**	**There is no point in**
Exactly. Certainly.	**This is the case.**	**denying that...**
I agree to it.	**I hold a similar**	**I see no point at all**
	view...	**to disagree that ...**

1. It was the Greeks' mathematical rigour that forced them to use line segments to express numbers as $\sqrt[3]{2}$ in their geometrical algebra. 2. Diophantus's approach to indeterminate equations is clever but he did not develop a general method of solving them. 3. The Arabs treated algebra geometrically like the Greeks and numerically like the Hindus. 4. Al-Khowarismi's work was not as good as that of Babylonians and the Hindus. 5. Omar Khayyam (c. 1100) was not only a Persian poet but a first-class mathematician of the Arab world. 6. It was principally through the Arabs that algebra entered Europe. 7. Elementary algebra (unlike geometry) emphasized method rather than logical (axiomatic) foundations. 8. G. Cardano published the results of others. 9. There cannot be any general formulas for solving polynomial equations of degree higher than four. 10. Complex numbers obey the same arithmetical rules as the real numbers. 11. Complex numbers are indespensable in many branches of physics and engineering.

IV. *Express your appreciation of both the idea and the word "function". Give the definition of a function. Summarize the discussion.*

The Babylonians of c. 2000 B. C. might be credited with a working definition of "function" because of their use of tables like the one for n^3+n^2, $n=1, 2, \ldots, 30$, suggesting the definition that **a function is a table of correspondence** (between n in the left column and n^3+n^2 in the right column).

More explicit ideas of function could have emerged about the time of **R. Descartes** (1637). The great French mathematician R. Descartes is credited with first introducing the use of Hindu-Arabic numerals as exponents on a given base. To any modern schoolboy the idea of writing $x \cdot x \cdot x$ as x^3 seems so obvious that it is quite natural for one to feel that Descartes probably hit upon this idea without help from his many predecessors in mathematics. But that was not the case! Ingenious inventions very often result from the insights of men who have learned from the trials and errors of others, such was the case with Descartes's use of exponents. Italian mathematicians **R. Bombelli** (1552) and **P. Cataldi** (1610) and others contributed to the development of exponential notation and the search of facile symbolisms for expressing mathematical concepts. Nevertheless R. Descartes may have been the first to use the term "function", he defined a function to mean any positive integral power of x, such as x^2, x^3, ...

G. W. von Leibnitz (1692) thought of **a function as any quantity associated with a curve,** such as the coordinates of a point on a curve, the length of a tangent to the curve, and so on.

Johann Bernoulli (1718) defined a function as **any expression involving one variable and any constants.**

L. Euler (1750) called functions in the sense of Bernoulli's definition "analytic functions" and used also a second definition, according to which **a function does not have an analytic expression but can be represented by a curve,** for example. It was Euler who introduced the now standard notation $f(x)$.

J. L. Lagrange (1800) restricted the meaning of function to a power series representation. **J. J. Fourier** (1822) stated that an arbitrary function can be represented by a trigonometric series.

P. G. L. Direchlet (1829) said that y is a function of x if y possesses one or more definite values for each of certain values that x may take in a given interval, x_0 to x_1.

More recently, the study of point sets by **G. Cantor** and others led to a **definition of function in terms of ordered pairs of elements not necessarily numbers.**

V. *Debate the origin and the significance of the phrase "Symmetric functions".*

A symmetric function of two or more variables is a function that is not affected if any two of the variables are interchanged. Perhaps the most familiar symmetric functions are those met in elementary theory of equations where for cubic equation $x^3+C_1x^2+C_2x+C=0$ we have $r_1+r_2+r_3=-C_1$; $r_1r_2+r_1r_3+r_2r_3=C_2$, $r_1r_2r_3=-C_3$. These last three equalities express the coefficients of the cubic equation as symmetric functions of the roots r_1, r_2, r_3.

When **F. Viète** made his first tentative discoveries concerning symmetric functions in the late sixteenth century the very notion of the roots of an algebraic equation was incomplete, in large measure because of an incomplete understanding of negative and imaginary numbers. Viète himself worked only with positive roots.

A. Girard was interested in extending Viete's result. He considered all roots — those he called "impossible" (i. e., imaginary) as well as negative and positive roots. He studied the sums of their products taken two at a time, then three at a time and so on. But Girard was also interested in obtaining expressions for the sums of given powers of the roots; these sums constituted a different set of symmetric functions than the one Viète had essentially pioneered. He published his results in 1629, which contained the statement that if $x^n-Ax^{n-1}+Bx^{n-2}-Cx^{n-3}+\ldots$ $\ldots=0$, then

$$\left.\begin{array}{l} A \\ A^2-2B \\ A^3-3AB+3C \\ A^4-4A^2B+4AC+2B^2-4D \end{array}\right\} \begin{array}{l}\text{will be the} \\ \text{sum of}\end{array} \left\{\begin{array}{l}\text{solutions} \\ \text{squares} \\ \text{cubes} \\ \text{biquadrates}\end{array}\right.$$

Girard stated this result rather casually. Perhaps because of this and perhaps also because seventeenth-century mathematicians were not ready, Girard's remark went unnoticed until it reappeared, without proof, in **I. Newton's** "Arithmetica Universalis" (1707) and became famous. It also became one of several theorems that are usually referred to as "Newton's theorems" in algebra. For a hundred years after Newton many mathematicans, including C. Maclaurin, L. Euler, J. L. Lagrange, concerned themselved with proofs and generalizations of this theorem.

VI. *Comment upon the achievements of the mathematicians mentioned and summarize them in one-two sentences.*

G. Cardano was the first to exhibit three roots for a particular cubic. He suspected that there should be three roots for every cubic, but he was puzzled by negative roots and imaginary roots. 2. Recognized at least in some sense, negative roots which he called "fictitious". 3. Had the intellectual curiousity to see what happens if one operates with such numbers as $5 \pm \sqrt{-15}$, which we now call "complex" or imaginary. 4. Recognized the "irreducible case" in the solution of a cubic. 5. Removed the x^2 term from a cubic equation. This reduction was more "theoretically general" than he realized. 6. Stated that the sum of the three roots of a cubic is the negative of the coefficient of x^2.

F. Viète in his work: 1. Introduced letters as general positive coefficients and contributed notably to algebraic modern symbolism. 2. He was the first to perceive the theory of symmetric functions of the roots of an equation. 3. Gave transformations for increasing or multiplying the roots of an equation by a constant. 4. Indicated awareness of relations between roots and coefficients of a polynomial equation. 5. Stated a transformation that rids a polynomial of its next-to-highest-degree term.

VII. *Reproduce the text.*

Hilbert's 10th Problem

Hilbert's famous list of 23 major problems has inspired generations of mathematical investigators and led to brand-new mathematical theories. Some of Hilbert's problems are still unsolved. The most recently conquered of Hilbert's problems is the 10th which was solved in 1970 by the 22-year-old Russian mathematician **Juri Matyasevich.** Hilbert's 10th problem is easily described. It has to do with the simplest and most basic mathematical activity: solving equations. The equations to be solved are polynomial equations, that is, equations such as $x^2-3xy=5$, which are formed by adding and multiplying constants and variables and by using whole-number exponents. Moreover, Hilbert specified that the equations must use only integers, that is, positive or negative whole numbers. No irrational or imaginary numbers or even fractions are allowed in either the equations or their solutions. Problems of this type are called Diophantine equations after Diophantus of Alexandria, who wrote a book on the subject in the third century.

Hilbert's **10th** problem is: **Give a mechanical procedure by which any Diophantine equation can be tested to see if solutions exist.** In Hilbert's words: "Given a Diophantine equation with any number of unknown quantities and with rational integral numerical coefficients: to devise a process according to which it can be determined by a finite number of operations whether the equation is solvable in rational integers". Hilbert does not ask for a process to find the solutions, but merely for a process to determine if the equation has solutions. The process should be a clear-cut formal procedure that could be programmed for a computer and that would be guaranteed to work in all cases. Such a process is known as an algorithm. If Hilbert's problem is simply stated **Matyacevich's solution** is even more simply stated: **No such process can ever be devised; such an algorithm does not exist.** Worded in this way, the answer sounds disappointingly negative. Matyasevich's result, however, constitutes an important and useful addition to the understanding of properties of numbers and the development of the theory of equations. The term "Diophantine equation" is slightly misleading, because it is not so much the nature of the equation that is crucial as the nature of the admissible solutions. For example, the equation $x^2+y^2-2=0$ has infintely many solutions if one does not think of it as a Diophantine equation. If we consider the problem as a Diophantine equation, however, there are only four solutions: 1) $x=1$, $y=1$; 2) $x=-1$, $y=1$; 3) $x=1$, $y=-1$; and 4) $x=-1$, $y=-1$. Suppose the equation is changed to $x^2+y^2-3=0$. There are still an infinite number of solutions if it is treated as an ordinary equation, but no solutions at all if it is treated as a Diophantine equation. This difference is central to Hilbert's 10th problem. A famous family of Diophantine equations has the form $x^n+y^n=z^n$, where n may equal 2, 3, 4 or any larger integer. If n is equal to 2, the equation is satisfied by the lengths of the sides of any right triangle and is called the Pythagorean theorem. One such solution is the set of numbers $x=3$, $y=4$, $z=5$. If n is equal to or greater than 3, the equation is what is known as Fermat's Last Theorem; it is probably the oldest and most famous unsolved problem in mathematics. Diophantine equations are easy to write down but hard to solve. They are hard to solve because we are so exclusive about the kind of numbers we accept as solutions.

VIII. *What do we mean when we say.*

Models. 1. When we say that **the quadratic equation** $ax^2+bx+c=0$ **has a general algebraic solution,** we mean that each of the two roots can be expressed in terms of a finite number of additions, subtractions, multiplications, divisions, and root extractions performed on the coefficients a, b, c. Thus the two roots are $(-b+\sqrt{b^2-4ac}/2a)$ and $(-b-\sqrt{b^2-4ac}/2a)$.

2. When we say that **the equation is solved by false position** we mean a method of solving equations by assigning a value to the unknown, if, on checking the given conditions are not satisfied, this value is altered by a simple proportion.

1. An equation has as many roots as its degree. 2. The three roots of the cubic equation (and the four roots of the quartic) yield to the same treatment as the two roots of the quadratic. 3. No general algebraic solution is possible for the polynomial equation of degree greater than four. 4. The Fundamental Theorem of Algebra. 5. The equation is solved by the "**completing the square" method.** 6. The equation is solved by the **parametric method.** 7. The equation is solved by **the method of elimination.** 8. Arabic algebra used **the rules of false position** and of **double false position.** 9. In algebra x always stands for number. 10. $a+b$ is the same as $b+a$. 11. A complex number $a+b\sqrt{-1}$ is essentially a pair of real numbers (a, b). 12. Analytic geometry made geometry a branch of algebra.

IX. *Suppose that the statement seems to you insufficient and you want to add. Repeat the statement and add your own reasoning, thus developing the idea further. Use the following phrases.*

There is one more point to be noted... Moreover... I might as well add that... More than that... Bearing in mind... In this connection one more aspect is interesting to mention...

Models. 1. **Even today there is some lack of uniformity in the use of symbols in mathematics.**
This is the case. For example, Americans write "3,1416" as approximation for π, while many Europeans write "3.1416". In some countries "$\div$" means minus.

2. **The symbol $f(x)$ is a short notation for "a function of x", it does not stand for the product of f and x.**
I might as well add that we use $f(x)$ for the simple reason that the word "function" begins with an f, but we can as well use $g(x)$, $h(x)$, $F(x)$, $G(x)$ or even $\theta(x)$, $\Phi(x)$, etc., if the given problem involves several equations.

3. **It is unfortunate that the term "imaginary" is used in connection with complex numbers.**
More than that. Complex numbers are, actually, as "imaginary" or "unimaginary" as other numbers. They are abstract mathematical objects which like all other mathematical objects are defined by means of postulates characterising their arithmetical rules. The name "imaginary" was given to them originally because it was difficult to attach to them any practical interpretation.

1. The standardization of algebraic notation was being made during the two (XVI—XVII) centuries. 2. Who invented a particular symbol is a question requiring detailed research. Often it cannot be determined with certainty. 3. Algebraists of the seventeenth century made many improvements over F. Viète's algebraic notation. 4. The theory of symmetric functions of the roots of an equation, first perceived by Viète, was established by Newton. 5. Newton gave a method for finding approximations to the roots of numerical equations. 6. A functional relationship between two variables need not be always expressible as an algebraic equation. 7. One way of classifying equations is by the number of unknowns which are involved. 8. Equations of the form $ax+b=0$ are referred to as linear equations in one unknown. 9. Every linear equation in two unknowns has infinitely many solutions. 10. There are several systematic ways in which systems of simultaneous linear equations can be solved. 11. The method of elimination works in the solution of systems of simultaneous linear equations in more than three unknowns.

X. *Reproduce the text.*

Fermat's Last Theorem

The history of Pythagorean triples goes back to 1600 B. C., but it was not until the seventeenth century A. D. that mathematicians seriously attacked, in general terms, the problem of finding positive integer solutions to the equation $x^n+y^n=z^n$. The French mathematician **P. Fermat** (1601—1665) conjectured that there are no positive integer solutions to this equation if n is greater than 2. Fermat's now-famous conjecture was inscribed in the margin of his copy of the Latin translation of Diophantus's "Arithmetica". The note read: "**To divide a cube into two cubes, a fourth power or in general any power whatever into two powers of the same denomination above the second is impossible and**

I have assuredly found an admirable proof of this, but the margin is too narrow to contain it".

Despite Fermat's confident proclamation the conjecture, referred to as "Fermat's last theorem" remains unproven. Fermat gave elsewhere a proof for the case $n=4$. It was not until the next century that **L. Euler** supplied a proof for the case $n=3$, and still another century passed before **A. Legendre** and **L. Dirichlet** arrived at independent proofs of the case $n=5$. Not long after, in 1838, **G. Lamé** established the theorem for $n=7$. In 1843 the German mathematician **E. Kummer** submitted a proof of Fermat's theorem to **Dirichlet.** Dirichlet found an error in the argument and Kummer returned to the problem. After developing the algebraic "theory of ideals", Kummer produced a proof for "most small n". Subsequent progress in the problem utilized Kummer's ideas and many more special cases were proved. It is now known that Fermat's conjecture is true for all $n<4{,}003$ and many special values of n, but **no general proof has been found.**

Fermat's conjecture generated such interest among mathematicians that in 1908 the German mathematician P. Wolfskehl bequeathed *DM* 100,000 to the Academy of Science at Göttingen as a prize for **the first complete proof** of the theorem. This prize induced thousands of amateurs to prepare solutions, with the result that Fermat's theorem is reputed to be the mathematical problem for which the greatest number of incorrect proofs was published. However, these faulty arguments did not tarnish the reputation of the genius who first proposed the proposition — P. Fermat.

XI. *Discuss the following text.*

A Triumph of the Intellect

For 300 years number theorists have been haunted by a few words that the great French mathematician, **Pierre de Fermat,** scribbled on the margin of a book. He claimed to have "discovered a truly marvelous proof" that there were no solutions to certain mathematical equations, but unfortunately the proof was so long that "this margin is too small to contain (it)". This statement soon came to be known as Fermat's Last Theorem, and ever since it has baffled mathematicians.

But now, when mathematicians were suspecting that Fermat was pulling their legs, a young West German researcher has taken a step toward proving this theorem. **Gerd Faltings** of Wuppertal University near Dusseldorf has written a **40-pageproof** of a theorem so sweeping that it even sheds light on Fermat's Last Theorem. Known as Mordell's conjecture, after the 20th-century British mathematician Lewis Mordell, it claims that **certain equations, including those considered by Fermat, have at most a finite number of rational solutions.** Faltings proved the conjecture in a paper so impressive that researchers have interrupted their summer to attend hastily arranged seminars about it. "Absolutely great — it's glorious", says mathematician Barry Mazur of Harvard University.

The problem Faltings solved lies in the realm of number theory, reputedly the "purest" branch of mathematics since it has so few practical applications and is the one that investigates the subtle and often peculiar properties of ordinary numbers. One such property is familiar to any high-school student: there are many numbers that are both perfect squares and the sum of two other perfect squares. (Perfect square is a number that is the product of a whole number multiplied by itself.) For example, 25 (the square of 5) is the sum of 9 (the square of 3) and 16 (the square of 4). But the properties of numbers that are perfect cubes (which are the product of an ordinary number multiplied by itself twice) or "quartics" or "quintics" or high numbers are more complicated. Fermat's Last Theorem states that **there are no perfect cubes (other than zero) that are the sum of two nonzero perfect cubes, nor any perfect quartics that are the sum of two other quartics, etc.** In proving Mordell's conjecture, Faltings finally proved that there are at most a finite number of these "magic" numbers — not necessarily zero, as Fermat claimed, but not an infinite number either.

Juggling: Mathematicians have been seeking a proof of Mordell's conjecture for decades, but Faltings found it using nothing more than pencil, paper and 18 solid months in which he thought of almost nothing else. His proof combines innovative work of his own with a lot of clever juggling of results by other mathematicians. Much of it relies on new theorems in geometry, which few mathematicians imagined would apply to Mordell's conjecture. And now what Faltings has done might be applied to Fermat's Last Theorem itself.

Faltings: A "glorious" proof finally breaks the four-minute mile of mathematics.

XII. *Reproduce the text in English. Remove the parentheses.*

Model. Так как решением двучленного уравнения (вида?) является радикал (какой?), то уравнение высших степеней с одним неизвестным (вида?) решается в радикалах, если его можно свести к цепи двучленных уравнений.

As the solution of a two-termed algebraic equation $x^m = A$ is the radical $\sqrt[m]{A}$ then the higher degree equation in one unknown (variable) of the type $x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots \ldots + a_{n-1}x + a_n = 0$ is solved in radicals if and only if it can be reduced to a series (array, chain) of two-termed polynomial equations.

Все уравнения 2-й, 3-й и 4-й степеней решаются в радикалах. Уравнение 2-й степени (вида?) было решено в глубокой древности по общеизвестной формуле (какой?). Уравнения 3-й и 4-й степеней были решены (когда?). Для уравнения 3-й степени (вида?), к которому можно привести всякое уравнение 3-й степени, решение дается так называемой формулой **Дж. Кардано** (какой?), опубликованной (когда?). Вопрос о том, была ли эта формула найдена самим Кардано или же заимствована у других математиков (каких?), нельзя считать вполне решенным. Метод решения в радикалах уравнений 4-й степени (вида?) был указан **Л. Феррари** (когда?). В течение трех последующих столетий математики (какие?) пытались найти аналогичные формулы для уравнений 5-й (вида?) и высших степеней. Наиболее упорно над этим работал **Ж. Лагранж** (когда?). Последний рассматривал особые линейные комбинации корней (так называемые резольвенты Лагранжа?), а также изучал вопрос о том, каким уравнениям удовлетворяют рациональные функции (какие?) от корней уравнения высших степеней. **К. Гаусс** создал полную теорию решения в радикалах двучленного уравнения (вида?), в которой свел решение такого уравнения к решению цепи двучленных же уравнений низших степеней (каких?) и дал условия, необходимые и достаточные для того, чтобы это уравнение решалось в квадратных радикалах (какие условия?).

С точки зрения геометрии, последня задача заключалась в отыскании правильных n-угольников, которые можно построить при помощи циркуля и линейки; поэтому это уравнение называется уравнением деления круга (почему?). **Н. Абель** показал (когда?), что общее уравнение 5-й степени (вида?) и тем более общие уравнения высших степеней не решаются в радикалах (почему?). С другой стороны, Абель дал решение в радикалах одного общего класса уравнений, содержащего уравнения произвольно высоких степеней, так называемых **абелевых уравнений** (вида?). Когда **Э. Галуа** начал свои исследования (в возрасте?), в теории алгебраических уравнений было сделано уже много, но **общей теории**, охватывающей все возможные уравнения высших степеней еще не было создано. Основные задачи были: 1) установить необходимые и достаточные условия, которым должно удовлетворять такое уравнение для того, чтобы оно решалось в радикалах; 2) выяснить к цепи каких более простых уравнений может быть сведено решение заданного уравнения; 3) каковы условия для того, чтобы корни уравнения можно было построить геометрически с помощью циркуля и линейки. Все эти вопросы Галуа решил в своем «Мемуаре об условиях разрешимости уравнений в радикалах», найденном в его бумагах после смерти (когда?) и впервые опубликованном **Ж. Лиувиллем** (когда?). Свое условие **(критерий Галуа?)** разрешимости заданного уравнения в радикалах Галуа формулировал в терминах теории групп. В современном понимании **теория Галуа** — это теория, изучающая те или иные математические объекты на основе их групп автоморфизмов (что это значит?), так например, возможны теории Галуа полей, колец, топологических пространств и т. п.

XIII. *Debate the given statements. It is advisable that the group be divided into two parties, each party advocating their viewpoint. Use the following introductory phrases.*

I will start by saying (claiming) that... What I mean to say is... You are free to disagree with me but... My point is that... Much depends on who (when, what, how) ... I'd like to make it clear...

1. How fortunate for mathematics that the travelling salesman Fibonacci (Leonardo of Pisa) had intellectual and mathematical interests. 2. Public contests wherein mathematicians challenged one another in problem solving were the main stimulus and impetus for mathematical discoveries in the XV—XVI cc. 3. Niccolo Tartaglia did not circulate his method of solving two types of cubic equations for the only reason, viz., to vanquish challengers. 4. G. Cardano — a physician, astrologer, suspected heretic — was far in advance of his time in mathematics. 5. Descartes's exponent notation and his "rule of signs" are considered as mere trifles in today's mathematics. 6. Newton did not contribute notably to aglebra at all. 7. There is all the justification to call Gauss "Prince of Mathematics". 8. Many of the greatest accomplishments of science and art were conceived first by very young men. 9. In the field of mathematics this precocity is particularly obvious. 10. Abel and Galois are founders of modern mathematics. 11. The lives of Abel and Galois are a monument of misunderstanding, unrecognition and negligence. 12. Abel had barely reached the age of twenty-two and Galois was not yet twenty, when they made two of the most profound discoveries which have ever been made. 13. Galois had accomplished his task and very few men will ever accomplish more. 14. It is safe to predict that Galois's fame can but wax, because of the fundamental nature of his work. 15. He had conquered the purest kind of immortality.

XIV. *Translate the following text into English.*

Математические работы Галуа составляют шестьдесят страниц. Никогда в истории математики труды столь малого объема не доставляли автору такой славы. Галуа испытывал отвращение к громоздким выкладкам, поэтому его формулировки предельно сжаты. Каждая из его работ — это смелый бросок вперед; прозрения Галуа ослепительны. Хотя Галуа много занимался теорией уравнения высших степеней, он был не просто выдающимся алгебраистом. Галуа интересовали общие идеи, определяющие и направляющие логический ход мысли. Его доказательства основываются на глубокой теории, объединяющей все достигнутые к тому времени результаты и определяющей развитие науки надолго вперед — **теории групп.** Одна из задач Галуа — решение алгебраических уравнений произвольной степени. Решить уравнение — это значит найти, чему равны его корни. Уже в случае уравнений третьей степени это не просто. Первое открытие Галуа состояло в том, что он установил некоторые из «свойств» этих корней. Второе открытие связано с методом Галуа — вместо того, чтобы изучать уравнение, Галуа изучал его «группу». Одна из важнейших заслуг Галуа состоит в том, что он дал **критерий разрешимости уравнений в радикалах.** Теория Галуа включает целую систему новых важных алгебраических понятий — перестановка, группа, поле, кольцо, автоморфизм и т. д. Не все из этих понятий имелись у Галуа, а те, что были, только позже получили точную современную формулировку, но это не умаляет значения его первопроходческих работ. Может возникнуть недоумение: «Как можно манипулировать перестановками корней, когда сами корни неизвестны? А если корни будут найдены, то никакие перестановки не понадобятся. В чем здесь достижение?» Оказывается, что группу Gal (f) действительно можно вычислять, не зная корней уравнения $f=0$, а **пользуясь лишь соображениями симметрии.** Непреходящее значение работ Галуа состоит в осознании того, что идея симметрии, связывавшаяся ранее исключительно с геометрией, на самом деле играет фундаментальную роль во всей математике и естествознании («группой Галуа» классической механики является группа Галилея, а механики теории относительности — группа Лоренца). Современная теория групп — основной и наиболее развитый отдел алгебры, изучающий в общем виде глубокую закономерность реального мира — симметрию. Открытия Галуа принадлежат не только алгебре и даже не только математике, но общечеловеческой культуре.

XV. *Recast sentences given below using the following phrase openings.*

I will start by saying that... It is a well-known fact that... It seems reasonable to assume that... It will be seen that...

Model. The laws of algebra are fundamental, independent and irreducible.
It is a well-known fact that the laws of algebra are fundamental, independent and irreducible.

1. Arithmetic originated with the question: "How many?" 2. While answering the question man must have faced up and developed basic arithmetic operations. 3. The ancient Egyptians and Babylonians tacitly assumed and used commutative, associative and distributive laws. 4. The distributive property has a way of turning up (appearing) in unexpected situations. 5. Struggling with more elaborate dealings with numbers Greek mathematicians took the jump from the finite to the infi-

nite. 6. The bold notion of infinity opened up wast possibilities for mathematics, and it also created paradoxes (Zeno's paradoxes). 7. In the nineteenth century mathematicians began to work in specialized fields but Gauss was an exception to this rule. 8. The word "algebra" was gradually being expanded to include new systems. 9. The broadening of the meaning of the word "algebra" has had many important implications. 10. Modern algebra has become multicompartmentalized but each separate branch cannot be treated in isolation. 11. Several different meanings can be attached to the expression $A+B$ in modern algebra. 12. Current higher algebra is occasionally pursued without reference to anything in particular. 13. Modern algebra has discarded several of the basic conventions of the elementary algebra.

XVI. *Summarize orally the topic "Emergence of Algebraic Structures" using the given statements.*

1. In the early nineteenth century algebra was considered simply as symbolized arithmetic. 2. It seemed inconcievable that there could exist an algebra different from the common algebra of arithmetic. 3. Instead of working with numbers, as we do in arithmetic, in algebra we employ letters which represent these numbers. 4. The following **five basic properties** always hold in the algebra of positive integers: the commutative law for addition, the commutative law for multiplication, the associative law for addition, the associative law for multiplication and the distributive law for multiplication over addition. 5. The **consequences** of these five properties constitute an algebra applicable to the positive integers and to many other systems. 6. That is to say there is a **common algebraic structure** (viz., the five basic properties and their consequences) attached to many different systems. 7. The five basic properties may be regarded as postulates for a particular type of algebraic structure. 8. Any theorem formally implied by these postulates will be applicable to any interpretation satisfying the five basic properties. 9. Considered from this view, algebra is separated from arithmetic and becomes a purely formal hypothetico-deductive study. 10. The earliest glimmerings of this modern view of algebra appeared about 1830 in England. 11. The members of the British school of aglebraists — **A. Cayley**, **J. Sylvester**, **G. Peacock**, **D. F. Gregory** and **A. DeMorgan** — contributed to an understanding of the foundations of algebra and advanced the modern concept of the subject. 12. In their work one can trace the emergence of the idea of algebraic structure and the preparation for the postulational programme in the development of algebra. 13. Soon their ideas spread to Europe where they were scrutinized with great thoroughness. 14. The remarkable works of **Abel**, **Galois**, **Hamilton** and **Grassmann** led to the liberation of algebra in much the same way that the discoveries of non-Euclidean geometries liberated geometry. 15. Hamilton and Grassmann were forced by physical considerations to invent an algebra in which the commutative law of multiplication does not hold. 16. By developing algebras satisfying different laws than those obeyed by common algebra, these algebraists opened the way for the study of innumerable algebraic structures. 17. One more noncommutative algebra — the **matrix algebra** was devised in 1857 by the English mathematician **A. Cayley**. 18. There is another law of common algebra, besides the commutative law of multiplication that is broken in matrix algebra and this is the cancellation law of multiplication. 19. In the middle of the nineteenth century algebras in which multiplication is nonassociative were developed. 20. In special **Jordan algebra** although multiplication is nonassociative, it is obviously commutative.

21. In a **Lie algebra** multiplication is neither associative nor commutative. 22. A desire for the general theory of structure (beginning with the solution of polynomial equations) led to the "completion" of the complex number system. 23. The extension of the complex numbers to hypercomplex numbers created new structures.

XVII. *Clarify what we mean by the phrase "algebraic structures", e. g., groups. The given statements may prove helpful.*

1. It took centuries of development and sophistication in mathematics to state explicitly the existence of algebraic structures. 2. Addition of numbers, multiplication of numbers, addition of vectors, composition of rotations, etc., exemplify algebraic structures. 3. Some algebraic structures belong to the second type, called structures of order. The set of real numbers is ordered. 4. The mainstream in the development of algebraic structure followed a parallel and concurrent stream in the development of the complex-number system. 5. One cannot have a complete and final list of types of structures on hand. 6. Several new structures in mathematics have been discovered in the last 20 years and one can expect new discoveries of that kind. 7. The majority of algebraic structures are associative. The theory of nonassociative algebra has recently attracted the attention of mathematicians. 8. The process of axiomatic inquiry in algebra has brought forth new concepts such as "groups", "rings" and "fields". 9. Each of these new concepts has its own set of rules of operation and its own characteristic theory. 10. The term "group" stands for a special kind of mathematical system and it has nothing to do with the colloquial meaning ordinarily attached to the word "group". 11. Anyone who is familiar with the theory of groups knows the working mechanism of such structures. 12. The nature of objects forming a group may vary but they share the same structure of group, defined by the groups themselves. 13. On the basis of the four rules characterizing a group it is possible to derive a wealth of mathematical theory. 14. A polynomial equation is solvable if and only if its group over the coefficient field is solvable. 15. A group G which satisfies the commutative law $ba=ab$, for all $a, b \in G$ is called an Abelian group. 16. By introducing addition as a second operation into the group, a more elaborate algebraic entity has been created whose elements are the linear combinations of group elements. 17. The basic entities in many applications of group theory are vectors in a linear vector space.

XVIII. *Agree or disagree with the given statements and keep the conversation going where possible. Use the following opening phrases.*

That's right.	**Quite the contrary.**
Exactly. Quite so.	**Not quite. It's unlikely.**
I fully agree to it.	**Just the reverse.**
	I don't think this is the case.

1. A vector is graphically regarded as a directed line segment or arrow. 2. Vectors are objects that can be added or subtracted and multiplied amongst themselves. 3. Vectors can be multiplied by real numbers. In each case the result is another vector. 4. Today vectors are studied only from the geometric point of view as directed line segments in three dimensions. 5. From the algebraic point of view vectors are treated as n-dimensional minifolds. 6. Vector addition is commutative and associative. 7. Vector multiplication is noncommutative and nonassociative. 8. Vector multiplication is distributive over vector addition. 9. In the se-

venteenth century mathematicians freed their thinking from three-dimensional Euclidean space. 10. They discussed manifolds of n dimensions and developed algebras for such systems. 11. The complex numbers remained on the purely manipulative level until the nineteenth century. 12. In the early nineteenth century $i(\sqrt{-1})$ was interpreted as a series of arbitrary operations on pairs of numbers. 13. C. Wessel represented the complex numbers geometrically and developed the theory of functions of a complex variable. 14. Hamilton gave an algebraic interpretation of complex numbers. 15. This enabled him to consider the extension of complex numbers to hypercomplex numbers.

XIX. *Dispute the following statements. Say what you think about each of them. The phrases given below may be helpful.*

I will start by saying that...
My own viewpoint is that...
It's worth considering (appreciating)...
I should like to make it clear that...
It is hardly likely that...
One cannot say that...
One must admit that...
While I accept that...

1. The most remarkable property of the quaternions is that the commutative law of multiplication does not hold. 2. A significant stride forward was made by **Hamilton** when he gave up the property of commutativity of multiplication. 3. Hamilton suddenly conceived a **way of** multiplying triplets. 4. Hamilton was bold enough to sacrifice the commutative property of multiplication. 5. The passerbys on Brougham Bridge must have been mystified when they read' $i^2=j^2=k^2=ijk=-1$ carved on the stone. 6. Quaternions were Hamilton's greatest creation. 7. The news of discovery spread quickly, and led to a wave of interest among people of rank and fashion like the later boom in General Relativity. 8. Quaternions were a first rather than a last noncommutative algebra. 9. In quaternion algebra two new symbols, i and j were introduced with the rules $i^2=-1$, $j^2=-1$ and surprisingly $ij=-ij$. 10. Hamilton was convinced that the theory of quaternions held the key to many new ideas in mathematics. 11. There was opposition to Hamilton's ideas, perhaps, because of the complexity of the algebra involved. 12. Other mathematicians tried to develop their own substitutes for it. 13. **J. W. Gibbs** developed an excellent departure from quaternions with his vector analysis. 14. **H. Grassmann's** more general n-tuples and the theory of tensor calculus were destined to play a key role in the theory of Relativity. 15. The theory of Relativity vindicated Grassmann's work. 16. Einstein showed that Grassmann had been 50 years ahead in his thinking.

XX. *Expand the given statements and develop them into a paragraph.*

Model. **A matrix can be regarded as a rectangular array of numbers.**

Exactly, $\begin{vmatrix} 1 & 6 & 7 \\ 2 & 0 & 4 \end{vmatrix}$ is a regular matrix and I'd like to add that the entire array is thought of as an entity in its own right. It is possible to define algebraic operations of addition, subtraction, multiplication and division for matrices. The result is a system of objects whose behaviour resembles that of ordinary numbers. One thing more should be noted: matrices are of great utility in both pure and applied mathematics.

1. The **discriminant** is a combination of constants which vanishes if at least two factors of a function are the same. 2. A **determinant** is a number associated with a given square matrix. 3. The product of two determinans each of order n is itself a determinant of the same order. 4. **Matrix** of orders m and n means a set of mn numbers arranged in rectangular array with m rows and n columns. 5. A matrix whose leading diagonal elements are each entity and all other elements are zero is the unit matrix. 6. Linear combinations of matrices with scaler coefficients obey the rules of ordinary algebra. 7. Matrix algebra is not commutative but associative. 8. The formal operation leading from x to x' and also from x' to x is called **transposition.** 9. A matrix and its transposed have the same rank. 10. Non-singular matrices of the same order form a group for multiplication, wherein the identical element is 1. 11. Not every system of two equations has a solution. 12. The equations are solvable unless all the determinants of the coefficient matrix m are zero.

XXI. *Discuss the statements given below. Summarize the discussion. Use the following phrases.*

There is no point in denying that...
It must be admitted that...
I will start by saying that...
All I mean to say is that...
To begin with, my point is that...
I am all for... but...
It's too much to say that...
That doesn't sound convincing enough. I doubt it.
Summarizing the discussion...

1. Current mathematics does not restrict itself to numbers and shapes. 2. Mathematicians have had to supplement ordinary numbers by new symbols to produce "complex" and "hypercomplex" numbers. 3. The new idea about algebra emerged in the XIX c. One of the stimuli was the concept of the square root of minus one $\sqrt{-1}=i$. 4. Algebra is changing constantly and rapidly. 5. In all departments of modern algebra significant research work is always in progress. 6. Algebras have been developed that discarded and sacrificed several of the basic conventions of the ordinary algebra. 7. Anyone is now free to invent his own algebra. 8. Contemporary algebra is any system of handling symbols according to prescribed rules. 9. A particular system of algebra will be productive and fruitful if it yields results and contributes to the rest of mathematics. 10. Vector algebra and matrix algebra are two subjects that have already made significant contributions to both mathematics and science. 11. Associative and non-associative algebras.

XXII. *Say it in English.*

Современная алгебра — чрезвычайно широкая и разветвленная область математики. Она объединяет большое число самостоятельных научных дисциплин. Их общим предметом являются алгебраические операции, представляющие собой абстракции операций элементарной алгебры. Эти операции определяются в многообразных множествах. Так выделился класс алгебраических структур, наибольшее значение среди которых приобрели поля, кольца и группы. Алгебра взаимодействует с другими областями математики, участвуя в образовании новых, «пограничных» дисциплин (топологическая алгебра, теория групп, алгебра Ли и т. п.). Столь общие воззрения на природу и состав алгебры сложились лишь в XX в. Вплоть до XIX в. основной задачей алгебры являлось решение алгебраических уравнений, понимаемое как нахождение корней уравнения с помощью рациональных операций и операции извлечения корня. В поисках общей формулы математики перепробовали множество методов и к концу XVIII в. были вынуждены прибегнуть к фактическому рассмотрению полей и групп, еще не вводя этих понятий явно. На рубеже XVIII—XIX вв. в алгебре были сделаны открытия необычайной важности. Они сопровождались введением в эту науку ряда новых понятий (в первую очередь понятия групп), легших в основу современной алгебры. Эти открытия привели к преобразованию всей алгебры в течение XIX в. Мы имеем в виду результаты **К. Ф. Гаусса, Н. Г. Абеля** и **Э. Галуа**, относящиеся к доказательству основной теоремы алгебры, доказательству неразрешимости в радикалах уравнений степени $\geqslant 5$ и созданию теории Галуа. В 1799 г. К. Ф. Гаусс опубликовал доказательство основной теоремы алгебры' **уравнение $P(x)=0$ может иметь столько корней, сколько единиц содержит его степень**. В связи с последующим введением комплексных чисел $a+bi$ это понимание переросло в уверенность, что корней уравнения $P(x)=0$ (где n — степень уравнения) будет именно n действительных и комплексных. Спустя много лет Гаусс вернулся к этой теореме и дал (в 1815, 1816 и 1849 гг.) три новых доказательства. Другое из замечательных алгебраических открытий начала XIX в. — доказательство неразрешимости в радикалах уравнений пятой степени. Первый реальный успех в поисках подходящей формы иррациональности для решения того или иного класса алгебраических уравнений выпал на долю скромного молодого норвежского математика **Н. Г. Абеля** (1802—1829). За время своей короткой жизни он успел сделать так много открытий в математике, что по праву может считаться одним из наиболее выдающихся математиков XIX в. Начав с доказательства невозможности решения в радикалах уравнения пятой степени, Абель произвел вслед за тем основополагающие исследования в области теории аналитических функций. В своих работах Абель доказал ряд теорем, относящихся к теории Галуа, и исследовал структуру нескольких конкретных классов разрешимых групп. Фактически Абель исследовал структуру коммутативных групп. Он показал, что эти группы являются произведениями циклических групп. Однако понятие группы у него еще не было выделено. Абель не смог дать общий критерий разрешимости уравнений с числовыми коэффициентами в радикалах. Решение этого вопроса принадлежит **Э. Галуа** (1811—1832). Галуа доказал, что задача разрешимости уравнения в радикалах может ставиться только по отношению к определенной области рациональности. Галуа связал с каждым уравнением группу подстановок его корней. Он ввел (1830) термин «группа» — адекватное современному, хотя и не столь формализованное определение. Структура группы Галуа оказалась связанной с задачей разрешимости уравнений в радикалах. **Чтобы разрешимость имела место, необходимо и достаточно, чтобы соответствующая группа Галуа была разрешима**. Аппарат, введенный Галуа, в значительной степени опирается на понятие группы. Однако в первой половине XIX в. факты теории групп играли еще вспомогательную роль, главным образом в теории алгебраических уравнений. Складывающаяся теория групп была еще преимущественно теорией конечных групп — групп подстановок. К середине века выяснилось, что понятие группы имеет более широкое применение. К концу XIX в. теория конечных групп сформировалась настолько, что для нее приобрела актуальность проблема классификации. Однако в общем виде эта проблема не разрешена до сих пор. Главные достижения здесь принадлежат **К. Жордану, Ф. Клейну** и **С. Ли**, которые предприняли систематическое изучение теории групп и ее возможных обобщений и приложений. История алгебры XIX в. будет неполной, если не упомянуть формирование **линейной алгебры**, выраставшей из теории систем линейных уравнений и связанной с ней **теорией определителей** и **матриц.** Следует также указать на важность **гиперкомплексных числовых систем Гамильтона** и **Грассмана**, созданных в 1830—1840 гг., и **теории векторных пространств**, играющих теперь столь важную роль в исследовании математических теорий.

COMPOSITION

I. *Choose one of the given words and write a "marked" paragraph, illustrating the concept.*

Model. **Equations** **Equations** are expressions of equality between two quantities connected by the sign = To be more exact, an equality which is not true for all the values of the letters in it is referred to as an equation, e. g., $x-7=0$ is true only if $x=7$. Much of maths is concerned with equations. The first and primary problem is finding solutions. To solve an equation means to find the values of the unknowns that satisfy the equation, i. e., to reduce it to an identity. Next, there is a problem of proving the existence of algebraic solutions. Finally, one must draw definite conclusions about the nature of the solutions from the **equations** themselves.

Groups, fields, rings, functions, quaternions.

II. *Write a composition: "Evariste Galois — one of the greatest mathematicians". Describe Galois's discoveries in detail.*

On the day preceding the duel **E. Galois** wrote three letters. The last letter addressed to his friend was a sort of scientific testament. Its seven pages, hastily written, dated at both ends, contain a summary of the discoveries which he had been unable to develop. This statement is so concise and so full that its significance could be understood only gradually as the theories outlined by him were unfolded by others. It proves the depth of his insight, for it anticipates discoveries of a much later date. At the end of the letter, after requesting his friend to publish it and to ask Jacobi or Gauss to pronounce upon it, he added: **"After that, I hope some people will find it profitable to unravel this mess".** The sentence is rather scornful but not untrue and the greatest mathematicians of the century have found it very profitable indeed to clear up Galois's ideas. This note was attached to what Galois thought were some new theorems in the theory of equations; these turned out to contain the essence of the **theory of groups**, so important today.

COMPREHENSION EXERCISES

Questions

1. When did algebra originate? 2. What were the first algebraic symbols? 3. What types of equations do you know? 4. How can rhetorical algebra be characterized? 5. Are Diophatine equation a typical example of syncopated algebra? 6. What caused the Greeks to give their algebra geometrical formulation? 7. Are the laws of algebra (associative, distributive, commutative) independent, or can one be derived logically from

another? 8. Are they really fundamental or could they be reduced to a more primitive, simpler and more elegant set of laws? 9. How did unsolved problems influence the development of mathematics? 10. What was the most remarkable unsolved problem in algebra? 11. Who managed to solve the problem? 12. What did Galois prove? 13. Are groups the only algebraic structures in modern algebra? 14. Does there exist General Theory of Equations? 15. Does algebra evolve as a unique and integral subject? 16. What does modern algebra deal with? 17. Is modern algebra the most sophisticated subject in mathematics?

Problems

I. *Use the method of elimination to solve each of following systems of simultaneous linear equations in two (three) unknowns. Explain in English each operation performed.*

$$\begin{cases} x+y=3 \\ 2x-y=3 \end{cases} \qquad \begin{cases} 3x-5z=-7 \\ 3x+5y=3 \\ 3y-3z=-2 \end{cases}$$

Permutations and Combinations

II. a) 1. In how many ways can 5 students be seated in a row of 5 seats? 2. How many different numbers of 3 different digits each can be made from the digits 1, 3, 5, 7, 9? 3. How many different symbols each consisting of 4 letters in succession can be formed from the letters *a*, *b*, *c*, *d*, *e* (repetitions are permitted)?

b) In how many ways can we select: 1. A committee of 3 from a group of 10 people? 2. A set of 3 books from a set of 7 different books? 3. A set of 3 mathematics books and 5 physics books, all different?

III. *Which of the following systems of numbers constitute groups with reference to the operations indicated in parentheses?*

a) The integers (ordinary addition).
b) The integers (ordinary multiplication).
c) The fractions (ordinary multiplication).
d) The pure imaginary numbers (ordinary addition).

Discussion

1. *Discuss the mainstreams in the flow of algebra*:
1650 **Egypt** (Rhind papyrus), **Babylonia** (clay tablets).
500—300 B. C. **Greece** (Pythagoras, Euclid, Archimedes, Appollonius) stop
250—320 A. D. **Greece** (Diophantus, Pappus)
825—1100 **Arabian Empire** (al-Khowarismi, Omar Khayyam)
628—1100 **India** (Brahmagupta, Bhaskara)
1202 **Europe** (Fibonacci's Liber abaci; 1450 Printing; 1494 Picioli's Suma; 1545 Ferrari, Tartaglia, Cardano; 1572 Bombelli; 1600 Viete; 1700 Newton).

2. In both the Babylonian and the Egyptian civilizations computations were handled by a small and exclusive group of experts, frequently the priests. Their special and carefully guarded skills and knowledge

gave them influence and power. The Pythagoreans may simply have followed their example. Your viewpoint.

3. Stories about the innovators in algebra — such as Cardano's exploration of imaginary numbers, Abel's search for the general solution, Galois's genius, Cayley's invention of matrices by noticing coefficient patterns in equations — may well serve to excite the curiosity and the spirit of adventure of the modern mind. Your viewpoint.

4. While the inventors of important applications, whose practical value is obvious, receive quick recognition and very often substantial rewards, the discoverers of fundamental principles are not generally awarded much recompense. They often die misunderstood and unrewarded. But while the fame of the former is bound to wane as new processes supersede their own, the fame of the latter can but increase. Prove it.

5. "The four rules of arithmetic may be regarded as the complete equipment of the mathematician" (J. C. Maxwell). Agree or disagree.

6. Mathematics is forever and again a study of different functions. Prove it.

7. The compexity of a civilization is mirrored in the complexity of its numbers. Illustrate the statement.

8. The number systems employed in mathematics can be divided into: a) the system of positive integers only; b) positive, negative numbers and zero; c) the rational numbers; d) the real numbers, which include the irrational numbers; e) the complex numbers. Why did mathematicians have to extend the number concept? The complex number system need not be enlarged further. Why?

9. Complex numbers feature the "quantity" $\sqrt{-1}$ (i), which, when multiplied by itself produces —1. It took centuries of development and sophistication in mathematics to accept this oddity. — i is a number that cannot be called either positive or negative. Discuss its significance and applications in mathematics.

10. The irrational numbers have a long history. The world of mathematics is far richer in irrational numbers than it is in rational ones. Prove it.

11. "The imaginary number is the wonderful creature of an ideal world almost an amphibian between things that are and things that are not" (Leibnitz). Characterize Leibnitz as a philosopher and a mathematician.

12. For many years Hamilton brooded over the fact that the multiplication of complex numbers has a simple interpretation as the rotation of a plane. The way Hamilton generalized this idea.

13. The major stumbling block in the extension of complex number system and its effect on the theory of equations.

14. Hypercomplex numbers: (4-tuples) Quaternions and n-tuples of Grassmann. Their roles and applications in modern science.

15. Algebraic structures: Groups, Rings, Fields.

16. Vector analysis and the theory of matrices.

17. The departments of modern algebra: Linear Algebra, Lie Groups, Homological Algebra.

LESSON EIGHT

INTRODUCTION TO CYBERNETICS

Grammar:
1. The Sequence of Tenses Rule. Reported Speech.
2. Direct and Indirect Questions.

LAB. PRACTICE

Repeat the sentences after the instructors.

1. Of all known forms of life on earth man is the only one to have developed a systematic procedure for storing up useful information and passing it from one generation to the next. 2. Calculation began with two problems, one concerned with the needs of practical living, the other with the requirements of science. 3. As problems became more complex, more powerful methods of computation were being discovered. 4. Man's techical progress is reflected in the tools he has invented. 5. Throughout the centuries man has refined the ability to record, process, and communicate information. 6. Ultimately came the period of mechanical invention, which began to flower in the early years of the twentieth century. 7. With the advent of automatic digital computers man has created devices that can solve complete problems without the need for human intervention during the course of solution. 8. With the invention of servo mechanisms, vacuum tubes, memory units, transistors, and the like, man suddenly discovered he had constructed an instrument that far transcended many of his own powers. 9. Many jobs, hitherto performed by human energy, were turned over to the new machines. 10. Computer can perform prodigies of deductive reasoning and solve complicated problems formulated in terms of Boolean algebra. 11. Computer can solve the most difficult equations, remember complicated chains of operations, form images and draw pictures of them. 12. When the information in an image is expressed in digital form it can be manipulated mathematically rather than optically. 13. The digital computer is now an essential tool in many areas of image processing. 14. Computer can be adapted with some success to the problem of translating languages. 15. The prospect of automation has become a reality in many lines of human endeavour. 16. Computer fulfils four types of functions: 1) input-output; 2) storage; 3) arithmetic, and 4) control. 17. The way in which these functions are executed, differs among the various computers. 18. A programme (routine) is a complete set of instructions for doing a particular task. 19. The process of preparing such a programme is known as programming. 20. Cybernetics is concerned with the design and construction of electrical or electronic analogs capable of performing processes carried out within a living entity including the selection and evaluation, as well as the storage of information.

Key Grammar Patterns

The Sequence of Tenses Rule

Though there is a tendency to use the Tense-Aspect forms of the verbs absolutely in today's English, the well-established tradition of the relative use of Tenses is still holding ground. The rule of the sequence of tenses (the relative use of Tenses) is mainly observed in **Object clauses.** The choice of the Tense-Aspect form in the subordinate clauses is usually free after a Present or Future Tense form in the principal clause; this is not the case with **the Past Tense-form**, which requires a strict observance of the Sequence of Tenses Rule.

The Principal Clause	**The Subordinate Object Clause**
I. I say shall (will) say that	he solves (solved, will solve, is solving, was solving, will be solving, has solved, has been solving, etc.) his problems.
II. I **said** (that) Present → Past Past Indef. → Past Perfect Future → Future-in-the Past *now → then*; *today → that day*; *this → that*; *these → those*; *here → there*; *ago → before*; *tomorrow → the next day*; *yesterday → the day before*	he **solved** (**was solving**) his problems. (simultaneous actions) he **had solved** (**had been solving**) his problems. (preceding or prior actions) he **would solve** (**would be solving, would have solved, would have been solving**) his problems. (following actions)

I. *Translate the following sentences into Russian and turn Direct Speech into Indirect (Reported) Speech.*

Model. Plato said, "Whatever we Greeks receive, we improve and perfect".
Plato **said** (that) whatever **they** Greeks **received** they **improved** and **perfected.**

1. Plato advised, "The principal men of our State must go and learn arithmetic, not as amateurs, but they must carry on the study until they see the nature of numbers with the mind only". 2. The often-repeated motto on the entrance to Plato's Academy said, "Let none ignorant of Geometry enter here". 3. Aristotle said in his "Metaphysics", "In their theory the Pythagoreans supposed that the whole nature is modelled on numbers, and numbers are the first things in the whole of nature; the elements of numbers are the elements of all things". 4. Kepler affirmed, "The reality of the world consists of its mathematical relations. Mathematical laws are the true cause of phenomena". 5. Descartes, father of modernism, said, "All nature is a vast geometrical system. Thereafter I neither admit nor hope for any principles in Physics other than those which are in Geometry or in abstract Mathematics, because, thus all the phenomena of nature are explained and some demonstration of them can be given". 6. In Descartes's words, "Give me extension and motion and I will construct the universe".

The Sequence of Tenses Rule may not be observed when the speaker believes that he is dealing with facts, statements or opinions which are true of all times, are a kind of general truth.

II. *Refer all the statements of the text to the Past. Begin with; I said that ...*

Algorithms

Twenty or more years ago the word "algorithm" was unknown to most educated people; indeed, it was scarcely necessary. The rapid rise of computer science, which has the study of algorithms as its focal point has changed all that; the word is now essential. There are some other words that almost, but not quite, capture the concept that is needed: procedure, recipe, process, routine, method, rigmarole. Like these things **an algorithm is a set of rules or directions (instructions) for getting a specific output from a specific input.** The distinguishing feature of an algorithm is that all vagueness must be eliminated; the rules must describe operations that are so simple and well-defined that they can be executed by a machine. Furthermore, an algorithm must always terminate after a finite number of steps.

A computer programme is the statement of an algorithm in some well-defined language, although the algorithm itself is a mental concept that exists independently of any representation. Anyone who has prepared a computer programme will appreciate the fact that an aglorithm must be very precisely defined, with attention to detail that is unusual in comparison with other things people do. Programmes for numerical problems were written as early as 1800 B. C. when Babylonian mathematicians gave rules for solving many types of equations. The rules were as step-by-step procedures applied systematically to particular numerical examples. The word "algorithm" itself originated in the Middle East, although at a much later time. Curiously enough it comes from the Latin version of the last name of the Persian scholar Abu Jáfar Mohammed ibn Musa **al-Khowaresmi** (Algorithmi) whose textbook on arithmetic (c. 825 A. D.) employed for the first time Hindu positional decimal notation and gave birth to algebra as an independent branch of mathematics. It was translated into Latin in the 12th century and had a great influence for many centuries on the development of computing procedures. The name of the textbook's author became associated with computations in general and used as a term "algorithm".

Originally algorithms were concerned solely with numerical calculations; Euclid's algorithm for finding the greatest common divisor of two numbers — is the best illustration. There are many properties of Euclid's powerful algorithm which has become a basic tool in modern algebra and number theory. Nowadays the concept of an algorithm is **one of the most fundamental notions** not only in mathematics but in science and engineering. Experience with computers has shown that the data manipulated by programmes can represent virtually anything. In all branches of mathematics the task to prove the solvability or unsolvability of any problem requires a precise algorithm. In computer science the emphasis has now shifted to the study of various structures by which information can be represented and to the branching or decision-making aspects of algorithms, which allow them to fall on one or another sequence of operations depending on the state of affairs at the time. It is precisely these features of algorithms that sometimes make algorithmic models more suitable than traditional mathematical models for the representation and organization of knowledge.

Although numerical algorithms certainly have many interesting features, there are non-numerical ones and in fact algorithms in Cybernetics deal primarily with manipulation of symbols that need not represent numbers. Algorithm-designing is both pure and applied branches of Cybernetics. Current algorithms are becoming more and more refined and sophisticated. Algorithms for searching information stored in a computer's memory, such as Sequential-Search, Binary-Search, Tree-Search, etc., may illustrate several important points about algorithms in general: an algorithm must be stated precisely and it is not an easy task to do that as one may think. When one tries to solve a problem by computer, the first algorithm that comes to mind can usually be greatly improved. Data structures such as Optimum-Binary-Search-tree are important tools for the construction of efficient algorithms. When one starts to investigate how fast an algorithm is or when one attempts to find the best possible algorithm for a specific application interesting issues arise and one often finds that such questions have subtle answers. Even the "best possible" algorithm can sometimes be improved if we change the ground rules. Since computers "think" differently from people, methods that work well for the human mind are not necessarily the most efficient when they are transferred to a machine.

Questions

Direct	**Indirect**
General	
He asked, "Is the computation of square (cube, etc.) roots of special concern of this type of programmes?"	He asked **if (whether)** the computation of square (cube, etc.) roots **was** a special concern of **that** type of programmes.
Special	
I asked, "When did the early mathematicians deal with the problem of extracting cube roots?"	I asked **when** the early mathematicians **had dealt** with the problem of extracting cube roots.

III. *Turn Direct questions of the paragraph into Indirect ones. Begin your Indirect questions with: I asked (he asked, you asked, etc.).*

What Is Programming?

The following items are included in the programming: a) **Consideration of the problem.** Is the problem completely defined? Can we find a method of solution? Will the method fit the computer we use? Will we have enough time, both to prepare the solution on the computer and to run out the answers? b) **Analysis of the problem.** Does the algorithm that we can use exist? Are there "canned" routines that we can apply? That is, are there parts of this problem for which we may already have the computer solution? How much accuracy do we want? How well we assure ourselves that the solutions are correct? Can we construct test data to check the computer solution? Thus, **propramming** covers all activities from the start of the job up to the end and including flowcharting.

IV. *Turn Indirect questions of the text into Direct ones.*

Boolean Algebra

One should know when and where the idea of laying down postulates for the manipulation of abstract symbols (not necessarily numbers) first occurred and who was the creator of such an abstract algebra (=who the creator of such an abstract algebra was). It occurred first in England and at about the time of **George Boole** (1815—1864) the English mathematician and logician. One can doubt whether Boolean algebra, i.e., the algebra of sets, studied largely by means of truth tables, has anything to do with computers; whether basic laws of ordinary algebra (commutative, associative and distributive) hold in Boolean algebra, etc. The most recent development in connection with Boolean algebra is its application to the design of electronic computers through the interpretation of Boolean combinations of sets as **switching circuits**. The answer to the question of what part of Boolean algebra is used so widely in Cybernetics may surprise the uninitiated — the limited special type of Boolean algebra having only two elements in it which at first sight may seem impractical at all is the very one used so widely nowadays. The basic electronic device in the early computers was the vacuum tube which was turned off or on by the electric current entering the tube. Boolean logical product of two sets corresponds to a circuit with two switches **in series.** It is easy to realize when electricity flows in such a circuit — only if both the first and the second switches are closed. The logical sum of two sets corresponds to a circuit with two switches **in parallel.** The question whether electricity can flow in such a circuit has the following answer: electricity flows in such a circuit if either one or the other or both switches are closed. Telephone circuits and electronic computers are basically designed upon a system of Boolean symbolic logic.

The fundamental components of any digital computer are, thus, **switches** capable of two different states of transmission. The speed of the computer in its calculations is limited by the time required for a switch to change states, among other factors. In general it is desirable that a switch should consume as little power as possible. No switching computer circuit acts instantaneously. One may wonder what it means in practice. It means that there is a brief **delay** between the instant incoming signals appear on the input leads and the instant an outgoing signal appears on the output lead. In the days of vacuum-tube circuitry the delay was about 10 microseconds (millionths of a second). Some of today's circuits have a delay of the nanosecond (a billionth of a second). Large or small switching delay is a factor that affects the speed of a computation. Called delay time, it is represented by Δt. One may inquire how fast modern computers can add. Today they can add at a rate of 10 million calculations per second; it is not a limit of course. In a current electronic computer virtually all the switches are transistors, and even the fastest transistors now in use cannot be made to change states in less than about a nanosecond, or a billionth of a second. An optical device analogous to the transistor that has recently been developed can switch from one transmission state to the other in about a picosecond or a thousandth of a billionth of a second.

The three basic functions of a computer — arithmetic operations, logical operations and the storage of information or memory — are all done by devices that have two stable states. In arithmetic operations the two states represent the numerals 0 and 1 of the binary number system. In the evaluations of logical propositions the two states stand for **true** or **false.** The memory of the computer stores the results of arithmetic and logical operations in devices that occupy one of the two states. With the binary algebraic system a computer can evaluate the truth of propositions by making use of just three logical functions, which are usually referred to as the **ANDfunction**, the **ORfunction** and the **NOTfunction.** In the ANDfunction a statement is taken as true if **all** its components are true. In the ORfunction a statement is taken as true if **any** of its components is true. In the NOTfunction the thruth value of a statement is reversed. More elaborate logical operations can be built out of the three basic functions, and so can arithmetic operations such as addition. Thus a computer requires a device that can represent the values 0 and 1, or true or false in physical form that can be assembeld into largescale devices that perform the three logical functions. By combining transistors and other circuit elements, structures that carry out the AND, OR, or NOT functions can be assembled.

Reasonable operations are logical and mathematical operations. Mathematical operations include addition, subtraction, multiplication, division, taking square root, etc., and also more advanced mathematical operations such as raising to a power, finding derivatives and integrating. Logical operations include comparing, selecting, sorting, matching, determining the next instruction which is to be performed, etc. We may question **what information is**. In the discussion of computers, the word information has a rather special definition: **Information (data) is a set of marks that have meaning.** In a large automatic electronic computer information may be recorded and manipulated as sequences of minute electrical pulses which are about a millionth of a second apart; and the presence or absence of a pulse in a position where either may occur is the basic code which represents information.

THE INTRODUCTORY TEXT

CYBERNETICS

The word "cybernetics" originated from the Greek "kybernetike", the Latin "gubernator" and the English "governor" all meaning, in one sense or another, "control", "management" and "supervision". More recently **Norbert Wiener** has used the word to name his book, which deals with the activity of a group of scientists engaged in the solution of a wartime problem and some of the mathematical concepts involved. Nowadays the word has become associated with the solution of problems dealing with activities for computers. As such, the discipline must rely on the exact sciences as well as sciences such as biology, psychology, biochemistry and biophysics, neurophysiology and anatomy.

Before studying computer systems it is necessary to distinguish between computers and calculators. These terms have, by connotation, two distinctly different meanings. The term **calculator** will refer to a machine which (1) can perform arithmetic operations (2) which is mechanical (3) which has a key-board input (4) which has manually operated controls. (Examples: Adding machines, desk calculators). The term **computer** will refer to automatic digital computers which can (1) solve complete problems, (2) are generally electronic, (3) have various rapid input-output devices, (4) have internally stored control programmes (routines). Speed and general usefulness make a computer equivalent to thousands of calculators and their operators. The ability of electronic computers to solve mathematical and logical problems, thereby augmenting the efficiency and productivity of the human brain, has made the sphere of their application practically boundless.

It is difficult to say what the future holds in store for Cybernetics. Every day we learn more and more about the penetration of Cybernetics into the most widely differing spheres of human activity. The lauching of sputniks and the delivery of our space rockets to their orbits with such high accuracy could have been hardly possible without computers. This, however, does not mean, that a machine can ever become "cleverer" than its creator. The point is that the machine does not replace man, it only increases his work outupt and multiplies his power over the forces of nature. It should be always remembered that the machine serves man, and not the other way round. Without man, even the most perfect machine would be only a useless heap of metal.

Man's technical progress is reflected in the tools he has invented. From early times he has been ceaselessly creating and improving devices to assist his brain in completing tasks difficult or otherwise impossible. Throughout the centuries man has developed and refined the ability to record, process, and communicate information. With the advent of automatic digital computers, man has created devices that can solve complete problems without the need for human intervention during the course of solution. Although operations performed by computers are the very basic ones (addition, subtraction, multiplication and division) great speed of operation is more than compensation. The principal use of computers has been in the area of applied mathematics. The application of computers to scientific problems has become later than the original business applications. Nowadays computers have become increasingly important as basic tools for analysis. This operation requires highly refined and flexible techniques.

The contributions of the scientists to the progress of Cybernetics consists of the evaluation, measurement and description of the capabilities and of structural and functional attributes of living organisms. Such studies involve the methods of communication, feedback and control in the living entity. Hence an important aspect of the work in Cybernetics for mathematicians deals with the mathematical theory of communication.

In terms of computer development Cybernetics is concerned with the design and construction of electrical or electronic analogs capable of performing processes carried out within a living entity, including the selection and evaluation, as well as the storage of information. In terms of understanding the operation of the human nervous system, Cybernetics contributes new insight into a wide range of processes such as learning, regulation and the emotional behaviour of individual human beings as well as societies. Specifically the problems of decision-making, thinking and synthesis, imagination and creative endeavour of people, come under the scrutiny of Cybernetics.

It is anticipated that the future developments of automated industries and societal functions will be based on the theorems developed from Cybernetics which thus far has made significant contribution to the technology of guided missiles, business and scientific computer applications, communications and automatic control. Cybernetics is a young science and yet it is increasingly applied in various branches of industry and research. Invading a wide range of fields in human activity, Cybernetics endeavours to find the answer to two major questions: the best way of controlling this or that process, and the best way of utilizing a machine (if possible) for controlling this process.

ACTIVE VOCABULARY

1. to access
2. to assist
3. to augment
4. to commit
5. to complete
6. to console
7. to consume
8. to delay
9. to entail
10. to execute
11. to iterate
12. to pro'cess
13. to recur
14. to reside
15. to resort (to)
16. to retain
17. to simulate
18. to steer
19. to store
20. to supervise
21. to sustain
22. to switch
23. to transcend
24. to transpose

Read and translate the text. Generalize its main ideas following the plan: finger counting — the abacus — logarithms computers — modern techniques of computation. How can one assess the creation and development of computers?

TEXT ONE

DEVELOPMENT OF COMPUTING MACHINES

The art of computation originated in the basic needs of human life and thus is found in the earliest records of the race. It was practised by the ancient Egyptians and the Babylonians. It appears in the oldest traditions of the Orient. The origin of computation or calculation, is indicated by the origin of the very word "calculate", which is derived from the Latin **calculus** meaning a small stone or pebble. The ancient Greeks wrote, and calculated with pebbles, by moving the hand from left to right; the Egyptians did the opposite.

The earliest device known for carrying out the ordinary operations of arithmetic is the **abacus.** The abacus, in time, became the principal computing machine of Western nations. Its use also spread into the Orient. In China the abacus is called the **suanphan** (arithmetic board); and in Japan, the **soroban.** The modern abacus differs little in its essential features from its Roman prototype. One weakness of calculation by the abacus is that each step erases the preceding step, so that there is no way to check the answer except by recomputing. Multiplication and division are performed by the repeated addition and subtraction.

The second calculating machine was undoubtedly a device for multiplication made by the discoverer of logarithms. There is no exaggeration in the often-repeated statement that the invention of logarithms, by shortening the labours, doubled the life of the astronomer. The introduction of logarithms which completely revolutionized the art of computation, occurred near the beginning of the seventeenth century and is universally attributed to **John Napier**. The next step in mechanical computation was made by **Edmund Gunter**, who constructed in 1620 a logarithmic scale two feet in length and multiplied numbers by means of a pair of dividers.

The first computing machine that might be called the prototype of those in use today was invented by **Pascal** in 1642. His machine was designed to do addition and subtraction. **Leibnitz**, another versatile genius, designed a computing machine in 1671 and completed it in 1694. One important innovation introduced by Leibnitz was the sliding carriage. Others were the delayed carry, rotations in different directions for addition and subtraction, latches to provide protection against overrotation, and a mechanism for "erasing".

The most ambitious project undertaken during the nineteenth century in the making of computing machines was that of **Charles Babbage**, who was the Lucasian Professor of Mathematics at Cambridge but in 1839 he resigned his position to work on his "difference engine". The Government contributed £ 17,000 to the project, and Babbage used much of his own resources (£ 6,000) in the quest. He was unsuccessful and never completed his machine.

However, after a new government subsidy had been denied him in 1842, Babbage, instead of being discouraged, enlarged his ideas and began to work on what he called his "analytic engine". This machine also was never finished; but his son completed part of the engine in 1906 and published 25 multiples of π to 29 figures as a specimen of its work. His failure was not so much in the design of his machine, rather Babbage failed because he lacked the machine tools, electrical circuits and metal alloys so essential in modern machines. The engine, sometimes referred to as "Babbage's Folly" should have been called much more accurately "Babbage's Vision". A new dimension was given to computing machines in 1888 by **W. S. Burroughs**, who designed a machine that printed its figures. **Herman Hollerith** developed in 1880 the forerunner of tabulating and sorting machines. He invented a machine for sorting cards. The cards contained holes which permitted their distribution by electromagnetic relays activated by contacts made through the holes.

From this time on, progress was rapid. Because machines working on **the relay principle** were relatively slow, they were replaced in 1944 by electronic mechanisms in the form of **vacuum tubes.** Unfortunately, the number of such tubes made the machines bulky; and the heat generated, when many of them were in operation, presented a serious problem. In 1948 the invention of **the transistor** was announced. This revolutionary device is a small crystal that operates in the same manner as a vacuum tube. But it is much smaller, has a long life, uses less current, and consequently generates almost no heat. In the most modern computers (since 1961) vacuum tubes have been replaced by transistors, and this change has resulted in much greater efficiency of operation.

The enormous calculating power of these new machines is derived very largely from their memories (storage capacity) and their great speed. Their development has been so rapid that one machine is rendered obsolete by another almost before it has been completed. Except for the slide rule, all the computing devices just described are what are called "digital calculators". The slide rule belongs to a second class, called "continuous variable computers". Since the numerical answers of the latter are read from graphs or scales, their accuracy is much less than that of the digital calculators, which can deliver answers to many decimal places. Subsequent invention in this field has led to the development of what are called "analog computers", in which the analogue between electrical circuits and the mechanisms of mechanical devices is utilized to transfer the computation to machines operated electronically.

Thus, the history of computation with numbers is one of extent. The art began with the ancient civilizations and has had a steady growth through the centuries. How have modern mathematicians extended analytical methods and what are a few of the newer techniques? Unfortunately, new analytical methods are too technical to be described readily. We shall content ourselves by giving the names of them as follows: the discovery of the differential and integral calculus near the close of the seventeenth century, the calculus of finite differences, the method of relaxation, the rapid development of nomography (the use of diagrams) to obtain rapid approximation solutions from complex analytical expressions, analytical continuations, Laurentz series, continued fractions, asymtotic series, Fourier series, iterative methods and inversion formulas.

The question of the impact of the great calculators upon modern society was first presented in a systematic way by **Norbert Wiener** in a work entitled "Cybernetics". The amazing analogy between the computing machine and the human brain suggested that in certain areas the machine could do what man could do, and in many instances do it as well or better. Subsequent developments have led to a social revolution. With the invention of servomechanisms, vacuum tubes, memory units, transistors, and the like, man suddenly discovered he had created an instrument that far transcended many of his own powers. Science took a vast step forward. The machine could perform prodigies of deductive reasoning, in the sense that it could solve complicated problems formulated in terms of Boolean algebra. It could remember complicated chains of operations; it could form images and draw pictures of them; it could be adapted with some success to the problem of translating languages. The machine could solve the most difficult equations; in fact, it could do most of those things that man requires both in his daily living and in his highest scientific endeavour.

The impact of such a machine upon society was not long in being felt. One application after another was found for it, far beyond the dreams of the original inventors. Many jobs hitherto performed by human energy were turned over to the new machines. The prospect of autotation became a reality in many lines of human endeavour. And all of this has come about as a direct consequence of man's first calculations, his struggle with the mystery of fractions, his mathematization of measurement, his study of the planets. Our story of computation with numbers must thus conclude without an ending, for society does not know what the end will be.

Read and translate the text. Using the vocabulary of the text describe a particular type of the computers you deal with. Emphasize the points of their likeness and similarity and the main differences between them.

TEXT TWO

WHAT IS AN ELECTRONIC COMPUTER?

An electronic computer is a device that can accept information, store it, **pro′cess** it, and present the results of the processing in some acceptable form. A most important adjunct to this definition is that a computer is told how to process the information by **instructions** which are stored in coded form inside the computer. A computer thus differs radically from a calculator, which can do the same thing that a computer does, except that the instructions are not stored inside the machine. The coded instructions are called a **programme** (modern usage prefers the word **routine**). We therefore speak of a computer as an **internally-stored-programme-device**.

Modern electronic digital computers have many attributes in common. They are usually built in several units, only one of them is a computer or "processor". The other units are control, storage, and input-output devices. The modern machine is more often called a computing system. These systems use semiconductors and include magnetic-core and magnetic-tape storage. Almost every digital computer has been found capable of doing more than it was originally designed to do.

Any computer or calculator contains devices for five main functions: input, storage, arithmetic, control and output. **Input** refers to the process by which information is put into the machine; **Output** is the process by which the results are moved out of the machine. **Storage** refers to the mechanism that can retain information during calculation and furnish it as needed to other parts of the machine. The **arithmetic unit** is that part of the machine which can carry out one or more of the basic arithmetic operations on the information held in storage. Finally, the **control** refers to those parts of the machine that dictate the functions to be performed by all the other parts.

In a computer, four of the five functions are, in principle, the same as in a calculator. All computers are electronic, so that the practical details of these functions are somewhat different. Input to the computer is provided by such things as **punched cards** or **punched papertape.** Storage is provided by a device such as **a ratating magnetic drum** or by **magnetic cores.** Arithmetic is carried out by various electronic circuits, as a part of the control function. Output is provided by such devices as **punched cards, punched paper tape**, a **typewriter**, or **a printer** which can print a complete line of information at a time. The main difference is that the instructions telling the computer what to do must be placed in storage before the computer proceeds with the solution of a problem. These instructions, which are made up of ordinary decimal digits are placed in the same storage device that holds the data.

Computers differ not only in size but in the type of equipment employed to carry out the various functions, especially with regard to storage, speed, and input-output. Most computers are equipped with devices for reading and punching cards and have line printers. A few machines have cathode-ray tube (TV) output devices, which are able **to display** lines, points and characters on their faces. These devices are often equipped with cameras so that permanent records of the output can be made. Many computers have **buffers** in connection with their input-output devices. The idea of a buffer is to provide a better **match** between the speed of internal electronic operations and the slower input and output operations, all of which (except the cathode-ray tube) involve some mechanical element. A buffer is thus a storage device which compensates for the widely disparate speeds of other devices.

The arithmetic unit contains **adders**, **multipliers**, **dividers**, and **subtractors.** These calculation devices are electronic and are made up of transistors, vacuum tubes, diodes or the combination of these. Also in the arithmetic unit are **delay lines**, **amplifiers**, and **the generators** which initiate the signals utilized. It is in the arithmetic unit that calculations are performed at incredible speeds as compared to the ordinary desk calculators. The control section of the computer contains **electronic gating circuits**, **electronic control circuits**, and in some computers, **mechanical relays** which are tripped by electric pulses. The control section must interpret the information and instructions which are retained in the storage section and it must transmit the results of such interpretations to the computer sections for initiating the calculation processes.

The results of the calculations or other processes performed by the computer are obtained from the output devices. The output devices may consist of **punched cards** which retain the information resulting from the calculations, or of **paper tape** which is also punched according to a specific code that interprets the results. The placement of information into the computer by means of the input devices is known as **read-in** and the procurement of information from the output devices is known as **read-out.** The devices are **card readers**, **card punchers**, **line printers**, **type writers**, **magnetic tape units**, and **disk storage drives.** The activity of each device is governed by a **control unit.** In some cases, the control unit is built into the device; and is physically indistinguishable from the device; in other cases, notable with disk and tape drives, the control unit is completely separate, with one control unit governing several devices. Physical motion such as punching holes in a card or rewinding a tape is always performed by the device. The determination of what holes should be punched or what should be recorded on a moving tape is made by the control unit. Generally purely physical movement can be carried out by the device without detailed **supervision** by the control unit, but any operation involving the transfer of data requires the cooperation and constant supervision of the control unit.

Programming. Even in scientific computations, the most difficult part of programming is not the **coding**, or actual writing of the instructions for the computer, but the technical analysis of exactly what is to be computed and how it is to be computed. To illustrate consider the case where a large problem involves, at some state, the solution of a quadratic equation. There are several methods of solving such an equation with the pencil and paper, but what method is preferable on the computer? Preferable in what sense — in the sense of fastest execution time for the calculation, in the sense of fitting into the least possible amount of core storage; or perhaps in the sense of being easiest to code. The programmer must take these decisions. The programmer does not make all his decisions at once. He usually begins with an overall view of the entire system and the computer is considered as just another unit in the system. As he plans the system, he draws a **system flowchart**, a graphic and easily understood picture of what happens in the system. He then breaks each part of the system flowchart down into as much detail as needed, producing **detailed flowcharts.** For the computer processing, he draws a special **programme flowchart outlining**, in general, the essential steps in the computer programme. This is refined to produce detailed programme flowcharts from which the coding begins. The coder, who may or may not be the same person who did the system analysis, translates each block of the flowchart into one or more instructions for the computer.

Multiprogramming. One reason for using an operating system is to increase **throughout** the amount of useful work the computer performs in a given time period. In many jobs, the computer spends most of its time waiting for the completion of Input-Output operations, particularly printing. If the computer has enough core storage and sufficient Input-Output devices, it allows for **multiprogramming**. Multiprogramming means that two or three different and unrelated programmes are placed in storage, with each programme having its own set of Input-Output files. The Supervisor gives control to the highest priority programme and it continues to be executed until it reaches a point where it can go no further until some pending Input-Output is completed. At this point, the Supervisor saves the status of the programme and transfers control to the next highest priority programme. When Input-Output operation is completed, the Supervisor halts the programme which was running and returns control to the first programme. Processing, continues in this way with the computer entering to wait state only when all the programmes are waiting. Although the amount of time taken for the computer to complete any one programme is increased, the total time for all programmes will usually be reduced substantially.

Read and translate the text. Reproduce the text.

TEXT THREE

MICROCOMPUTERS

It is now more than 25 years since the electronics industry learned to make miniature electronic circuits on a "chip" of silicon substrate by altering processes of masked etching and diffusion. In the early 1960's the commercially avaialble intergrated circuits incorporated at most a source of components such as diodes, transistors and resistors. Production yields (the fraction of circuits that worked) were low and packaging technology did not allow the realization of practical devices with more than a dozen leads or connections. The basic technology, however, was so amenable to improvement and the rivalry among many branches was so keen that every year since then the number of components that could be economically placed on a single chip has doubled. The chips less than a quarter of an inch on an edge can incorporate well over 20,000 components. As a result, the cost per component has dropped by a factor of more than 100. The steady increase in component density, combined with parallel advances in circuit organization and complexity has predictably led to the **microcomputer**, a full-fledged general-purpose machine whose logic and memory circuits can be mounted on a single plastic card that would fit comfortably inside a cigar box. Where space is as special premium the complete microcomputer can be squeezed onto a substrate two inches square.

The microcomputer is a direct descendant of the **minicomputers** — small parallel data processors introduced in 1963. They became known as minicomputers primarily because of the physical size, not because of limitations in their performance. Nearly as powerful as much larger computers costing several times more, they were soon widely imitated. Within a decade they had given rise to an entire industry concerned not only with "hardware" (the computers themselves), but also with "software" (the programmes) and with innumerably perepheral devices and auxiliary services.

Minicomputers rapidly found their way into existing systems. To them they brought, in addition to cost reductions, the flexibility and simplified design of stored-programme machines. More important, they made possible a wide range of new application that called for an inexpensive resident computer. Much of the success of the minicomputer industry was due to the dramatic advances made in microelectronics throughout the 1960s—1970s. These advances had a twofold effect. The performance of minicomputers improved steadily, and the size and cost of the systems decreased at a rate averaging 30 percent per year. It was clearly just a question of time until the further integration of microscopic components would lead to a microcomputer, a machine that would have tens of thousands of components on a single chip.

Working microcomputer is contained in a small desk-top unit to which various input and output devices can be attached. For the most part microcomputers are used not as a substitute for a larger computer, but rather as a specially programmed subsystems that serve as "resident" computers in a compex device or system, such as a process-control instrument or a traffic-light-control system. Microcomputers are usually classified according to the number of bits that can be handled by their central processing unit. Their performance is judged by the richness of their instruction set, by the efficiency of their programme and by the speed with which they execute typical programmes, and how they compare with machines many times bigger and more expensive, by their impact they have had on the design of electronic systems.

In the future some cyberneticians claim a lot of information is going to be available only via microcomputers. Nowadays pulbic libraries are moving into electronic age, their catalog room — the index of knowledge for readers — are being computerized; drawers and cards are being replaced by a central memory bank and low-slung terminals. In some libraries instead of thumbling through stocks of 3-by-5 cards in search of a book the readers will now type a title or topic on a keyboard and watch the pertinent information flash onto a screen before them. Such computerized catalogs are only the most visible sings of the changes that electronic technology is making. Libraries are purchasing microcomputers, the most forward-looking ones are plugging their new terminals into computer networks and giving card-holders access to remote electronic data banks. Many US public library systems have microcomputers for cardholders to use and terminals among the computer stocks. A few libraries have even ventured into the world of computer communications. Elements of the library of the future are already in place. Any branch equipped with a terminal can retrieve the full text of scores of newspapers, magazines and professional journals through data-base services. Library planners and managers envision the day when a reader sitting at a branch library terminal will be able to call up books and articles stored in microcomputers all over the world.

Unfortunately, electronic information services are still luxuries beyond the budget of all but the best-funded library systems. Then, too, some librarians resist commercial data-banks on philosophical grounds. Many are reluctant to offer a service for which they will have to attach a fee. Besides, the advent of the computerized library has also brought new problems. Computers have a way of making simple research task more difficult — for example, when a casual user needs computer instruction just to find a book, an awkwardly phrased query can quickly lead to information overload, generating hundreds of responses. Even trained librarians say there is an art to performing an efficient data-base search. Electronic data are certainly easier to store and cheaper to move from place to place than printed material. On the other hand, most readers prefer browsing through books and magazines to reading little green words on a video screen. And nobody is eager to take on the Herculean task of transcribing into bits and bytes the vast body of knowledge already stored in printed volumes.

Translate the text. Discuss the advantages and applications of supercomputers.

TEXT FOUR

SUPERCOMPUTERS

American computers now reaching the market in volume can perform a few hundred arithmetic operations per second. At the other end of the scale are the powerful machines known as **Supercomputers**, of which there are now only several dozen in the world, whose peak computing speeds exceed 100 million operations per second. For a machine to be qualified as a **supercomputer in 1982**, it should be able to sustain average rates of 20 megaflops (the megaflop-one million floating-point operations per second) for a range of typical iterative problems that have a data base of a million or more words. The first commercial electronic computer Univac I (1951), was about three times faster than today's home computers and thousands of times more massive. Since Univac the speed of large-scale scientific computers has doubled on the average every two years. These increases of speed have entailed large increases in memory capacity, necessary for storing data and results. The latest supercomputers are the **Cray-1** and the **Cyber-205**, they are fully operational and offered for sale.

The doubling and redoubling of computer speeds in recent years has been made possible in large part by the steady decrease in size of microelectric circuits. The number of transistors that can be fabricated on a chip of solicon measuring a fraction of an inch on a side increased from a dozen or so in the early 1960s to several thousand in the early 1970s and has now reached several hundred thousand. These strides in semiconductor technology made it possible for the first time to build large, fast memories at tolerable cost. A second benefit of increased circuit density has been a reduction in the time needed for each cycle of logic operations. Concentrating, however, a large amount of circuitry into a small volume to minimize the length of wires creates a serious problem: the removal of the waste heat generated by electrical energy conversions.

Digital circuits are subject to errors caused by the failure of components and by random electrical noise. Even though chips have become increasingly reliable, error rates grow as supercomputers evolve, because the error rates of a complex system are at least proportional to the number of its parts. In order to protect the user from flawed results supercomputers incorporate mathematical codes capable of detecting and correcting errors. This is done by adding redundant bits of information to the minimum word length required. Circuitry for generating and checking the redundancy bits must be included at several error-control points in the machine. All supercomputers of the current generation incorporate error-control systems. To be sure, error control adds appreciably to machine cost and does not eliminate the possibility of undetected errors (errors of three bits or more), although it does reduce their rate to acceptable levels.

Although the current performance levels of such machines owe much to the rapid advance of microelectronics, new concepts in computer architecture have been equally important. The term **architecture** refers to the logical organization of the computer as it is seen by the programmer. The architectural innovations of greatest significance are those that enable the machine to carry out many similar operations in parallel, or concurrently. Whereas early computers obliged the programmer to break his computational problem down into a sequence of elementary steps, which are to be executed one at a time, the latest supercomputers allow him to specify that many different elementary steps be executed simultaneously.

A modern American supercomputer can be regarded as simplified model of both the Cray-1 and the Cyber-205. Although supercomputers are still few in number, they are to become widely available with the development of national and international high-speed data networks that link the scientific centres. Some major centres of scientific computation have developed elaborate data-communications networks linking several supercomputers, front-end processor (=input-output systems), other general-purpose computers, mass-storage data banks and graphic-display stations so that each user has rapid access to the most suitable resources for his problem. Supercomputer users demand ever increasing power for attaching the computational problems at the frontiers of their various disciplines. Computer architects have differing ideas for satisfying user demands.

It is not unreasonable to ask why anyone wants a computer 50 times faster than most of the fastest computers that are being manufactured today. There are some computational tasks in which speed is important, viz., long-range weather predictions, also cryptology demands can benefit directly from higher speed. To attain higher speed a new-microelectronic technology replaces transistors with superconducting switches. These computers can carry out a **billion** elementary operations per second. Such an ultrafast supercomputer can and will be employed for the routine processing of insurance premiums, tax returns and payroll records. It will serve multiple users and multiple functions in the central computing departments of universities. In most such applications it makes no difference whether a task that takes a million machine cycles is completed in one millisecond or 50 milliseconds. The point is that a computer running 50 times faster can do 50 times more, perhaps per the same cost.

Soviet supercomputer. The USSR has started production of PS-2000 computers (PS stands for "parallel system") capable of performing 200 million operations per second. Any computer performs many functions: in addition to number crunching the computer supervises the whole computing process, and determines the sequence in which information is to arrive at its processors. In fact, the computer takes only one-tenth of its total operating time to do the computing, the function it is designed for. What is called for, therefore, is a computer with computing elements free from all other functions. This becomes possible if supervision is assigned to special processors acting as managers. Operating in conjunction with computing circuits these processors give instructions and run the queue of information to enter the processor from storage, etc.

This new computer concept attracted interest from the American Control Data Corporation (CDC), and resulted in the production of the Cybers, the fastest computers in the USA. It works like an assembly line with individual processors performing individual operations. Data have to pass through the entire length of the conveyor, no matter how many processors are there to process them. The Soviet approach differs radically from the above. It suggested a principle whereby all processors respond to a single control command which leaves them a certain margin of freedom, with the possibility to sort out their data independently. Receiving a common "command" they all start off doing similar operations, later switching over to successive operations until the whole problem is solved. The efficiency of this parallel system is obvious.

First, similar operations can be handled at any speed as it depends on the number of processors involved. Second, a single control system for all processors is simple and, consequently, lowcost. CDC was ready to take part in realization of the project, but this was not to be. The White House imposed restrictions on sales of computer technology to the USSR, thus freezing both trade and cooperation in this sphere.

As a result CDC lost a lucrative contract and the USSR has now started commercial production of the PS-2000. So far there are no computers in this class in the West. It is high-speed and low-cost. It can be used in weather forecasting, to predict patient's post-operation states or the behaviour of an airfoil which only exists in a blue-print, and also help to pinpoint drilling sites for oil and gas by plotting cross-sections of the earth's crust to a depth of up to six kilometers by making use of geological survey data. Incidentally, the American Cyber-72 takes nearly two hours to plot an eartn's crust cross-section to a depth of 25 kilomethers, an operation the Soviet computer performs in just ten minutes.

It is no use denying the advantages of the CDC computers. But **the concept of single control for parallel processors** catches on in the USA as well. Similar computers would have been produced earlier on in the USA (in Britain and France, too, where they are being designed), but for the tough line taken by those circles which want to hamper contacts between East and West in science, technology, trade and the economy. The new restrictions recently sanctioned by the US Coordination Committee, a body in charge of exports to the socialist countries, cannot stop the progress of electronic technology in the world of socialism.

Discuss the main points of the given text. When will the optical computer be manufactured, to your mind?

TEXT FIVE

THE OPTICAL COMPUTER

The development of the digital computer in the past 40 years is so closely associated with the development of electronic technology that people tend to think of the computer as the inherently electronic device. Actually the operations carried out by a computer are logical and arithmetic ones, and they can be done by any of several means. Since the mid-1970s it has become apparent that the potential exists for constructing a computing device in which **signals are transmitted by beams of laser radiation rather than electric currents.** There is a powerful incentive for developing such an optical computer: it may operate 1,000 times faster than an electronic computer. **An optical analog of the transistor** is the basic element of a computer based on beams of light.

A computer based on beams of light rather than electronic currents might be capable of a trillion operations per second. The crucial component, an optical analog of the transistor, has already been built. Each optical transistor can be the site of many simultaneous switching operations carried out on parallel laser beams; an electronic device operates on one signal at a time. Moreover the crystals can switch to successively higher levels of transmitted power with successive increases in the incident beam; in contrast an electronic transistor of the kind employed in computers has only two output states. The adoption of devices with more than two stable states may lead to a new system of computer logic.

An experimental version of the optical transistor that is switched by a small change in the intensity of an incident beam of laser radiation has been developed. The optical transistor (called a "**transphasor**" because its operation is based on controlling the phase of the light within it) has been designed on a property of certain crystals. An increase in the intensity of light causes a change in the crystal's refractive index, a measure of the extent to which light is slowed as it passes through the material. In a new optical transistor switching times of approximately a few picoseconds have been obtained. The starting point of the optical transistor is an ingenious and widely used piece of optical apparatus known as the Fabry-Perot interferometer, invented by the French physicists in 1896 to measure the wavelength of various colours of light. The optical transistor relies on two precisely adjusted laser beams focussed on the front face of a Fabry-Perot interferometer containing a substance with a non-linear refractive index.

The "constant" beam is strong and unvarying. The "probe" beam is weaker and can be modulated. The low transmission or "off"-state can stand for the value 0 in the binary logic of the computer; the high transmission or "on"-state can stand for 1. In this respect, the optical thansmitter is analogous to the electronic transmitter.

Optical switching systems need not be thought of merely as rapid substitutes for electronic devices. On the contrary, the greatest benefits of optical switches may come from applications that cannot be duplicated by other means. Optical fibers which have the capacity to carry prodigious amounts of information, are being utilized increasingly for communications, including communications among computers. Optical switching is a natural candidate to mediate between electronic systems and optical ones. On the other hand, if the computation is done optically to begin with, optical fibers can be employed as direct links between computing systems. In future the capabilities of the optical transistor will transform the architecture of the computer itself.

Constructing an optical computer will require a variety of circuit elements in addition to the optical transistor. Nevertheless many of the elements needed to make an integrated optical circuit have already been demonstrated in experimental form. Formidable technical difficulties must be overcome before such a circuit can be manufactured. However, the optical computer is an intriguing and realistic prospect for the relatively near future, once an optical transistor has been constructed. New laser and new non-linear materials will undoubtedly make it possible to improve the speed and efficiency of optical switches. The question of appropriate materials is at the centre of the practical problems of building an optical computing machine.

Sum up the main ideas of the text. Reproduce the summary obtained in class. Speak on the topic: "Modern Applications of Computers".

TEXT SIX

IMAGE PROCESSING BY COMPUTER

A digital computer processes information in discrete numerical units: digits. Most images, of course, do not come in such units. An ordinary photograph is an analog representation of a scene, the information is recorded in continuous gradations of tone (and in some cases of colour) across the two dimensional surface of the film. Processing a photograph by means of a computer therefore requires that the analog image first be converted into a digital one. A number of direct ways of digitizing an image are now available, e. g., a photograph image can be sampled by a scanning microdensitomiter by the flying-spot scanner; more advanced image-sampling systems, based on semiconductor technology have been devised in recent years.

An image that has been reduced to a set of binary numbers can be stored on magnetic tape or transmitted directly to the computer. The data can then be processed to obtain a new set of digital values, which in turn can generate a revised image. The new image is usually displayed on a cathode-ray tube, where it is either viewed directly or photographed. Once the image has been digitized and transmitted to a computer, various mathematical operations can be carried out on the data in order to enhance the visual quality of the image and thereby allow the information in the image to be more readily and accurately perceived by the human visual system.

When the information in an image is expressed in digital form, it can be manipulated mathematically rather than optically. By mathematical methods a blurred photograph can sometimes be restored to clarity. The digital computer is now an essential tool in many areas of image processing. An out-of-focus photograph or one streaked by movement of the camera can now be digitally processed to improve its resolution and to restore lost details. In many cases the cause of the blur can be determined from the blurred image itself. The techniques for deblurring images can be applied not only in scientific research and medicine but also in fields such as criminology and military intelligence. Investigators are currently working on digital methods for compressing the information in an image, a capability that can lead to greater efficiency in TV transmission.

The greatest application is the automatic recognition of certain recurrent patterns in large number of images, which makes it possible to extract more information from satellite images of the earth. Semiconductor image-sampling devices offer many advantages besides their direct digital output. Accurate and reliable devices for sampling and quantizing an image or for displaying an image reconstructed from digital data have been commercially available for more than a decade. Without them digital image processing could not become the active field it is today. The increase in the number of companies that specialize in the manufacture of devices for getting images into and out of a computer manifests the growth of the activity.

What is responsible for the recent surge of activity in digital image processing? First, an image embodies a prodigious amount of information, and so most image processing is done with a large and powerful computer. As the continuous revolution in microelectronics has reduced the cost of such machines while increasing their capacity they have become more widely available. Second, the same advances in microelectronic technology have also led to the construction of improved devices for converting an image into digital data and vice versa. Third, the development of sophisticated algorithms or mathematical procedures that can be executed by a computer has made it possible to carry out image-processing operations that once had been impossible or impractical.

Scientists are familiar with some of the more spectacular products of digital image processing; enhanced satellite photographs of the planets (including the earth), colourful representations of more distant celestial objects and images assembled from *X*-ray data to show a cross-section of the human body. Perhaps the most important application of digital image processing is in the area of remote sensing particularly in the analysis of satellite images of the earth. The satellites are equipped with scanning devices that record images of the earth passing under them in a series of strips. Each strip is 185 kilometers wide, and it is usually subdivided (by computer processing on the ground after the data are received) into a series of images 185 kilometers on a side. More than 90 percent of the earth's surface has been surveyed under cloud-free conditions at this resolution. By combining images made at different wavelengths it is possible to enhance the information display.

The potential of remote sensing for helping to solve human problems is great. For example, the growth of a crop such as wheat is accompanied by predictable changes in its multispectral image. The onset of various crop diseases, such as blight in corn or rust in wheat, can be detected from changes in the multispectral image. Experiments of this kind have led to the suggestion that it might be possible to monitor food crops throughout the world and to predict the world harvest from satellite images. Besides, there is much interest in the development of computer programmes for the recognition of patterns in images. Because most deposites of mineral resources are associated with characteristic geological features, efforts are also being made to automatically isolate and extract from remotely sensed images such geological details, as faults and escarpments. The expectation is that the automatic extraction of information from such images will some day be one of the most active areas of digital image processing.

Read and translate the text. Give answers (in writing) to the questions posed in the text. Combine all your answers to make an outline of the main ideas of the text. Reproduce your outline in class.

TEXT SEVEN

ARTIFICIAL INTELLIGENCE

Are we intelligent enough to understand intelligence? One approach to answering this question is **Artificial Intelligence** (A. I.), the field of computer science that studies how machines can be made to act intelligenty. "Artificial intelligence" is the ability of machines to do things that people claim require intelligence. A. I. research is an attempt to discover and describe aspects of human intelligence that can be simulated by machines. For example, at present there are computers that can do the following things:

1. Play games of strategy (e. g. Chess, Checkers, Poker) and (in Checkers) learn to play even better than people.
2. Learn to recognize visual or auditory patterns and perform image processing.
3. Find proofs for mathematical theorems.
4. Solve certain, well-formulated kinds of problems.
5. Process information expressed in human languages, etc.

The extent to which computers can do these things independently of people is still limited; machines currently exhibit in their behaviour only rudimentary levels of intelligence. Even so, the possibility exists that machines can be made to show behaviour indicative of intelligence, comparabe or even superior to that of the humans. Alternatively, A. I. research may be viewed as an attempt to develop a mathematical theory to describe the abilities and actions of things (natural or man-made) exhibiting "intelligent" behaviour, and serve as a calculus for the design of intelligent machines. As yet there is no "mathematical theory of intelligence", and researchers dispute whether there ever will be. A. I. is also the study of ideas which enable computers to do the things that make people seem intelligent. But then, **what is human intelligence**? Is it the ability to reason? Is it the ability to acquire and apply knowledge? Is it the ability to manipulate and communicate ideas? Surely all of these abilities are part of what intelligence is, but they are not the whole of what can be said. Indeed a definition in the usual sense seems impossible because intelligence is an amalgam of so many information-processing and information-representation talents.

Nevertheless, one can define the goals of the field of A. I. **The central goals of A. I. are to make computers more useful and to understand the principles which make intelligence possible.** Since one goal is to make computers more useful, computer scientists and engineers need to know how A. I. can help them solve difficult problems. And since the other goal is to understand general intelligence better for its own sake, psychologists, philosophers, linguists, and other people who want to understand human intelligence also need to know and evaluate what is learned. A. I. enthusiasts believe that using computers to understand central issues and the dimensions of intelligence is a powerful addition to the traditional methods of social sciences, psychology, philosophy and linguistics. There are some reasons for their commitment viz., computers are ideal experimental subjects: they exhibit unlimited patience, they require no feed-ing. Moreover, it is usually simple to deprive a computer programme of some piece of knowledge in order to test how important that piece really is. It is impossible to carve up animals' brains with the some precision. Computer models are precise and enable more powerful thinking about thinking.

Note that wanting to make computers **be** intelligent is not the same as wanting to make computers **simulate** intelligence. **A.I.** excites people who want to uncover principles that apply to all intelligent information processors, not just those that are made of wet nervous tissue instead of dry electronic gadgetry. Consequently, there is neither the obsession with mimicking human intelligence nor a prejudice against using methods that are involved in human intelligence. The overall result is a new point of view which brings along a new methodology and leads to new theories. One result of their new point of view may be ideas about how to help people become more intelligent. Just as psychological knowledge about human information processing can help make computers intelligent, theories observed purely with computers in mind often suggest possibilities for how to educate people better. Said another way, the methodology involved in making smart computer programmes may thansfer to making smart people.

The classical experiment proposed for determining whether a machine possesses intelligence on a human level is known as **Turing's test**, after A. M. Turing, who pioneered research in computer logic, undecidability theory and A. I. This experiment has yet to be performed seriously, since no machine yet displays enough intelligent behaviour to be able to do well in the test. Still Turing's test is the basic paradigm for much successful work and for many experiments in machine intelligence. Basically the test consists of presenting a human being *A* — human interrogator with a typewriter-like or TV-like terminal, which he can use to converse with two unknown (to him) sources, *B* and *C*. The interrogator *A*, is told that one terminal is controlled by a machine and that the other terminal is controlled by a human being he has never met. *A* is to guess which of *B* and *C* is the machine and which is the person. If *A* cannot distinguish one from the other with significantly better than 50% accuracy, and if this result continued to hold no matter what people are involved in the experiment, the machine then, it is claimed, **simulates** human intelligence.

It is clear that the intellectual capabilities of a human being are directly related to the functioning of his brain. Surprisingly little is known concerning the limitations of human intelligence. No one has made any complete survey of the problems that can be solved by human beings. The ability to solve certain types of problems has been studied and made the basis of "intelligence" tests, but the generality and validity of these tests is disputable. I. Newton, for example, might have scored low on such tests when he was an adolescent; yet he is estimated by some researchers to have had an **Intelligence Quotient** (I. Q.) near 200. One of the shortcomings of these tests is that they predict little concerning the development of a person's intelligence, especially what problems he could learn to solve. What is **natural intelligence**, after all? Many definitions of "intelligence" may be summarized in one phrase, viz., **"intelligence" is the ability to "act rightly, in a given situation".** Although one can imagine an entity that always behave "rightly", without making any errors, A. I. research is more concerned with the concept of partial success with building machines that can make mistakes, but which can also change their behaviour with time and perhaps stop making mistakes.

Intuitively, A. I. research is concerned with building machines that can "adjust" or "adapt" to certain environments, and which in effect **learn to solve problems** within these environments. This corresponds with the ordinary conception of human intelligence — that it is limited, but that it can learn and thereby improve its performance of certain tasks with time.

Though people have succeeded in constructing machines that can "learn" to produce solutions to certain specific intellectual problems superior to the solutions that people can produce our knowledge of the natural intelligence limits is not sufficient to determine whether the attainment of a **general artificial intelligence** is within the bounds of computational ability. The term general artifical intelligence refers loosely to a machine (procedure) that has aptitudes for general problem-solving, general game-playing, general theorem-proving, general pattern-recognizing, general language-understanding, and also has aptitudes enabling it to display all other kinds of intelligent behaviour normally exhibited by people. A. I. researches still do not have enough evidence to decide whether machines can be made as intelligent as human beings.

Any discussions of artificial intelligence raise the question of how the study of thinking machines should or could influence our philosophical and psychological concepts. Philosophy poses two types of questions for artificial intelligence. One is, "How should **thought** be represented by a mechanical device?" This question leads to a consideration of the appropriate internal symbolic representation of the external world and its problems. Similarly, we may want to know how we can represent inside a computer the fact that a particular action may have multiple effects besides those intended when the action was made. Such questions are aspects of the more general question: **How should a machine think?** which presupposes an answer to a prior question: **Can a machine think?** which is a more exciting question. Unfortunately, the question is not easy to answer directly. What is the answer to **How do you decide that anything is thinking?** This is not at all trivial. Some philosophers and many psychologists influenced by them claim that the concept of "thinking" is not a useful one. Instead they prefer to talk about specific behaviour which, if observed, may be characterized as "thinking", for purposes of a shorthand code, but without any committment to belief in some common underlying process. In fact most of these writers decidedly prefer not to use the word at all. If this is to be an accepted criterion, then the question of **machine thought** reduces to a series of empirical tests. Can theorem provers solve problems as well as professional mathematicians can? Certainly not. Can computer programmes be written to solve the pattern-recognition problems found in intelligence tests? Yes. Can we today write programmes that match human comprehension of natural language? No. Will we ever? May be. It seems that the answer to the question, **Can a machine think?** is an arbitrary one. Is there a general theory of thought? No. Is one on the horizon? Doubtedly. It is generally agreed that intelligent behaviour can only be produced by a system of information processing elements, feature detectors, theorem provers, and decision makers. There are non-trivial laws about how such systems must be organized. Modern computers are much faster, more complex, multifunctional and useful than most people dreamed of 50 years ago. They display intelligence and can be made to show more in future. Genius is yet undefined.

I. *Learn the meanings of the given words and translate the following sentences. Try to justify your choice of Russian (English) equivalent.*

intelligence(cy) *n.*
1) ум, интеллект, умственные способности;
2) сведения, известия, информация, сообщение;
3) разведывательные данные, разведка;
4) сношения, связи

intelligent *adj.* (Intelligential)
умный, разумный, понятливый, смышленый, понимающий, информирующий, осведомленный

intelligently *adv.*
умно, разумно, с пониманием дела

Intelligence test
испытание умственных способностей

intellectual(s) *n.*
интеллигент; мыслящий человек; интеллигенция.

intelligence Quotient (US)
коэффициент умственного развития

intellectual *adj.*
умственный, интеллектуальный, мыслительный, мыслящий, разумный, интеллигентный.

intelligence Service (Engl.)
разведовательная служба

intellectually *adv.*
интеллектуально, умом, рассудком

intelligence Police
контрразведка

Разум; рассудок; интеллект; ум; умственные способности, мозги.
Reason; Mind; Intellect; Intelligence; Brains; Wits.

умственный	— mental; brain; intelligence.
интеллигентный	— well-educated; intellectual.
интеллигентность	— intellectuality; cultural level.
разумный	— reasonable; intelligent; intellectual.
культурный	— cultured, cultivated.
рассудочный	— cerebral.

1. The human **brain** is so complex that the exact prediction of its behaviour is essentially impossible. 2. The tools of Cybernetics give us the power to increase our **mental** capabilities. 3. Artificial **Intelligence** is the field of computer science that studies how machines can be made to act **intelligently**. 4. Can machines (robots) be made as **intelligent** as human beings? 5. Man knows his **mind** insufficiently. A man supplied with **intelligent** cybernetics devices — the products of man's **reason** — thinks deeper and wider than a man who has to resort only to the primitive means of his own **intellect**. 6. Cybernetics deals with the fantastic world of the future peopled by robots and electronic **brains**. 7. The Cyberneticians today turned their attention to the study of the higher **cerebral** functions and the intricacies of **intelligence**. 8. All arithmetic operations performed by human **brain** differ radically from the physical processes used in analog computers. 9. Since the time when computer programmes managed to process symbolic data one could speak of computer aptitude to solve **intellectual** problem and the origin of the artificial **intellect** research.

10. The techniques for processing images can be applied not only in scientific research and medicine but also in fields such as criminology and military **intelligence**. 11. Is the exhaustive analysis of human **reason** into rule-governed operations on discrete, determinate, context-free elements possible? Is the approximation to the goal of artificial **intelligence** even probable? The answer for both questions is No. 12. The problem of testing a machine to see whether it is **intelligent** was first discussed by the great British logician and computer pioneer, Alan Turing, noted for his **wits**.

II. *Observe the meaning of the verbs: remember — recollect — recall — memorize using illustrative examples from the text.*

The Brain

This century man has made many discoveries about the universe — the world outside himself. But he has also started to look into the workings of that other universe which is inside himself — the human brain. Man still has a lot to learn about the most powerful and complex part of his body — the brain.

In ancient times men did not think that the brain was the centre of mental activity. Aristotle, the philosopher of ancient Greece, thought that the mind was based in the heart. It was **not until** the 18th century **that** man realized that the whole of the brain was involved in the workings of the mind. During the 19th century scientists found that when certain parts of the brain were damaged men lost the ability to do certain things. And so people thought that each part of the brain controlled a different activity. But modern research has found that this is not so. It is not easy to say exactly what each part of the brain does.

In the past 50 years there has been a great increase in the amount of research being done on the brain. Chemists and biologists have found that the way the brain works is far more complicated than they had thought. In fact many people believe that we are only now really starting to learn the truth about how the human brain works. The more scientists find out, the more questions they are unable to answer. For instance, chemists have found that over 100,000 chemical reactions take place in the brain every second!

Scientists hope that if we can discover how the brain works, the better use we will be able to put it to. For example, how do we learn language? Man differs most from all the other animals in his ability to learn and use language, but we still do not know exactly how this is done. Earlier scientists thought that during a men's lifetime the power of his brain decreases. But it is now thought that this is not so. As long as the brain is given plenty of exercises it keeps its power. It has been found that an old person who has always been mentally active has a quicker mind than a young person who has done only physical work. It is now thought that the more work we give our brains, the more work they are able to do.

Other people believe that we use only 1% of our brain's full potential. They say that the only limit on the power of the brain is the limit of what we think is possible. This is probably because of the way we are taught as children. When we first start learning to use our minds we are told what to do, for example, to remember certain facts, but we are not taught how our memory works and how to make the best use of it.

The Mystery of Memory

How does memory work? No one — but no one — is sure. It's that simple. What makes memory so hard to understand is the seeming caprice with which it operates. Sometimes our recollections are vivid and sharp sometimes they're blurred and murky. Sometimes we recall things in great sweeping overviews; sometimes we remember only minutiae. Research now indicates that the way information is stored depends upon the way it was learned in the first place. **Short-term memory (STM)**, our simplest memory-storage receptacle, serves as a kind of holding pen for data we may or may not want to retain.

Generally, the capacity of STM is limited to seven or eight chunks of information. These can range in complexity from a single digit to an elaborate sentence of thought. STM capacity can thus be enhanced by consolidating many individual bits of information into fewer, meaningful units. For exampe, it is difficult to memorize at first glance a string of 12 digits such as 1, 8, 6, 5, 1, 4, 9, 2, 1, 9, 6, 9. But the task becomes far easier if we recognize that the first four digits represent the year the Civil War ended, its next four the year Columbus discovered America and the last four the year men first landed on the moon. Twelve bits of data are thus compressed into three.

Unlike STM, **long-term memory (LTM)** has a comparatively limitless capacity and duration. In order for information to make the leap from STM to LTM, it must have some significance or association. Hence, a random license plate on a random car might be observed and quickly forgotten. But if the same car is screeching away from a robbery and the observer jots down the number to give to the police, chances are that those six or seven digits will be recollected and remembered for a lifetime.

Special expertise also facilitates memory. The layman examining a photo of two football teams in play might labour long and hard to memorize the exact location of each player. A quarterback, however, might glance at each picture, recognize the play and instantly memorize each athlete's position. Recall is also influenced by whether information is memorized in linear, beginning-to-end, fashion or all at once. Pictures, for example, are generally absorbed in a single swallow, so we can recall the entirety of some paintings or photos all at once. On the other hand, we probably learn a song or poem in tidy, start-to-finish order. Hence, if we try to remember a lyric from the middle of a time, we find ourselves rapidly reviewing the song from its opening bars, sort of singing our way to the words we're looking for.

Although observing the function of memory is easy enough, explaining its physiology is not. Just what goes on in the brain when we process a thought? Here opinions diverge. Some suggest that the structure of a nerve pathway changes when data are preserved, forming a neural road map of a thought. Others think the brain works holographically, each new piece of information being stored in all areas of the brain. The study of the physiology of memory is in its infancy and researchers must thus still rely on analogy, on terms like **storage** and **retrieval**, to explain how we remember. But even a rudimentary understanding is better than none at all, and science is now providing at least that much insight — a significant stride in a field of study that has mystified so many for so long.

III. *Don't mix them up!*

process	software	current	digit (1)	attain
processing	hardware	currant	digit (2)	sustain
data	search	match (1)	simple	
date	fetch	match (2)	single	

IV. *Give one Russian equivalent of the following groups of words.*

a) Facts — data — information — intelligence / sign — number — figure — character / change — alteration — modification / attributes — features — characteristics / match — correspondence / unit — part — device / user — consumer / display — show — exhibition / ability — capability — faculty — aptitude.

b) To replace — to substitute / to simulate — to mimick — to copy — to imitate / to surmount — to overcome — to conquer / to perforate — to make holes through or into — to penetrate — to pierce / to fetch — to go and bring — to come forth or out / to halt — to stop — to cease.

c) Bulky — large — enormous — prodigious / subsequent — later — following / vast — great — huge — extensive / idle — not employed — not in use — lazy / tolerable — bearable — endurable / desperate — reckless.

d) At the same time — simultaneously — concurrently / essentially — substantially — largely / rapidly — swiftly — quickly — speedily / incredibly — unbelievably.

V. *Give the Russian equivalents of the given word combinations and set phrases.*

General-purpose computers / front-end processors / vacuum-tube technology / magnetic-core and magnetic-tape storage / access to memory / the memory reference / the execution phase / storage capacity / processing speed / mass-storage data banks / the prodigious computing power / supercomputer installation / data communications networks / graphic display stations / semiconductor failure rates / redundant bits of information / concurrency provisions / real-time-image processing / floating-point registers / the data-stream computers / single-instruction-stream-multiple-data-stream computers.

VI. *Distinguish the meaning of the given words in man-to-computer conversational context.*

Say — tell — speak — talk — chat — converse — communicate

Man to computer: Speak! Talk soon. Chat later

Can people **converse** with the computers on any topic they might pick and in their native languages? It's unlikely, for the time being. People cannot so far **communicate** with computers in a natural human way. Can the computer respond to man's questions and say anything when it is **told** or asked? It will be able to, sure enough. The first generation of computers were operated by valves, then there were transistors. Next came the silicon chip and then an even better silicon chip. Now it is possible to look forward to the next generation of computers. These will be more intelligent. They will reason in terms of words rather than simply in terms of numbers. In a sense this new type of computer will copy the operation of the human brain, dealing with problems in parallel rather than simply one after the other. We will be able **to talk to** it — **but** using a very limited vocabulary of technical terms. The next step may be computers at telephone exchanges, doing actual translation work. You want, perhaps, to ring up someone in Tokyo. You speak English — the machine translates for you — and the other person hears a Japanese version of what **you said**. That will be a major step forward. But as for having a nice **chat** over a cup of coffee, that will have to wait till later.

Dutch booksellers who telephone the Netherlands' central book warehouse hear the voice of a pleasant operator who answers questions about titles and prices and takes their order. The operator never misses a day of work — unless he overloads a circuit. "He" is a computer called Boektel, which owes its voice to a new speech-synthesis system. The Voice Response System (VRS) can be programmed **to speak** in any language in a clear, if slightly metallic, voice. Although it does not understand human speech, callers can give it instructions using a standard Touch-Tone telephone, making the system ideal for airlines, banks and other firms that must get answers or disseminate up-to-date information over the phone. First the VRS may say, "If you want to know about title availability, press 2". The user then simply presses the phone key marked "2" to signal the VRS to continue. Up to 48 telephone lines can be connected to one VRS; the Boektel system, which has 32 lines, handles as many as 500 calls each day. 5,000 VRS units have already been installed around the world. A similar system has been recently developed that is a veritable computer mimic. It has the ability to sound like a middle-aged woman, an elderly man, a child or a deep-voiced man. The system is contained in a box about the size of a large typewriter.

VII. *Translate the text at sight.*

Computer Memory

The organization of computer memory has received much attention over the years. There are two general ways of partitioning memory, which can be called **vertical** and **horizontal**. The incentive for structuring memory in a vertical or hierarchical manner is that fast memories cost more per bit than slower ones. Moreover, the larger the memory is, the longer it takes to access items that have been stored randomly. The processing units in most large computers communicate directly with a small, very fast memory of perhaps several hundred words. Data can be transferred to or from one of these disk units at a maximum rate of half a million words per second, and in practice it is possible to maintain data flow between central memory and several disk units simultaneously.

The maximum rate of transfer of information to or from a memory device is known as a **bandwidth**. In order for the average computing speed not to be dominated by the smaller bandwidth of the lower memory levels, programmes must be arranged so that as much computation as possible is done with instruction and data at the higher levels before the need arises to reload the higher level from one below. This is an important consideration in programming vector operations for supercomputers, whose central-memory bandwidth is small in relation to the megaflop rate that can be sustained for data held in the register set.

Several multiprocessing supercomputers currently under development incorporate a number of independent parallel memory modules that linked to an equal number of independent processors through a high-speed programme-controlled switch so that all the memories are equally accessible to all the processors. For pipelines processors still another kind of horizontal partitioning of central memory has been devised: the memory is divided into a number of "phased" memory **banks**, so described because they operate with their access cycles out of phase with one another. The rationale for the scheme is that random-access central memories are relatively slow, requiring the passage of a certain minimum number of clock periods between successive memory references. In order to keep vector operands streaming at a rate of one word per clock period to feed a pipeline, vectors are stored with consecutive operands in different banks. The phase shift that "opens" successively referenced banks is equal to one processor clock period. The supercomputers — the Cyber 205 has 16 phased banks; the Cray-1 has 8 or 16, depending on the memory size. In both supercomputers mentioned the bank cycle time is equal to four processor cycles.

The memory of the modern supercomputers — the Cray-1 and Cyber 205 — **is organized hierarchically.** The two register memories are the smallest, followed in capacity by central memory, extended semiconductor memory and disk memory. The extended semiconductor memory has just begun to appear in supercomputer installations because rotating-disc technology has not kept pace with the increasing speed of processors. The largest such memory, with the capacity of 8 million words is in the Cray-1 supercomputer. Here all the functional units are "pipelined"; meaning that tasks are broken into elements that can be executed at peak speed and reassembled in a continuous flow, thereby achieving one floating-point operation per clock period: 20 nanoseconds in the Cyber 205 and 12,5 nanoseconds in the Cray-1.

All the functional units can run concurrently, but not all can run at top speed concurrently because they share common resources, such as data paths or memory access cycles. Moreover, conditional branches in the programme interrupt the smooth flow of instructions through the instruction processor. Before the processor issues an instruction, it must wait until it is clear that all the resources needed for the execution of the instruction will be available when they are needed. In the Cray-1 the register memories incorporate further hierarchical structure, and the vector processor holds additional register memory. The vector processors of the Cray-1 and the Cyber-205 also differ significantly in other respects. The disc-based secondary storage systems of both the Cray-1 and the Cyber-205 are too slow to allow any feasible solution of continuous-field simultation whose iterative data base is too large to fit in the machines central memory.

LAB. PRACTICE

Grammar Rules Patterns

The Sequence of Tenses Rule

I. *Analyze the analogy between the Russian and English Tense forms in the Principal clause.*

Present		Future			
Я говорю	I say I am saying I have been saying	Я буду говорить	I'll say I'll be saying I'll have been saying	Я скажу	I'll say I'll have said

Мне говорят	I am told I am being told	Мне будут говорить	I'll be told	Мне скажут	I'll be told I'll have been told

As a rule, the choice of the Tense form in the subordinate Object clause is free after a **Present** or a **Future** Tense form in the Principal clause.

Past

Я говорил	I said I was saying I had been saying	Я сказал	I said I have said I had said
Мне говорили	I was told I was being told	Мне сказали	I was told I have been told I had been told

After one of the Past Tense-Aspect forms in the Principal clause only **Past Tense-Aspect** forms should be used in the subordinate Object clause.

The Subordinate Object Clause

He said that ...

Present Indef. → Past Indef.	Past Indef. → Past Perfect
Present Cont. → Past Cont.	Future... → Future-in-the-Past
Present Perf. → Past Perf.	shall... → should...
Present Perf. Cont. → Past Perf. Cont.	will... → would...
	must... → must...

II. *Join the two simple sentences to make a complex sentence. Mind the Sequence of Tenses Rule.*

Model. I was told. Systems of regulation imply the notion of feedback.
I was told that systems of regulation **implied** the notion of feedback.

1. I said. The successful use of computers has been reported in economy. 2. I have been saying up to now. Man-and-Computer system is capable of accomplishing a lot. 3. I was being told. With machines that can write poetry and compose music Cybernetics has gained a foothold in the arts. 4. I'll be told. An exhaustive analysis of human intelligence by Cybernetics models is impossible. 5. I had already said. The next generation of computers will not operate with digit numbers. 6. I was told. A theorem-proving programme is solving an open problem in lattice theory. It is both sophisticated and challenging. 7. I'll have said at the end. We must not abandon work on artificial intelligence until the day we construct such an artificial man. 8. I was saying. Supercomputers will display wonders.

III. *Turn the following statements into Indirect Speech. Make all the necessary changes.*

Model. Current computers are capable of doing intellectual tasks.
What have I said? You have said (You've said) that current computers are capable of doing intellectual tasks.
What did I say? You said that current computers **were** able of doing intellectuat tasks.

What have I said? You've said that... **What did I say?** You said that...

1. N. Wiener — the "father" of Cybernetics — is the author of 200 scientific papers and 11 books. 2. His mathematical prodigy helped him obtain his doctorate in science at the age of 19. 3. He has laid the foundations of the new science and coined the title "Cybernetics". 4. The use of the word Cybernetics however, goes back to Plato. 5. Plato was the first to use the term to describe the science of steering ships. 6. The French scientist Ampère (XIX c.) designated study of the control of society by the same term "Cybernetics". 7. N. Wiener's definition of Cybernetics is still generally being accepted. 8. Current tendency is to regard Cybernetics either as computer science or philosophical approach of many pure and applied sciences. 9. There is no realm of human activity in which Cybernetics will have no role to play in the future. 10. It is difficult to say what the future holds in store for Cybernetics. 11. Cybernetics bears all the hallmarks of an explosive science. 12. Man has been building more and more powerful computers since 1940s. 13. Nevertheless, man has remaind computer's slave as he has still to control them. 14. The Third Industrial Revolution with computers capable of controlling themselves is looming on the horizon. 15. One is justified to call Cybernetics a veritable 20th century Queen of Sciences. 16. The social sciences will have much to gain from Cybernetics in the future.

IV. *Turn the following General questions into Indirect ones. Make all the necessary changes.*

Model. Has man been improving devices to assist his brain in calculations for a long time?

What have I asked? You have asked (You've asked) **whether** man **has been improving** devices to assist his brain in calculations for a long time.

What did I ask? You asked whether man **had been improving** devices to assist his brain in calculations for a long time.

What have I asked? You've asked whether... **What did I ask?** You asked whether...

1. Are operations performed by a computer very complicated? 2. Will the term "computer" refer to only digital computers? 3. Has the principal use of computers been in the field of applied maths? 4. Did engeneers contribute to the progress of Cybernetics? 5. Is Cybernetics a young science or an old one? 6. Does the word "calculus" mean a pebble in Cybernetics? 7. Was the abacus the earliest calculating device? 8. Are Cybernetics techniques being applied to large systems analysis? 9. Are logarithms really labour-saving and time-saving means? 10. Has man been using the slide-rule for a long time? 11. Was Pascal's computing machine designed to solve differential equations? 12. Was Leibnitz a versatile genius only in mathematics? 13. Has Babbage's analytic engine been the true prototype of current computers? 14. Will computing mathematics have accomplished great results by the end of this decade? 1. Was transistor technology a great modification in computer science? 16. Are transistors and cerebral neurons functioning in a similar way? 17. Does there exist any analogy between the computer hardware and human brainware? 18. Has a computer transcended any of human powers? 19. Will Artificial Intelligence soon be created? 20. Should Artificial Intelligence research be abandoned?

V. *Turn the following Special questions into Indirect ones. Make all the necessary changes.*

Model. How can a current supercomputer be defined?

What have I asked? You've asked how a current supercomputer can be defined.

What did I ask? You asked how a current supercomputer **could** be defined.

What have I asked?	What did I ask?
You've asked what (when, where, why, etc.) ...	You asked what (when, why, where, etc.) ...

1. What is a supercomputer? 2. What do we mean by the term "multiprocessing"? 3. Where have the first supercomputer been built? .4 Why does man seek to create smarter computers? 5. How can a superclass computer be visualized? 6. What computers can be exploited to design better computers? 7. What algorithms must be developed to exploit supercomputers? 8. How can communications between a supercomputer and the external world be carried out? 9. What have centres of scientific computation been developed for? 10. What computers are such centres equipped with? 11. Why can a modern supercomputer be regarded as a model of the Cray-1 and the Cyber-205? 12. How many front/end Processors does a supercomputer possess? 13. Where do mass-storage data banks reside? 14. Where can a consumer have rapid access to the resources for his problem? 15. What is a megaflop amount to? 16. What does the term Artificinal Intelligence designate? 17. When did the development of robots originate? 18. Where are commercial robots being utilized?

VI. *Translate the sentences expressing facts, common (general) notions, actual present, etc., in which the Sequence of Tenses Rule is not observed.*

1. The first world-Artificial-Intelligence-Conference-participants in 1956 **claimed** that no large bank, insurance companies, research lab. or education institution **can survive** without using computers. 2. Some specialists of Artificial Intelligence **insisted** that some day computers **will have** the same faculties as human beings. 3. A. I. specialists **disagreed** on the means and routes they **must take** and follow to attain their cherished goals. 4. Some A. I. enthusiasts **said** that the next stage of computer evolution **is** the creation of Artificial Intelligence. 5. However, they **admitted** that the only obstacle in this evolution is the failure to produce humour on the computer. Humour, they **acknowledged, plays** a major role in human thinking. 6. Many researchers **believed** that computers **will help** men in many of their complex perceptual and problem-solving activities. 7. Some critics of A. I. **urged** that this research **should be abandoned** as it is useless and ridiculous. 8. Nevertheless, they **failed** to deny that today computers **have permeated** our society. 9. They **witnessed** that superclass computers and robots **are** already **beginning** to move from the laboratory to the state and engineering establishments. 10. A. I. specialists **believed** and **claimed** that a new generation of supercomputers and the Artificial Intelligence of the future **are not** a dangerous threat, but rather a promising hope for the mankind.

VII. *The word "whether" may perform different functions in the sentence. It may be both a pronoun and a conjunction introducing different subordinate clauses. Translate the following sentences identifying the function of "whether".*

Models. Cybernetics is the entire field of control and communication theory, **whether** in the machine or in the animal. (a pronoun)
The question is **whether** a machine can think (a conjunction, Predicative clause)
We asked **whether** (=if) computer could be of any benefit to mankind. (General Indirect Question)
The problem of testing a machine to see **whether** it is intelligent was first discussed by A. Turing. (a conjunction, Object clause)

1. C. Shannon was not sure **whether** his automaton can be rated a thinker. 2. There are a number of functions which must always be carried out in data processing, **whether** done by manual, mechanical or electronic methods. 3. What is meant by the "logical abilities of computer"? These logical abilities include the determination of **whether** a number is zero, **whether** one number is larger than another, and **whether** a quantity is plus or minus. 4. The question is **whether** and how rapidly the Supercomputers will be exploited by business. 5. Laymen wonder **whether** Cybernetics models display, though imperfectly, a brain-like behaviour. 6. It is reasonable to ask **whether** Cybernetics is not a trivial and meaningless pursuit. 7. Since Cybernetics was born it has been variously defined, **whether** science or art. 8. It is difficult to say **whether** one branch of Artificial Intelligence is more important than another. 9. Enthusiasts claim that Artificial Intelligence study provides a solid foundation for other fields of learning. **Whether** this is actually the case is debatable. 10. Cybernetics' spectacular achievements are obvious **whether** in automatic steering for cars on motorways or automatic landing of the aircraft.

CONVERSATIONAL EXERCISES

I. *Substantiate the following statements with proofs or examples.*

1. **Fundamental maths** is a cornerstone of modern science and engineering. 2. It is one of the noblest activites of the intellect and certainly one of the most rigorous and challenging. 3. It is in the Universities that most of the fundamental work in maths is carried out. 4. New mathematical ideas are often developed with no thought of application. 5. There is the time lag between the invention of new mathematical tools and their applications. 6. Pure mathematical theories have repeatedly had immense scientific, technological and economic benefits. 7. In spite of the increased application of computers in maths most mathematical work is still done with pencil and paper or with chalk or blackboard. 8. Computer technologies could not have existed without the prior abstractions in maths. 9. The abstractions of maths are adopted by scientists to understand the patterns of nature. 10. Computer was developed by the mathematician J. Neumann and his colleagues chiefly for the purpose of solving mathematical problems. 11. They were in turn indebted to the mathematical logicians (A. Turing, A. Church; K. Gödel) for the framework in which they carried out their investigations. 12. The algorithms that are automated on the computer have similar mathematical roots. 13. They can be traced to systematic work in the art of computation and numerical analysis begun by mathematicians — Newton, Euler, Gauss. 14. Mathematical analysis continues to be essential to the development of algorithms. 15. Each year there emerges a new algorithm devised by mathematicians for solving problems in linear programming.

II. *Debate the role of modelling in Science. Supply some illustrations.*

One of the most promising current directions in fundamental maths is the continued development of **the mathematical model**. Mathematical modelling of natural phenomena is hardly new. The models of science are judged by their ability to explain the observed properties of the universe, whereas mathematical models are appreciated by mathematicians according to the validity and beauty. Nevertheless, advances in numerical analysis and the development of computers have made it possible to simulate processes in ways that are much more complex and more realistic than ever before. Mathematical modelling in partnership with the computer is rapidly becoming **a third** element of the scientific method, coequal with more traditional elements of **theory** and **experiments.**

III. *Agree or disagree with the following statements. Use the introductory phrases.*

That's right.	**I don't think so. This is not the case.**
Exactly. Certainly.	**It's wrong, I am afraid. Quite the reverse.**
I fully agree to it.	**The definition is inappropriate.**

1. There are many myths about Cybernetics and computers. 2. Computer stories and scientific fiction in periodicals come to the attention of the public at large. 3. They leave myriad paradoxical and misleading impressions. 4. Some people believe that a computer is a monster that will ultimately destroy us. 5. Others see in the powers of computers an Arabian genie that will create new wonders and a higher living standard for the human race. 6. Computers are mysterious, incomprehensible, stupid, frightening, slavish, many people claim. 7. Computing maths specialists are not abstract-minded persons, they are applied mathematicians. 8. Cybernetics is the theory of computers and communication processes. 9. Cybernetics is the means of studying the analogues existing between machines and living creatures. 10. Cybernetics is the philosophical doctrine that seeks to probe the ultimate mystery of life. 11. Cybernetics is the fantastic world of the future peopled by robots and electronic brains. 12. Cybernetics is the science of robots that allows to build machines with conditioned reflexes, which can learn and initiate life. 13. Cybernetics is the science which considers economy not as an economist, biology not as a biologist, engines not as an engineer. 14. Cybernetics is concerned with how systems regulate themselves, reproduce themselves, evolve, learn and organize themselves. 15. Cybernetics is the twentieth century Queen of sciences.

IV. *Say what Cybernetics is or rather what it is not. The following sentences may be helpful. Use the opening phrases.*

I'll start by saying that ...	**My point is that ...**
What I mean to say is ...	**It is common knowledge that ...**

1. Dealing with abstract and basically mathematical concepts, Cybernetics is essentially a mathematical branch of science. 2. It is no wonder that the main parts of Cybernetics, e. g., the general theory of algorithms have been developed in Mathematical Logic. 3. Cybernetics cannot, however, be reduced completely to mathematics. 4. Contrary to mathematics, Cybernetics employed both deductive and experimental methods.

5. Cybernetics has a field of investigation of its own, viz., abstract concepts of any form of transformers of a sufficiently general kind. 6. Cybernetics is concerned with complex systems, whose behaviour is beyond the description by analytic methods. 7. Cybernetics frequently resorts to simulating an object or a system on a computer. 8. Cybernetics deals with self-controlled systems. 9. Cybernetics is concerned with the General Laws of data processing in large complex control and information processing systems. 10. Among the most interesting objects involved in Cybernetics are abstract analogues of the human brain and models of thinking processes. 11. The Cybernetics Revolution is a great "leap forward" in the development of our civilization. 12. The Cybernetics Revolution is leading to a long-dreamed paradise on earth.

V. *Translate the text into English.*

Существует множество определений кибернетики, исходящих из основного положения о том, что это наука об управлении в системах различной природы (технических, биологических, экономических, общественных и т. д.). С течением времени круг систем, к которым считались применимыми кибернетические подходы, интенсивно расширяется. Уже можно говорить о нейрокибернетике, экономической, правовой, сельскохозяйственной кибернетике и т. д. За рубежом есть тенденция заменить термин «кибернетика» другими, например, «информатика», «теория систем», «компьютер сайенс» и др. Но ведь ясно, что кибернетика, вычислительная техника и математика — это не одно и то же. Они в их развитой современной форме — только база кибернетики, они слишком узки по сравнению с тем богатым содержанием, которое в настоящее время мы вкладываем в понятие «кибернетика». В разнообразных определениях понятия «кибернетика» подчеркиваются такие ее стороны, как системность, надежность, использование моделей и ЭВМ и пр. В различных источниках в качестве составляющих кибернетики указываются многие науки и научные направления, часто весьма слабо связанные друг с другом. Можно ли такой конгломерат относительно самостоятельных элементов считать наукой или научным направлением? Можно ли все эти направления, дисциплины, разделы других наук включать в понятие «кибернетика»? Видимо, ответ на эти вопросы может быть только отрицательным. Следует ли из этого вывод, что кибернетики не существует? Может быть кибернетика — это просто ставший модным синоним для современной теории автоматического регулирования и управления, теории, существующей и успешно развивающейся уже более 50-ти лет? Думается, что и на этот вопрос следует ответить отрицательно. **Кибернетика — это прежде всего научно-методологическое направление, рассматривающее весь объективно существующий мир с одной, а именно информационной, точки зрения.** При этом кибернетика намеренно отвлекается от материальной стороны мира и изучает только информационные процессы, т. е. получение, кодирование и обработку информации, в том числе ее передачу и использование. Информация в кибернетике понимается широко — это сведения, знания, факты — все то, что используется для принятия решений по управлению. Этим «кибернетическим мировоззрением» определяется значение кибернетических представлений, проникновение кибернетических подходов во все области современной науки и техники, во все формы человеческой деятельности. Именно в этом и проявилось в прошедшие годы основное значение кибернетики.

Кибернетика и ЭВМ. Одно из до сих пор встречающихся ошибочных мнений — отождествление проблем создания и использования

ЭВМ с кибернетикой. Многие считают, что если в какой-то области знаний применяются ЭВМ, то, следовательно, в этой области «используется кибернетика». В действительности это совсем не так. Использование информации и ее обработка не есть кибернетика, если при этом объектом изучения не являются сами процессы использования информации и методы ее обработки. ЭВМ, их возникновение и развитие действительно тесно связаны с кибернетикой. Во-первых, ЭВМ используется для переработки информации. При этом имеется в виду не техническое устройство ЭВМ — само по себе которое ничего не может делать, а ЭВМ вместе с заложенной в ней программой. Расширение областей применения ЭВМ тесно связано с кибернетикой и распространением ее идей и делает ЭВМ инструментом для кибернетических исследований. Во-вторых, сами ЭВМ, их логика, архитектура, программирование для них (исключая чисто технические вопросы их проектирования и изготовления) — объект рассмотрения с математико-кибернетической (информационной) точки зрения. Таким образом, ЭВМ выступают сразу в трех планах: как инструмент для вычислений и обработки информации; как средство для расширения областей применения кибернетических принципов и идей в разнообразных областях науки и практики; наконец, ЭВМ сами являются объектом приложения кибернетических идей и методов и используются для их совершенствования и развития. Именно это и привело к возникновению ошибочной тенденции относить все связанное с ЭВМ к кибернетике. Отличительная черта кибернетики — органическое соединение в ней теоретической и прикладной сторон. Кибернетика — это самая практическая из всех теоретических областей знаний. За короткий срок изменился подход к ЭВМ и возможностям их использования. Появились мини-, микро- и суперЭВМ. Сейчас не говорят о вычислительных машинах как таковых, говорят о системах, в которых объединены многие ЭВМ, зачастую расположенные не только в разных городах, но и в разных странах и на разных континентах. В такую систему — вычислительную сеть — входят несколько десятков вычислительных центров. Их суммарная вычислительная мощность может составить миллиарды операций в секунду. Помимо выполнения сложных расчетов, ЭВМ может служить средством решения информационных и управленческих задач. **Проблема кибернетики — это создание методов и средств, обеспечивающих высокую скорость сбора, передачи, обработки и выдачи данных в форме, удобной для работников производства и аппарата управления.** Однако без программы, которая переводит информацию на язык, понятный машине, ЭВМ мертва. Поэтому нужен программист, выполняющий роль посредника между окружающим ЭВМ миром и самой ЭВМ. Стоимость программного обеспечения ЭВМ в несколько раз превышает стоимость самой машины.

VI. *Computer Generations. Reproduce the text in English and speak about each generation in greater detail, specifying the equipment, innovations, computer provisions, etc.*

Computers that are able to do many problems concurrently — **transputers** — have numerous advantages. Are they being manufactured plentifully?

Появление ЭВМ — результат настойчивого поиска средств автоматизации вычислений. В прошлом компьютерные поколения отличались друг от друга оборудованием, дававшим им решающее преимущество перед их предшественниками. Сначала были электронные лампы (1946—1957), затем транзисторы (1958—1963), интегральные схемы

(1964—1975) и, наконец, интеграция в очень крупных масштабах «микроминиатюризация» (1975—1980). За последние сорок лет вычислительная техника достигла замечательных успехов главным образом благодаря быстрому и непрерывному совершенствованию элементной базы ЭВМ. Однако в последнее время это совершенствование замедлилось. Причина заключается в том, что скорость света накладывает абсолютный предел на быстроту переключения и передачи сигналов между блоками ЭВМ — и эта скорость оказывается недостаточной. Существует ряд задач таких, как метеорологические прогнозы или создание систем с «искусственным интеллектом», которые требуют гораздо больших скоростей обработки информации. В этих условиях естественный путь увеличений быстродействия ЭВМ — революционные перемены в их **архитектуре** и программном обеспечении. Сегодня (1986) в центре внимания специалистов находится разработка ЭВМ с параллельной, а не последовательной обработкой информации, когда множество вычислительных операций выполняется одновременно. Первые шаги в этом направлении уже делаются. Разрабатывается специальный микропроцессор — **транспьютер**, создан язык программирования Оккам для микропроцессорных систем, новые способы решения задач. В настоящее время специалисты ведут работы по созданию ЭВМ пятого поколения, способных решать по преимуществу логические задачи новыми методами. В этих «разумных» ЭВМ основным обрабатываемым элементом **будут знания, а не информация**. Их задача будет заключаться не столько в расчетах, сколько в том, чтобы делать заключения, обосновывать, строить гипотезы или учиться. К этим ЭВМ будут обращаться с просьбами типа «рассчитать достоверность или ошибочность таких-то предположений или сделать логические заключения».

VII. *Agree or disagree.*

1. An algorithm is a completely mental concept. 2. In principle any problem for which an algorithm can be devised can be solved mechanically. 3. If the problem is to be solved with the aid of computer an algorithm is indispensable. 4. Only those procedures that can be stated in the explicit and unambiguous form of an algorithm can be presented to a computer. 5. Instructions that are vague or that rely on intuition are unacceptable. 6. The algorithms always give a correct answer. 7. The algorithms often require an inordinate amount of a programmer's time. 8. Algorithm theory has not been designed and developed. 9. People do not deal with algorithms in their daily life. 10. In mathematics the task to prove the solvability or unsolvability of any problem requires a precise algorithm.

VIII. *In an age of computers algorithms for getting a specific output from a specific input are crucial. What search algorithm can you design for the given below problem? Draw a flowchart.*

The problem is to discover whether or not a certain word, x, appears in a table of words, stored in a computer's memory. The word x may be the name of person, the number of mechanical part, a word in some foreign language, a chemical compound or almost anything. Suppose n different words have been stored in the computer's memory. The problem is to design an algorithm that will accept as its input the word x and will yield as its output the location j where x appears. Thus, the output will be a number between 1 and n, if x is present; on the other hand, if x is not in the memory, the output should be 0, indicating that the search was unsuccessful.

IX. *What do we mean when we say*:

В математике часто встречаются задачи, решение которых не удается получить в виде формулы, связывающей искомые величины с заданными. О таких задачах говорят, что они не решаются в явном виде. Для их решения стремятся найти какой-нибудь бесконечный процесс, сходящийся к искомому ответу. Если такой процесс указан, то, выполняя некоторое число шагов и затем, по достижении заданной точности обрывая вычисления (их нельзя продолжать бесконечно!), мы получим приближенное решение задачи. Эта процедура связана с проведением вычислений по строго определенной системе правил, которая задается характером процесса и называется **алгоритмом**.

Алгоритм — это конечный набор правил для выполнения некоторой процедуры, удовлетворяющий трем основным требованиям: первое — массовость (universality). Это значит, что предписание должно обеспечивать выполнение не одной конкретной процедуры, а быть пригодным для реализации класса однородных процедур. Второе требование — детерминированность (determinancy). Это означает, что указания должны быть однозначно понимаемыми. И третье требование — результативность (terminateness). Это требование обеспечивает конечность применения указаний. Результат должен быть получен за конечное число шагов.

В математике понятие алгоритма используется очень широко. По существу, все математические процедуры, приводящие к решению задач, основаны на алгоритмах. Можно ли построить алгоритм, строящий необходимый алгоритм решения любой точно поставленной задачи? (Д. Гильберт). Ясно, что такой универсальный алгоритм существовать не может. Надо иметь точное математическое определение алгоритма, чтобы точно доказать это. Было придумано около двух десятков математических уточнений понятия «алгоритм». Затем было доказано, что все они эквивалентны между собой. Английский математик **Тьюринг** предложил уточнение понятия алгоритма, получившее название **«машины Тьюринга»**. Что это за машина? Другое уточнение этого понятия связано с именем выдающегося советского специалиста в области математической логики А. А. Маркова. Что значит **«нормальный алгоритм Маркова»**? Крупнейший советский математик **А. Н. Колмогоров** построил схему, лежащую в основе любого уточнения понятия «алгоритм», с помощью которой можно придумать много новых уточнений. Что это за схема?.

Алгоритмические процедуры — это образец четкости и ясности. Детерминированность и результативность регламентируют получение однозначности результатов при одинаковых исходных данных. Можно ослабить жесткие требования алгоритма, что позволит рассматривать более гибкие процедуры. Отказ от массовости ничего нового не дает. Отказ от детерминированности дает более гибкую процедуру, называемую **«вероятностным алгоритмом»** (probabilistic algorithm). Что это значит? Можно еще более ослабить жесткость задания процедуры и построить процедуры, похожие на алгоритмы, но не являющиеся ими — **квазиалгоритмы** (quasi-algorithms). Что это такое? Существуют квазиалгоритмы для описания процедур, присущих человеку. Это так называемые **«размытые алгоритмы»** (fuzzy algirithms). Что это значит?

X. *Express your views on the following statements trying to prove your point.*

1. Mathematics provides basic tools for much of computer science. 2. Knowledge of mathematics (especially calculus, functions, logic, sets) is presuposed in studying computer science. 3. Arithmetic has long been considered most natural for computers. 4. Addition is the major arithmetic operation performed by computer. 5. Next to arithmetic, algebra is the most widely used branch of mathematics. 6. The automatic solution of symbolic algebraic equations was one of the first computer applications. 7. Today computers perform high-speed, error-free algebraic calculations. 8. Since 1960 several non-numerical branches of mathematics yielded to computerization. 9. Versions of automatic theorem-proving systems have been developed for use in set theory. 10. Since 1962 computers could solve Integral Calculus problems. 11. Today computers use the working tools of Mathematical Logic. 12. The computer system may be engaged in solving differential equations.

XI. *Agree with the following statements, adding your own comments. Use the introductory phrases.*

That's right. There is no denying that ...
There is no point in disagreeing that ...

1. A computer is a combination of computer **hardware** and **software.** 2. The hardware of a computer consists of a central processing unit (a memory) and peripheral equipment (card-, tape-, paper-handling devices, sensors, effectors). 3. The peripheral devices enable a computer to interact directly with its physical environment. 4. The basic computer hardware can operate only upon numeric programme instructions. 5. Diverse tasks are done with the same collection of physical equipment — the hardware of the computer installation. 6. Computer technology is advancing at a rapid rate. 7. The software of a computer consists of sequences (strings) of instructions that may be expressed in a variety of programming languages. 8. Assembly Language is a convenient symbolic version of the machine language that is wired into a computer's circuitry. 9. Special translators called Compilers and Interpreters enable a computer to carry out programmes written in different problem-oriented languages. 10. Symbol-manipulation languages are expecially well-suited for programmes that process nonarithmetical data. 11. A special programme "Assembler" translates programmes from symbolic Assembly Language into the machine language. 12. The Assembler was not only the first step in the evolution of the programming languages, but a giant step in developing supercomputers of today.

XII. *Say whether the following statements are true or false. Justify your choice.*

1. Twenty years ago the major problem of computer science was to build a reliable computer (hardware). 2. The problem of programming — the design, construction and maintenance of the software — received very little attention. 3. Today in most computer installations programming costs have caught up with the hardware costs. 4. Experts predict that the value of the software typically will be doubled the value of the hardware. 5. The process of creating a large error-free programme is not today a slow, frustrating, largely trial-end-error activity. 6. Most computer programmes can never be thoroughly tested. 7. Programmes are designed to work with millions of possible inputs, only a fraction of which can be

tried out. 8. Generally, the programmer chooses a few test cases, makes the programme work properly on them and put it into general use. 9. When errors (bugs) turn up, the programmer must be called back for maintaining his programme. 10. Good progress is being made in the art of programming. 11. An outstanding feature of modern computers is the incorporation of elaborate error-detecting facilities. 12. New programmes are being developed to test and correct other programmes. 13. Another approach to the automatic programming is to create a programme that can develop other programmes. 14. If such a programme-writing programme is good enough, it produces absolutely correct programmes. 15. Automatic programme-testing and programme-writing programmes are still in their earliest development stages. 16. The difficulties of problem testing and correction have already disappeared. 17. Today's approach is to try to prove that a programme will do what it is supposed to.

XIII. *Discuss the following text.*

How to Debug a Code

We are going to discuss some general principles to follow in debugging a code; the same techniques apply regardless of the coding system used. It seems to be generally true that the computer routines do not run properly when tried the first time. In fact the name "gold-star code" is applied to those that do. Usually one or more things go wrong. The routine may "hang up" in an endless loop, there may be overflows, or the machine could stop from an impossible op-code.

The debugging problem is basically simple: given the symptoms of the trouble, how do we trace back to the errors? With a small machine we frequently do the job right at the machine (i. e., "console debugging"); with large machines such a procedure is out of the question because of the excessive costs. But in either case it is a reasoning process. We cannot tell you how to perform this reasoning but there are a few fundamental rules in the debugging check list worth considering.

1. Key the routine into storage with test data. In the early stages of debugging a code the test data need only be in the correct form; the values used are not too important.

2. Check to see that the arithmetic tables are in storage and are correct.

3. Cut a tape of the tables the routines keyed in, and the data. This tape is for insurance: you need not read it back at this time.

4. Set the console error switches to **stop** etc.

5. When you have discovered as many errors as your can, key the corrections into storage (if your routine has become garbled in storage, first reload the tape you cut at step). Immediately cut a new tape and **discard** the earlier one, etc.

6. Above all, think before acting ...

XIV. *Suppose that the statement seems insufficient and you want to add. Repeat the statement and add your own reasoning, thus deveoping the idea. Use the following phrases.*

There's one more thing to be noted ...	**Moreover ...**
I may as well add that ...	**More that that ...**

Models. 1. **A computer has essentially only three parts: a memory, an instruction processor and a data processor.**

... **but there's one more thing to be noted.** You've ommitted the input/output system that provides communication of the computer with the outside (external) world.

2. **A datum is anything that can be an operand.**

... **but I may as well add** that datum is anything that can be manipulated mathematically, and it includes previously computed results.

1. Memory holds both data and instructions. 2. Memory locations are referenced by unique addresses. 3. Instructions are coded patterns of bits that specify the operations to be done. 4. Instructions are organized into programmes, often called routines or codes. 5. The instruction set is the essence of the computer architecture. 6. Each instrustion can refer only to a small number of operands. 7. Fetching an instruction from memory and decoding it in the instruction processor takes several machine cycles. 8. Few computer programmes are simply linear sequences of instructions. 9. Computer programmes have loops or sequences of instructions to be repeated many times.

XV. *Confirm or deny.*

1. A single floating-point operation is the addition, subtraction, multiplication or division of two floating-point operands. 2. In modern computer designs the first three operations take about the same time. 3. Division is somewhat slower, but programmes can be written so that division is fairly rare. 4. Floating-point operations call for more computational work than the corresponding fixed-point, or integer operations. 5. "Floating-point" refers to the binary version of the well-known method for representing numbers called scientific notation. 6. Floating-point addition claims several steps that must be carried out in sequence. 7. A register is an electronic unit which holds information temporarily. 8. Floating-point-register in the IBM 360 computer systems is able to contain a double word. 9. Input is the term given to information from a source external to the computer into core storage. 10. Output is the term given to information sent from core storage to an external source. 11. The computer spends most of its time waiting for the completion of input-output operations, particularly printing.

XVI. *Observe the difference in meaning.*

Hardware — software — firmware

A fundamental issue in the design of any computer is how to control or steer the electrical signals that represent information. In the arithmetic and logic unit, where the actual processing of information is done, signals must be routed between various counters, adders and other components. The control system must also mediate the transfer of information between the central processor, the main memory units and the various input and output devices. In one approach the control system is completely "hard-wired", that is, it is laid down permanently in the processor's electrical circuitry. A second approach is more flexible and in many cases less expensive. The essential idea is to reduce the complexity of the control system by recording the detailed instructions for controlling the computer in a coded form. In other words, the sequence of paths is to follow is embodied in a programme, which is stored in a separate memory unit incorporated into the processor.

In the hierarchy of programmes that operate a computer the instructions executed by the control system occupy the lowest and most elementary level; each instruction specifies a **single** functional state of the machine. Because the control instructions are responsible for such fine details, the task of defining and encoding them is termed **microprogramming**, thereby distinguishing it from the writing of the higher-level programmes known generically as **software**. A set of control instructions — a microprogramme — is written in microcode. The idea of microprogramming was conceived more than 30 years ago soon after the advent of the first computers. At the time the **hardware** needed to implement the idea did not yet exist. The method has been adopted, however, in most computers that are being built today. Evolutionary successor of the minicomputer, the microcomputer, is a set of microelectronic "chips" serving the various computer functions. It has opened up new realms of computer applications. In recent years a good deal of confusion has arisen about the meaning of the term microprogramming, owing largely to the advent of the microprocessor (the computer on a chip) that is at the heart of the latest products of the progressive miniaturization of silicon-based semiconductor technology. It must be emphasized that microprogramming a computer is not the same as programming a microcomputer; in principle any computer, from the largest "mainframe" system to the smallest personal computer can be designed with a microprogrammed control system. To avoid such confusion microprogrammes are sometimes classified as "**firmware**", thereby signifying their intermediate status between hardware and software. In most modern computers the routing of information is controlled at the lowest level by a **microprogramme**, i. e., a set of stored instructions that function in place of a completely "hardwired" control system.

XVII. *Explain in detail in English what is meant by each type of computer provisions.*

1. Техническое обеспечение. 2. Математическое обеспечение. 3. Программное обеспечение. 4. Информационное обеспечение. 5. Лингвистическое обеспечение.

XVIII. *Make the right choice. Justify your choice.*

1. The volume and complexity of business and scientific data processing has been growing at a spectacular rate, and shows no signs of slackening. The solution of these problems is...

a) to hire more clerks; b) to improve the managerial control; c) to reduce the paperwork; d) to install computers; e) to perform data processing by supercomputers.

2. The term "data processing" encompasses the whole range of operations from input of data to output of results. It involves...

a) **data** origination, rearrangement and output; b) updating files; c) computing gross and net pay; d) economic forecasts; e) verifying data by comparison with facts already in files.

3. There are a number of functions which must always be carried out in **data processing** whether done by manual, mechanical or electronic methods. These functions are...

a) information must initially be recorded; b) information must be stored; c) information must be processed to put it in more useful form; d) the processed information must be made available to the ultimate users.

4. It is hard to conceive data processing problem without **files** which distinguish data processing from engineering or scientific computation. A **file** consists of ...

a) records; b) information; c) a set of accounts; d) stock items.

5. File information may be recorded on a great variety of media such as ...

a) perforated paper tapes; b) punched cards; c) photographic film; d) magnetic tape; e) magnetic discs; f) microelectronic circuits.

6. Information stored withing an electronic computer is ...

a) data to be processed; b) programming instructions; c) routines; d) Boolean algebra characters.

7. Computers with internally stored programmes operate in terms of simple elementary steps such as ...

a) select memory positions one by one to determine their operations; b) interpret the stored characters as a specified operation; c) convert the coded characters into a sequence of electrical signals.

8. The success or failure of any computer application depends upon:

a) the memory capacity; b) the speed of operations; c) the skill of a programmer; d) the perfection with which programmes (routines) are prepared.

XIX. *Explain in your own words the meaning and the correctness of the following statements. Mind, when talking about what computers can do, it is often appropriate to preface the claims with "To some extent..." as in most cases basic research is only becoming engineering practice.*

Computers can do the following tasks:

1. Computers free the researchers of lengthy and boring computations.
2. A computer perceives pure bookish knowledge and information more readily. A computer reads texts and summarizes them.
3. A computer reasons, converses, and even learn new words.
4. A computer thanslates from English into other languages and conversely.
5. A computer can do Geometric analogy intelligence tests.
6. A computer can do expert probem-solving and pattern recognition.
7. A computer can do useful industrial work.
8. A computer can model psychological processes.
9. A computer can understand drawings and make drawings itself.
10. A computer igeniously pursues scientific and engineering goals.

XX. *Complete and extend the given sentences.*

Model. **Computers can never ...**
Computers can never appreciate aesthetics. Computers can never possess consciousness, intuition, common sense, insight, ingenuity.

1. Computers can never do ... 2. Software can never equal brainware because ... 3. Intelligence can never be understood as ... 4. Computer can never model psychological processes precisely as ... 5. Computers can never play a champion chess game at a speed comparable to human because ... 6. Computers cannot produce idiomatic language translation as ... 7. Computers cannot carry on free and natural conver-

sations with humans as ... 8. Computers cannot help themselves in automatic programming in a proper way because ... 9. Computers can never attain the cultural level of humans because ...

XXI. *Dispute the USSR approach in computer-based sciences and production. The given statements may prove helpful.*

1. The modern computer is a very versatile and convenient machine that acts out at enormous variety of functional and structural roles. 2. The rate of introduction of computers affects the rate of economic development. 3. To lack a first-rate computer technology is to fall behind on a broad front of technological, economic and social progress. 4. The USSR is fully aware of the importance of computers and claims that the demand should be met from mostly domestic production. 5. The USSR is somewhat behind in numbers of computers in service, it is nevertheless using computers in virtually all fields of life. 6. Nowadays science and engineering are being computerized. 7. The inevitable microcomputers march to school and educational institutions has become apparent. 8. The first generation of the Russian computers the M-20 and the Ural-1 appeared in 1959. 9. In 1964 the USSR introduced Minsk-22, a small versatile scientific machine that soon became the most successful computer in eastern Europe. 10. The spectrum of computer Minsk-32 problems ranges from those related to everyday life to space explorations. 11. Most of the Russian computers at present are fourth generation units. 12. There is a considerable difference, however, in the Russian approach. 13. Business data processing in the USA, Western Europe and Japan is customar- or firm-oriented. 14. In the USSR the first priority is given to science and the needs of industry-computer management and production control. 15. The central planning network with its 1000 computer-centres is the apotheosis of the USSR approach. 16. A crucial step in computer-based sciences and engineering was made in 1972 with the decision of the USSR, Poland, Bulgaria, Hungary, Czechoslovakia and the German Democratic Republic to unite research and computer designing.

XXII. *Debate the given statements. It is advisable that the group be divided into two parties, each party advocating their viewpoint. Summarize orally the topic. Use the introductory phrases.*

As for me ... As concerns ...
As far as I am concerned ...
What I mean to say is ...

Summing up the discussion ...
In conclusion, I may say ...
To summarize the topic ...

1. The fruitful CMEA collaboration has resulted in the creation of a unique and conjoin system of manufacturing computers of the highest world's standards. 2. In the new computer models that are just appearing telecommunications, teletype, terminals and time-sharing capability have been designed and implemented. 3. Computer reliability is now taken for granted. 4. Current models compare favourably with the most powerful US machines. 5. Their precision is much higher, the size is smaller, productivity is greater and the guarantee is longer. 6. CMEA countries satisfy 70 per cent of their requirements in upgrading computer technology and tools designed jointly through reciprocal deliveries. 7. They intend to expand both trade with each other and mutually advantageous industrial cooperation. 8. Many capitalist firms readily cooperate with CMEA countries computer-tools producers. 9. Purchase negotiations with many West European firms are under way. 10. It is apparent that CMEA countries are able to compete with the West in decisive spheres of technical progress.

XXIII. *Agree with the following negative statements.*

Model. **No known programming language at present is satisfactory. No, it isn't.** You are right. There are now a considerable number of programming languages and they are by no means perfect.

1. As servants of man computers cannot understand or speak their masters' language. 2. Machine instructions cannot be written in natural spoken language. 3. Computers cannot work without precisely organized coded sequences of input data. 4. Computers cannot do what they please. 5. Not all computer programmes are simpy linear sequences of instructions, they have loops. 6. Each instruction cannot refer to a large number of operands. 7. Fetching an instruction from memory and decoding it in the instruction processor does not take one machine cycle, but several. 8. The programmer cannot always predict a computed result. 9. No signal can travel faster than the speed of light (=a foot per nanosecond). 10. The *n*th iteration cannot proceed at the same time as iteration $n+1$. 11. Many important large-scale problems cannot be organized into vector form efficiently. 12. Problems calling for much searching and sorting do not vectorize well.

XXIV. *Assess the latest innovations in manufacturing computer technology.*

The Next Step Beyond Transistors

An American scientist has developed a new microscopically small and fast-working electrical device that requires only 1 percent as much power as the most advanced semiconductors now being produced. The "quiteron" has signal-switching and amplifying characteristics. Like other superconducting circuits it works at extremely low temperatures (minus 273 degrees Celsius), eliminating the resistance that certain metals have to the flow of electricity. The quiteron works on a unique principle called "heavy quasi-particle injection-tunneling effect". Essentially, this means that a switching action takes place when an electrical voltage is passed through the device and changes the equilibrium of the circuit material.

IBM officials say that quiterons could eventually work as fast as 300 picoseconds (a picosecond is one-trillionth of a second) and measure a mere one-tenth of a micrometer (a micrometer is one-millionth of a meter). IBM research scientists say that it may be several years, however, before quiterons will be used for more than experimental applications.

A Superchip with Twice the Memory

Although the latest generation of superchips — 256K-bit memory circuits that can store more than a quarter-million individual pieces of computer data — are not even in use yet, a new experimental chip with nearly twice the capacity has been developed. Engineers at IBM's Essex Junction, Vt., laboratory have built the first batch of 512K-bit memory chips: 3/8-inch-square computer components that can store more than a half-million bits of electronic information.

Manufacturers are awaiting samples of the 256K-bit superchips this year so that they can begin making new computers with increased speed and power. The 512K-bit chips will not be commercially available for a number of years, but they should eventually account for an even more advanced and versatile breed of computers in the fields of robotics, communications and artificial intelligence. Described at an engineering conference, the 512K-bit memory chip has circuit patterns as small as 1.5 micrometers — about 1/50 th the diameter of a human hair. The chip performs reliably, the IBM engineers say, because of a technique called "plate pushing" that produces stronger electrical signals from the individual memory cells.

XXV. *Discuss the following statements. The opening phrases may come in handy.*

I have every reason to believe that...
The same problems arise when...
In view of all this...
I hold a similar view...
What is missing in the statement is...
Much depends on who (when, what, how)...
It will be seen that...
It is a well-known fact that...
You are free to disagree with me but...
What I mean to say is...
My point is that...
By specialists' criteria...

1. Each of the two supercomputers — the Cray-I and the Cyber-205 is (or was) superior for a particular class of scientific problems. 2. Supercomputer manufacturers estimated at 100 to 200 machines with enhanced capabilities over the next five years. 3. By 1988, if not before, the US and Japan may also demonstrate better supercomputers. 4. Performance levels above 100 megaflops claimed by the manufacturers of supercomputers are always peak values, attainable only in short bursts and under occasional conditions in actual practice. 5. Nothing happens is less than one cycle time or clock period. The computer designer tries to make the clock period as short as possible. 6. Cycle times much shorter than one nanosecond can be achieved with superconductor technology that is now being developed. 7. The cycle times of supercomputers are roughly proportional to their linear dimensions. 8. Signal-propagation speed is the limiting factor. 9. The parallel computation can be programmed for vector processing. 10. The progress toward making computers more intelligent and smarter has been slow.

XXVI. *Choose the definition of Artificial Intelligence which, to your mind, is the correct one. Justify your choice.*

1. A. I. — is a new type of psychology.
2. A. I. — is a science of robots.
3. A. I. — is an experimental science which employs computer as a means of modelling to perceive the nature of the human thinking.
4. A. I. — is a science that designs machines to make what a man considers intellectual when he is making the same.
5. A. I. — is the study of an electronic machine which is as general-purpose an object as a man is.
6. A. I. — is the science that is hard to understand as some knowledge of a great many fields is called for.
7. A. I. — is the area concerned with programming computers to behave in an intelligent way.
8. A. I. — is the field of computer science with many divisions and subdivisions, the most important of them are: problem-solving; game playing; language translation; pattern recognition; heuristic search theory; robotics; heuristic scene analysis; image processing.

9. A. I. — is the science whose chief application is industrial automation.

10. A. I. — is the science that asserts the essential unity of the animate and inanimate.

XXVII. *Disagree with the following statements.*

Model. **"Computer thinking"** is becoming a common phrase.
Computer hardware and human brainware are similar.
This is not the case. Although they are both general-purpose, symbol-manipulating devices they are not similar. Whether the human brain operates like a digital computer is a question that can be settled by neurophysiology. The human brain cells and neurons notably do not operate the terms of maths equations.

1. Man is a Turing machine. 2. The process of "thinking" is no longer regarded as mysterious. 3. The term "The black box" related to the human brain is inappropriate. 4. Man is an information-processing system functioning like a heuristically programmed digital computer. 5. One is justified in using computer models exclusively in psychology. 6. Within ten years psychological theories will take the form of computer programmes. 7. Computers possess subconscious faculties. 8. Human and mechanical information processing ultimately involve the same elementary processes. 9. The computer can be programmed to execute information processing quite like the human brain. 10. The human intelligence must be understood as processing information according to formalized heuristic rules. 11. Any form of information whatever can be processed by a digital computer. 12. The computer models explain what people actually do when they think and perceive. 13. There must be a digital way of performing human tasks. 14. Human intelligent behaviour is the result of information processing by a computer. 15. Human behaviour is described in a formalism which can be manipulated by a digital computer. 16. In "processing information" a human being actually follows formal rules like a digital computer. 17. People can make machines as intelligent as themselves within a few years.

XXVIII. *Express your personal views.*

Learning to Love Robots

Collins English Dictionary tells us that a robot is "any automated machine programmed to perform specific functions in the manner of a man". Usually our first response to the word "robot" is to picture a so-called tin man, who looks and behaves much like a human being. So at first sight this definition seems to be straighforward. However, by now most of us have built up a general idea of a computer, and seen if not used one. And in the real world, the first generation of usefully practical robots have computers to control their actions — these are industrial robots. But although we can now accept the idea of a robot as a machine, with little or no resemblance to a human being, we still find the fictional versions in our minds far more interesting and exciting.

Artificial intelligence is a fascinating concept, and it is not surprising that we still indulge our imagination with the fictional version of robots by reading about them and watching them on the screen. The majority of these are still human-like to look at, though there are others, more like mobile calculating machines and information banks. We also come across computer "brains" which have full control over space stations, spaceships, etc., and so do not have a "body" as such. The problems of actually constructing a "typical" fictional robot, though, are enormous, and at present our industrial robots are relatively "stupid". Their ability to interpret information is very limited.

The majority have no sight or hearing (and in those that do it is extremely limited), and as soon as anything crops up that deviates from their set programme in any way, the robot cannot cope. It will either stop altogether or continue working away regardless of whether, for example, the components have run out. This means also that a robot could be dangerous to humans — "because it cannot see anyone walking by it at the wrong moment. It could easily knock out or even kill a careless passerby. So most robots are fenced off behind security gates". All of which indicates that in future we will have to design robots to be more sensitive to variations on their set programmes. And robotics research is being carried out to that end.

XXIX. *Express doubt in responce to the statements given below. Use the introductory phrases.*

Model. **"An automaton can do whatever we know how to order it to perform"**. Ch. Babbage.
I doubt that an automaton can do only whatever it is ordered to. A current computer can do something of its own.

1. Today's robots can pass Turing's test satisfactorily. 2. A robot can be programmed of any actions, functions, processes and tasks. 3. It's possible to design a robot which makes its own decisions. 4. A robot with a large storage capacity is capable of carrying out functions exceedingly greater than those of a human being. 5. The robot's ability has no limits. 6. Robots have been built that can match a postoffice sorter in reading hand-written addresses. 7. Robots have been designed for the functions which take place subconsciously. 8. A robot works at a higher speed if its signal transformer is not complicated. 9. A robot works more intelligenty if its store is more accessible. 10. A robot is capable of idiomatic language translation if its storage capacity is large.

XXX. *Dispute the problem of engineering locomotive applications of computers.*

Machines That Walk

Many machines imitate nature; a familiar example is the imitation of a soaring bird by the airplane. One form of animal locomotion that has resisted imitation is **walking.** Can it be that modern computers and feedback control systems make it possible to build machines that walk? So far only two machines have been built. One has six legs and a human driver; its purpose is to explore the kind of locomotion displayed by insects, which does not demand attention to the problem of balance. The other machine has only one leg and moves by hopping; it serves to explore the problems of balance. The first kind of locomotion is called **crawling** to distinguish it from walking and running, which involve periods of flight as well. This research has helped to understand how people and other animals crawl, walk and run.

Unlike a wheel, which changes its point of support continuously and gradually while bearing weight, a leg changes its point of support all at once and must be unloaded to do so. In order for a legged system to crawl, walk or run, each leg must go through periods when it carries load and keeps its foot fixed on the ground and other periods when it is unloaded and its foot is free to move. This type of cyclic alternation between a loaded phase, called **stance**, and a unloaded phase, called **transfer**, is found in every form of legged system.

At present the research is being carried out that may eventually lead to the development of machines that crawl, walk and run in terrain where softness or bumpiness makes wheeled or tracked vehicles ineffective and thus may lead to useful industrial, agricultural and military applications. The advantage of legged vehicles in difficult terrain is that they can choose footholds to improve traction, to minimize lurching and to step over obstacles. In principle, the performance of legged vehicles can be to a great extent independent of the detailed roughness of the ground. It is clear that very sophisticated computer-control programmes will be an important component of machines that smoothly crawl, walk or run. Though locomotion on legs resists imitation, modern control technology should be able to solve the problem. Experiments with machines that hop and crawl has already illuminated the mechanisms of natural walking.

XXXI. *Express regret, reproach, obligation, advisability, desirability in the following statements using Modal Verbs (can, could, may, might, must, should, ought to, need) + the Perfect Infinitive to refer the actions to the Past.*

Models. Babbage's analytic engine was often called "Babbage's Folly".
It should have been called more accurately "Babbage's Vision".
A computer is nothing but a big fast arithmetic machine.
One ought to have said a computer is a big fast, general-purpose-symbol-manipulating machine.
Robotics **may (might) have been termed** as one of the promising branches of Artificial Intelligence.

1. Computers are not as intelligent as human beings. 2. Modern computers are much faster, more complex, multifunctional and useful than most people dreamed 50 years ago. 3. Thinking machines affect modern philosophical and psychological concepts. 4. Supercomputers display human intelligence. 5. Turing's test is the basic paradigm for determining the accuracy of computer simulation of human intelligence. 6. The validity of Intelligence Quotient tests is disputable.

XXXII. *Add the opening phrase, repeat the statement and keep the conversation going.*

It is not meant to imply that...	**The statement does not imply...**
I don't mean to say...	**Scientists do not claim...**
Mathematicians reject...	**It is too much to say that...**

1. Current supercomputers are smart. Smart computers are flexible, decision-making, problem-solving, image-processing and perceiving. 2. To relieve the supercomputers of managing large volumes of input-output traffic, most of them are equipped with front-end processors. 3. The front-end processors act as appointment secretary to the migthy supercomputer. 4. A supercomputer of 1982 is a collection of general-purpose processors where each processor is capable of executing tasks independently of the others. 5. Current supercomputers show a trend toward increased multiprocessing. 6. Computers that can handle multiple instruction streams as well as multiple data streams (M. I. M. D.) have been dis-

cussed for many years. The first models are being tried out. 7. The first superclass of M.I.M.D. computers will be fully operational within the next two years. 8. Nowadays it is widely recognized that computers can be exploited to design better computers. 9. Supercomputers can understand and process scientific fexts. 10. They are Artificial Intelligence functioning in real practice today.

XXXIII. *Agree or disagree.*

1. Diffferent levels of human intellect were being modelled. 2. Researchers neglected at first formal aspects of thinking. 3. For a long time mathematicians did not try to treat matters of perception mathematically. 4. A worker in Artificial Intelligence seeks to design sophisticated information-processing machines that parallel human intellectual behaviour or brain function. 5. The design of current A.I. parallels the structure of the human brain. 6. Robots — mechanical intelligence capable of operating in our own real-world environment — are widely employed in science and engineering. 7. There has been some success in Robotics.

XXXIV. *Say it in English.*

Возможно ли создание «искусственного разума»?

Обычные вычислительные машины — это всего лишь счетные устройства, быстродействующие, но абсолютно неразумные. Вся их программа содержит лишь список команд, которые они безошибочно выполняют. В некоторых научно-исследовательских центрах уже имеются другие вычислительные машины, внешне очень похожие на прежние, но в них заложены более сложные программы. Они начиняют машину информацией и учат ее «мыслить». Такие машины, наделенные «разумом», постепенно смогут имитировать многие наши способности, а в некоторых случаях даже превзойти их. В скором времени, возможно, это будут роботы, которые начнут рассуждать, понимать, приобретут способности учиться, а после этого попытаются изменить наши представления о жизни и даже о самих себе. Такие машины не хранят в своем запоминающем устройстве заготовленные и стереотипные фразы. Машина сама формирует ответы, аргументирует, «размышляет» и в этом она в какой-то степени уподобляется человеческому существу.

Машина «мыслит» своим особым способом. Это одно из недавних достижений науки получило название **«искусственный разум»**. Опираясь на психологию и информатику, ученые поставили перед собой по меньшей мере честолюбивую задачу: изучить образ мышления человека и его поведение, чтобы затем воспроизвести их искусственно. Некоторые ученые говорят: «В информатике вот-вот произойдет настоящая революция, начинается век машин, наделенных «разумом». Исследователи во всем мире занимаются этой проблемой вот уже в течение 25 лет. Во время второй мировой войны английский математик **Алан Тьюринг** изобрел машину-прародительницу современных вычислительных машин. Он мечтал о создании такой машины, которая способна учиться и стать разумной. То, что сейчас ошибочно называют первым «искусственным мозгом», родилось в проектах другого известного математика, **Дж. фон Неймана**. Усилия многих ученых и специалистов в настоящее время направлены на создание нового поколения ЭВМ, которые способны выполнять «мыслительные» функции, до сих пор доступные лишь человеку.

За время, прошедшее после первой международной конференции по проблемам искусственного интеллекта (США, 1969 г.), сделан огромный шаг вперед в слиянии разнородных исследований в этой области в единое научное направление. Одновременно специалисты, занимающиеся интеллектуальными системами, сумели исключить ряд направлений из сферы своих интересов. Сегодня искусственный интеллект уже может быть назван наукой. У этого направления исследований есть свой специфический предмет исследований — изучение психики и поведения человека с целью имитации их в технических средствах; свой научный язык и свои методы решения проблем.

XXXV. *Reproduce the text in English. Confirm or deny the forecasts concerned.*

Прогнозы

Всем известно, что научный прогресс столь стремителен, что всякий прогноз оказывается весьма неустойчивым и ненадежным. Однако можно назвать несколько цифр, с которыми более или менее согласны все крупные специалисты, работающие в области теории искусственного интеллекта и роботостроения.

К концу 80-х годов в вычислительной технике наряду с привычными ЭВМ большое место займут различные вычислительные системы, структура которых позволит организовать параллельное и асинхронное течение процессов. Такие системы очень перспективны для робототехники. Именно к этому времени роботы третьего поколения начнут проникать во все сферы человеческой деятельности; они будут уже способны к групповой деятельности и сами синтезировать речевые сообщения. Появятся роботы-экспериментаторы, роботы-исследователи, роботы-домохозяйки. Будут внедрены ЭВМ, в которых вместо электрических сигналов начнут использоваться световые сигналы. Скорость работы элементов таких ЭВМ приблизится к скорости света.

К концу 90-х годов. Благодаря оптоэлектроники ЭВМ смогут передавать черно-белые картины, подобные тем, которые мы видим на экранах телевизоров. Разрабатываются основы специальной картинной логики, позволяющей обрабатывать широкую информацию. Голография дает уникальную возможность хранить в памяти ЭВМ информацию о двумерных и трехмерных изображениях. Роботы, в которых одновременно действуют две системы распределенных вычислительных ЭВМ — традиционная и оптоэлектронная — получат основу для реализации метапроцедур понимания, общения и чувственного представления.

К 2000-му году человечество столкнется с популяцией разнообразных роботов и других систем, интеллектуальный уровень которых будет в сотни раз выше уровня современного промышленного робота. Эти роботы будут способны к полноценному общению в заданной проблемной области, смогут хранить об этой области знаний информацию, не меньшую чем специалист-человек, и выполнять, целесообразно планируя, все процедуры решения задач в этой области.

COMPOSITION

I. *Reconstruct the text, segmenting it into paragraphs. Translate your version of the text. Write 5 sentences-long abstract.*

The Field Computation

The Cray-1 and the Cyber-205 can perform 100 million arithmetic operations per second. This kind of "number crunching" is needed for the solution of complex problems such as those in fluid dynamics. Since every region of space contains infinitely many points, the complete description of a physical field calls for an infinite amount of data. Hence the first step in devising an approximate numerical method of computation is to **retreat** to a discrete description of the field by introducing a grid of finitely many points distributed throughout the region. Field values are attached only to the grid points. Mathematically speaking a system of partial differential equations is replaced by a large system of ordinary algebraic equations. The number of grid points needed depends in part on the number of spatial dimensions in the model, the complexity of the geometry of the region and the amount of spatial detail sought in result. Three-dimensional grids with as many as a million points have been needed in order to simulate turbulent aerodynamic flow in fairly simple geometries, and tens of millons of points will be needed in order to obtain solutions sufficiently detailed for engineering purposes in complex geometries and as the region surrounding an entire aircraft. For each grid point several numerical quantities must be stored in the computer memory and periodically updated in the course of the computation: the quantity for each physical field variable, perhaps several quantities to describe the geometry and often several intermediate computed results. The number of stored quantities per grid point in current supercomputer programmes for three-dimensional simulations of aerodynamic flow ranges from 5 to 30. The field computation proceeds through many iterations of a basic step in which new values for the field variables at every grid point are computed from the old values at the point and at neighbouring points. In a typical steady-state problem the iterated steps yield successively better approximations of the exact solution. The computation is usually launched by making some reasonable guess about the initial field conditions at each grid point. The computer then applies the governing differential equations at each step to introduce the corrections necessary to bring the current field description closer and closer to a true solution. The number of iterations needed for convergence to a sufficiently accurate approximation of the true solution varies strongly with the numerical method and can range from several hundred to several thousand. The overall effect is to increase the number of iterations needed to arrive at a useful answer.

II. *Write a composition on the topic "Robotics — fruitful field of Artificial Intelligence". The following statements may serve as the items of your plan. Add some more current information, evidence, proofs, illustrative examples, etc., thus developing and extending the main ideas. Your composition must be 3 pages long.*

Robotics

1. Robots have always fascinated humanity. 2. A robot is a mechanical intelligence capable of operating in our own real-world environment. 3. The successful construction of a robot entails the integration of most techniques developed in Cybernetics. 4. A robot must have a) some sort of sensing and perception system that allows it to detect pattern examples in the environment; b) a way of reasoning about its environment; c) a way of acting upon its environment. 5. All Artificial

Intelligence researchers have been so far only partially successful in giving computers sensing, perceiving and reasoning abilities. Still, there has been some success. 6. Research on robots is currently being undertaken in many countries. 7. All computer programmes can be looked on as part of the brain of a robot. 8. Modern robot studies follow along two general lines: scientific and commercial. 9. The robot sensing and perception systems so far investigated have been primarily **vision** and **touch**, with some attention to **hearing.** 10. The study of robots — Robotics — is arbitrarily defined as the actual building of automated devices. 11. The robot problem is a specialization of problem solving, pattern recognition and vision. 12. The only robot systems for acting on the environment yet developed are **mechanical hands** (and arms) and **locomotion systems**. 13. There is no robot that successfully combines all the system units performance. 14. The U. S. Stanford University robot had its brain in a computer, its ears in a conventional microphone, its single hand on a fixed shaft and its cyclops eye in a TV camera. One word for such a system was "robot" (1968). 15. What commercial robot designers are attempting to do is to produce easily programmable general-purpose object manipulators. 16. Current commercially made robots are fairly general devices. 17. There are now in existence machines which move about under control of radio signals from a computer. 18. They peer at their visual world through cyclopean TV camera eye, analyze the scene. 19. They see and reason about the path they should take to move from one point to another. 20. The robot's visual perception is based on analysis of discrete ones from the visual scene. 21. There is no attempt to develop a representation of the matrix of three-dimensional space. 22. Metric representations are extremely important if the machine's perceptions of and reasoning about space are to approach remotely human capability. 23. The robot's world is limited. 24. Robots are still "effectively blind" when compared to humans. 25. Robots are unable to "see" moving objects, even when they are moving quite slowly. 26. Robots can perform only relatively trivial tasks. 28. Several scientists have claimed that it is possible to place some sort of complex automated explorer on the moon or a planet in the near future. 29. Indeed, some quite complex devices have already been placed there. 30. Whether they are robots or not depends on one's definition of the term. 31. Psychological and physiological models have made a marked impact on robot design. 32. Many experts in Artificial Intelligence claim that robot projects provide a unifying theme for all aspects of the science A. I. 33. Fascination with robot building may have diverted attention from duller, but in the long run, more solid scientific problems.

COMPREHENSION EXERCISES

Questions

1. What can (can's) computers do? 2. Are computers slaves or masters? Who is really giving orders: man or machine? 3. Can we tell a computer how to learn, to create, to invent, etc.? 4. How can a machine be made to decide? 5. How can one determine whether or not a problem can be solved by a computer? 6. Why is the road from the conception of a programme to its execution by the computer so long and tiresome? Why is the software so costly and unsatisfactory? 7. What are we really trying to do in the process of communication? 8. What human abilities are irreplaceable by a computer? 9. Why is the number of possible com-

puter designs limitless? 10. Is there any limit to computer speed? 11. What enables modern computers to operate in millions of a second? 12. How can current computers be applied to the problem of computer science itself? 13. What is the role of the programmer in the transition of meaningful information to the strings of meaningless bits (information in the computer sense) with which a computer operates? 14. What results can be achieved by performing a sharply defined sequence of sharply defined actions? 15. Can a programmer be dispensed with in the modern supercomputers programming? 16. What is known about natural intelligence? 17. When can we justifiably call a machine intelligent? 18. How and to what extent do computers currently simulate intelligence or display intelligent behaviour? 19. How can machines eventually simulate intelligence? 20. How can machines and their behaviour be described mathematically? 21. Has the ambition of A. I. researchers to programme the computer to do the translating job itself been realized? 22. What uses can be made of intelligent machines? 23. Can a computer attain the cultural level of a child? 24. Why has this question divided all specialists into two "for" and "against" camps? 25. Does humour play any role in human thinking? 26. Can a computer be programmed to produce humour? 27. Are files containing personal information within easy access? 28. Is information about people's private lives in the US increasingly vulnerable to interception and misuse? 29. Is it because of the spread of computers as record keepers? 30. Are job applicants forced to take lie-detector tests in the USSR? 31. Will anyone's privacy be protected in future?

Discussion

1. The history of computation and computing devices: finger reckoning → the abacus → early computers → current microcomputers and supercomputers.

2. The rise of computers has been called "The Second Industrial Revolution", but it is far more than that. Explain.

3. Man and computer are able of accomplishing things that neither of them can do. Prove it.

4. The difference between Cybernetics in theory and in practice.

5. **N. Weiner's** — the father of Cybernetics — definition of the science "Cybernetics Is Control and Communication in the Animal and the Machine" (1948) is still generally accepted despite the appearance of other definitions of various lengths and complexity. Why?

6. Concentration of large amount of circuitry into a small volume to minimize the length of wires creates a serious problem: the removal of the waste heat generated by electrical energy conversions. How is this problem being solved?

7. Speed with which signals can be propagated or transferred from one part of the computer to another is an important feature of a computer. Why?

8. Digital circuits are subject to errors caused by the failure of the computer components and by random electrical noise. What should be done to reduce error-rates to your mind?

9. To err is human. Even when people do try, they make mistakes. It is impossible to be always perfectly accurate. If your pride cannot reconcile with this fact, you will never make a good programmer. Agree or disagree.

10. Communications between a computer and the outside world referred to as input-output (I/O) operations are very slow in relation to the computer's internal processing speeds because peripheral devices have mechanical components and human response time is slow. Are any modifications in this area possible?

11. Much effort is currently being given to the reexamination of algorithms for large computational problems aiming to maximize the number of operations that can be carried out concurrently in the new generation of supercomputers and with the help of new programming languages. Have any advance and progress been achieved in this field of research?

12. "The language of the brain is not the language of mathematics" **J. V. Neumann.** Discuss the difference between the mathematical mind, the perception mind, the computer mind.

13. "Why does Cybernetics — this magnificent applied science which saves work and makes life easier bring to us so little happiness? The simple answer runs: Because we have not yet learned to make sensible use of it" (**A. Einstein**). Agree or disagree.

14. Computers have permeated our society. No educational institution, research lab., large bank, insurance company can survive today without using computers. Is it really the case?

15. The mastery of handling a calculator, personal computer, sophisticated computer techiques of calculation ought to become one more compulsory educational task for every person nowadays. Universal **computer science competence and skillfulness** (компьютерная грамотность) is the nearest future reality. Agree or disagree.

16. The power of computers can be used for good and evil. Illustrate both. Computers, cable television and electronic-banking devices which make life .easier for many persons also help intruders gain access to private data. US Federal bureaucrats maintain a mass of dossiers and other files on citizens to track their behaviour and conduct. Amateurs, too, use equipment sold without restriction to bug a neighbour with tiny microphones and transmitters. The world in which all statements are "on the record" is frightening to contemplate. Your viewpoint on the relentless march of technology and threats to privacy in the United States.

17. "We may hope that machines will eventually compete with man in all purely intellectual fields" (A. M. Turing, 1950). Was Turing right?

18. **C. E. Shannon,** the inventor of Information Theory said: "Efficient machines for such problems as pattern recognition, idiomatic language translation, etc., may require a different type of computer that any have today. It is my feeling that this will be a computer whose natural operation is in terms of patterns, concepts, and vague similarities rather than sequential operations on ten-digit numbers". Your viewpoint.

19. **The software triad**: MATHEMATICAL MODEL — ALGORITHM — PROGRAMME. The significance of each component and of the three taken together.

Mathematical modelling. Computing experiment. What do these phrases mean? How can one construct a mathematical model to represent an object (system, process, phenomenon) under study? Scientific experimenting on the computer — is it a dream or today's practice?

It was in the Soviet Union that the first mathematical model of "nuclear winter" was designed and computed. **The integration** of mathematical modelling — designing — computing experimenting. Its aims and effects in science and industrial production.

20. **The contribution of Cybernetics to:** a) **Modification** of human mentality and scientific thinking; b) **Accelaration** of scientific and technological progress; c) **Modernisation** and updating of industry; d) **Restructure** of the management system; e) **Crash Changes** in economy to double productive output; f) **Implementation** of large-scale integrated programmes in the strategic areas; g) **Application** of intensive technologies in agriculture; h) **Advance** in socio-economic development of the Soviet society; i) All-embracing **international security system** building.

21. The first world conference in Artificial Intelligence was held in 1956. All the participants agreed to adopt the term "A. I." to qualify their field of research. They disagreed on the means and routes to attain their cherished goals. The scope and limits of A. I. research.

22. The basic problem facing workers attempting to use computers in the simulation of human intelligent behaviour is now clear: all alternatives must be made explicit. Is it possible?

23. Many ambitious research projects were launched with the goal of clearly demonstrating the learning capabilities of computers so that they could translate idiomatically, carry on free and natural conversations with humans, recognize speech and print it out, diagnoze diseases, etc. All these activities demand human qualities. What are the current achievements in all these fields?

24. The field of A. I. exhibits a recurring pattern: early dramatic success followed by sudden unexpected difficulties and failures. This pattern occurs in all basic areas of A. I. (problem-solving, game-playing, language-translation and pattern-recognition) in two phases, each lasting roughly five years. The reasons, to your mind?

25. In spite of grave difficulties and failures A. I. enthusiasts are not discouraged, in fact they are optimistic. They claim: "Computers will be able, within ten years of doing any work that a man can do". There must be a reason why they continue dogmatically to assert their faith in the progress. Your viewpoint.

26. The formalization of intelligent human behaviour must be possible as well as computers capable of communicating with people in common language. Agree or disagree.

27. Man is building more and more powerful machines but remains their slave as he has to control them. Coordination will be left to the machine itself in the future. Such predictions may belong to the realm of science fiction, or are these claims possible and realizable in practice?

28. The Third Industrial Revolution looms on the horizon. It will be brought about by the development of machines free from human control and its limitations, and capable of thinking for themselves, and organizing themselves into autonomous breeding units. When will this happen?

LESSON NINE

INTRODUCTION TO SET THEORY AND THE FOUNDATIONS OF MATHEMATICS

Grammar:
1. Moods.
2. Inversion.

LAB. PRACTICE

Repeat the given sentences after the instructors.

a) 1. **The foundations of mathematics** is a special branch of mathematics where fundamental mathematical concepts (such as number, function, algorithm, etc.) have been precisely defined and mathematical sciences made more rigorous, refined and sophisticated. 2. It is common knowledge that the greatest mathematicians and philosophers from Greek times on had been plagued with problems involving **infinite quantities.** 3. The subject of infinity has been difficult and disturbant; Georg Cantor made the first extensive assault upon it that could be called successful, although others (Galileo, for example) had had glimmerings of his ideas before. 4. Cantor was led to the study of transfinite numbers by his research in connection with functions and real numbers. 5. Cantor dealt with the concept of infinity with reference to **transfinite cardinal** as well as **ordinal** numbers. 6. His work provided a new foundation for much of mathematics and stirred up many controversies which are not yet settled. 7. The controversy over the use of infinite processes resulted in the formation of various schools of thought concerning the foundations of mathematics. 8. Research in the structural (topological) aspect of the real number system late in the XIX century gave rise to the mathematical concept of a "set". 9. It is the merit of Cantor who had extended his survey on Number Continuum to **infinite sets;** in doing so he founded a new branch of mathematics — the theory of sets. 10. In Cantor's set theory the sequence of natural numbers becomes an **actual infinite set of all natural numbers designated** by $\aleph_0$ (aleph-null) (=the letter *A* in the Hebrew alphabet) — the smallest transfinite number. 11. Dedekind, Frege and Cantor thought that they had established the foundations of mathematics satisfying the rigorous and exact logic. This was not the case, however. 12. The foundations of mathematics have undergone three crises; none of them easily or quickly settled. 13. The first crisis occurred in the 500 B. C. with the Pythagoreans' discovery of incommensurables. 14. This crisis was resolved in about 270 B. C. by Eudoxus's revised theory of magnitude and proportion. 15. The second crisis followed the creation of infinitesimal calculus by Newton and Leibnitz in the XVII c. 16. The second crisis had been overcome in the XIX c. thanks to Cauchy's precise method of limits and the subsequent arithmetization of analysis by Weierstrass. 17. The third crisis began in the latter part of the XIX c. with the revelation of the paradoxes in Cantor's set theory. 18. The paradoxes motivated the development of **axiomatic** and **abstract** set theories as well as intensive research in the foundations of mathematics. 19. No approach (intuitionism, formalism, logicism) has been completely successful in answering the fundamental questions concerning foundations. 20. What at first looked like security on closer inspection was only an illusion and failure. 21. The rigorous and self-consistent foundations of mathematics have yet to be built. 22. Despite the paradoxes and inconsistencies, **it is Cantor's intuitive set theory that holds the key position in modern mathematics.** 23. We ought now to recognize Cantorian set theory with **the Continuum Hypothesis** as one of its axioms and also various non-Cantorian set theories with other axioms substituting the Continuum Hypothesis. 24. **The Axiom of Choice** is generally accepted as one of the basic principles of mathematical reasoning. 25. Almost all modern mathematical theories are developed on set-theoretical basis and the Axiom of Choice.

b) 1. **Should** the problem of infinite quantities have no importance, both philosophers and mathematicians **would not have paid** so much attention to it. 2. It is not strange that the problem **(should) have originated** in Ancient Greece. 3. Mystical as **was** the nature of much of the Pythagoreans' study, they created a good deal of sound mathematics. 4. Of special importance **was** the Pythagoreans' discovery of incommensurables (irrationals). 5. Zeno of Elea **might be called** the forerunner of the concept of mathematical infinity. 6. **Only** by devising a good number of colourful and subtle puzzles of philosophical interest **did** Zeno manage to reveal the difficulties inherent in the concepts of number continuum, length, time, motion and infinite quantities. 7. It is essential (that) one **should remark** that we have no extant writings of Zeno and it is even possible that he **(might) have written** nothing at all. 8. **But for** the commentaries and criticism, that appeared early, scientists **could not have known** about Zeno's paradoxes. 9. Zeno had such art of speaking **as though** the same thing **should appear** to his listeners like and unlike, one and many, at rest and in motion. 10. Greek scientists wished they **could have resolved** Zeno's paradoxes. They failed. 11. Aristotle demanded that infinite quantities **(should) be prohibited** in mathematics and he claimed, "the natural numbers must be accepted without further analysis as the foundation of mathematics". 12. If the incommensurables **had not been discovered,** the first deep crisis in the foundations of classical Greek mathematics **might not have occurred.** 13. If Zeno's paradoxes **had not been unsolvable** and the discovery of the incommensurables **could have been concealed,** the classical Greeks **would not have tried** to avoid infinite processes altogether. 14. Nevertheless, Euclid was the first to prove that the set of all natural numbers is infinite. So **is** the set of all the points on a given line segment. 15. Not only **did** mathematicians of succeeding periods recoil from infinite quantities but they sought to dispense with the concept. 16. In the XVII c. Galileo claimed that there **should exist** another startling paradox of the infinite. 17. Galileo wished he **could compare** two infinite sets of numbers: whole numbers and whole even numbers. 18. Galileo suggested that infinite sets **should not be compared.** 19. Not until the end of the XIX c. **had** mathematicians **realized** that the concept of infinite **could not be dispensed** with. 20. It is important (that) the students **(should) know** that the first successful attack on the problems of the infinite was made by G. Cantor. 21. Scarcely **did** Cantor **investigate** the real number continuum, when he became involved with the problem of the nature of number. 22. Not only **had** Cantor **extended** the number system by inventing transfinite numbers, but he also developed cardinal and ordinal numbers theory and transfinite numbers arithmetic. 23. Cantor operated with infinite sets as if infinity **were** an actuality. 24. **Should** one be concerned with infinity, one **would have** to distinguish at least two kinds of infinity: the first — the infinity of natural numbers designated by $\aleph_0$. The sets with $\aleph_0$ cardinality are called countable or denumerable. 25. The second kind of infinity is the one represented by a line segment; Cantor denoted it by the letter *C* lest it **should be confused** with the former. 26. Any line segment has cardinality *C*. So **has** any rectangle in the plane, any cube in space. 27. Cantor wished he **could find** an infinite set of points on a line segment that is neither equivalent to the whole segment nor to the set of natural numbers. 28. Cantor conjectured that such a set **should not exist** and the problem acquired the title "the continuum hypothesis". 29. Paradoxes of set theory **might have not appeared had** Cantor **created** an axiomatic set theory. 30. Axiomatic set theory was developed early in this century by Zermelo, so **was** Russell's abstract set theory. 31. It is essential that though the theory of cardinal and ordinal numbers underwent some technical refinements, Cantor's conception of transfinite numbers **should remain** fundamental. 32. It **would seem** reasonable to pay tribute to Cantor's contribution to the development of Mathematical Logic and modern axiomatic method as well.

Key Grammar Patterns

Moods

Mood is the form of the verb which shows **the relation** between **the action expressed by the predicate verb and reality.**

The Indicative Mood

The **Indicative Mood** represents an action as a fact and is characterized by **all Tense-Aspect forms of Active and Passive Voice.**

1. In fact, when we **count** on our fingers we **are** merely **placing** some sets of objects in one-to-one correspondence with some set of fingers.
2. There are difficulties in mathematical concepts of length and time which **were** first **revealed** and **pointed out** by the Greek philosopher Zeno.
3. In his fight against the infinite divisibility of space and time Zeno proposed Achilles and tortoise paradox and the arrow paradox. These paradoxes **can be resolved** only in terms of the modern theory of infinite classes.

Real Actions

The Imperative Mood

The **Imperative Mood** expresses **commands and requests.** It may be used in the affirmative and in the negative form. The emphatic form makes a command or request more expressive.

1. **Let him suppose (Suppose)** an arrow is in its flight.
2. **Do admit** that at any instant the arrow is in the definite position and is at rest as well.
3. **Don't doubt** at the very next instant in its flight the arrow will be in another position.
4. **Let us ask** when the arrow goes from one position to the other.

} **Commands. Orders Requests. Warnings. Urging to actions.**

We find a great variety of forms expressing **unreality** in present-day English. Some forms represent an action as **problematic,** the others express action **contradicting reality,** i. e., actions which cannot be realized at all. The forms of the Oblique Moods (Subjunctive and Suppositional) serve these purposes.

The Subjunctive Mood

The **Subjunctive Mood** represents actions as unreal, doubtful, problematic, desirable, improbable and the like. The Russian particle "бы" is expressed by the model verbs "**should**", *"**would**" plus the forms of **the infinitive** (Indefinite or Perfect) according to the time reference (Present, Future or Past).

бы-	**should**	**Pr., Fut. + go**	**with modality added:**
	***would**	**Past + have gone**	could go (have gone) *мог бы...*
			might go (have gone) *может быть бы...*
			ought to go (to have gone) *следовало бы...*

Simple Sentences

1. Pythagoras **would have liked** to conceal the discovery of incommensurables; one **ought to know** it.
2. The first crisis **might not have occurred** without Zeno's paradoxes, we **should** (would) **say.** (=we'd say).
3. The set concept **could be used** in modern systems of teaching arithmetic.
4. Today's mathematicians **would prefer** to investigate number systems in their enterity and not individual numbers.

} **Desirable. Problematic. Doubtful actions**

Clauses

Pr., **Fut.** } →be, were, could, had, spoke, knew, wrote...

Past →had been, had spoken, had known, had written...

The Object Clause (after "**wish**").

1. **We wish we knew more** of Zeno's paradoxes. { Как бы нам хотелось... Как жаль, что мы не знаем...
2. We wish Cantor **were** our contemporary.
3. We wish Cantor **could witness** that his set theory holds the key position in modern mathematics.
4. Cantor wished he **could have proved** or **resolved** the Continuum Hypothesis. He failed.

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* **Would** is mainly the verb corresponding to the Russian „бы" in modern usage.

The Comparison Clause (after "as if", "as though")

1. Classical Greek mathematicians avoided infinite processes **as if** they **had not existed** at all. 2. The formalistic school (Hilbert and his students) took the attitude **as though** mathematics **were developed** purely formally. 3. In axiomatic set theory Zermelo regards a set **as if** it **were** simply an undefined object satisfying a given list of axioms.	**Unreality. Actions contrary to reality.**

The if-Clause

The If-Clause (Introduced by: "if, unless, in case, provided, providing, on condition that, even though, suppose (ing), if it were not for, but for, granting", etc.).

1. If (In case) you **ask** me, **I'll say** that the free use of Cantor's intuitive notion of a set (will) **can lead** to contradictions.	**Real condition.** The Indicative Mood is used.

Unreality

2. If you **asked** me now If you **should ask** me **Should** you ask me If I **were to be asked** **Were I to be asked**	**I'd** (=should, would) **say** that... I **could** (might, ought to) **say** that...	**Unreal conditions. Actions contrary to reality**
3. If you **had asked** me yesterday **Had** you **asked** me yesterday	**I'd** (=should, would) **have said...** I **could** (might, ought to) **have said...**	

The Suppositional Mood

The Suppositional Mood represents necessity, recommendations, suggestion, order, decision, etc.

***(should) + go (have gone)**

The subject clause It is essential nesessary important strange, etc.	that one **(should) make** a distinstion between finite and infinite sets.	Supposition Necessity. Probablity. Requirement. Order. Purpose. Advice.
The object clause He claimed demanded commanded suggested insisted proposed, etc.	that the number properties of infinite sets **(should) be** different from those of finite sets.	

* "Should" serves as an indicator of the Suppositional Mood only without any meaning of its own whatever; hence it may be omitted (should).

Clause of purpose (in order that..., that..., so that..., → may, might, can, could) (lest→should)

1. One matches the elements of a given set with those of its subset one-to-one, **so that** he **may (might, could) obtain** the number of elements in a set without counting. 2. Pythagoras tried to suppress the information about the incommensurables lest it **should discredit** mathematics in the eyes of general public.	Actions problematic not necessarily contradicting reality.

Inversion

Grammatical

Why **had** the greatest mathematicians from Greek times on **be plagued** with problems involving infinite quantities? **Did** Galileo abandon the idea of infinite quantities? **There are many ways** of settling this problem in modern mathematics.

Emphatic

Non-emphatic	*Emphatic*
1. **If the set concept were so simple as it may seem** the mathematicians might have made it explicit and applied much earlier.	**Were** the set concept so simple as it may seem... **Should** the set concept **be** so simple as it may seem...
If Zeno's paradoxes **had not been** so colourful, the literature and commentaries involved would not have been growing at such a good pace in antiquity.	**Had** Zeno's paradoxes **not been so colourful...**

2. **Only, never, little, hardly ... when, scarcely ... when, no sooner ... than, not only ... but also, not until, nowhere, neither ... nor**

Not until the time of Galileo, **do** people **begin** to look for the essence of things in number. (Pythagoreans' belief).

Hardly had XIX c. mathematicians **begun** the research in the structure of real number system, when it became clear that a more precise analysis of the structure as a whole was needed.

Only after great discoveries in topological aspect of real number system **did** Cantor make the first nontrivial discovery in set theory.

3. **So..., neither..., no..., no more...**

Cantor's intuitive set theory was created early in this century, **so was** Zermelo's axiomatic set theory. The logistic school did not fulfil their programme, **neither did** the formalistic school.

4. **Great..., significant..., strange..., incomplete...**

Great as **is** the genius of Cantor, it is almost certain that he was able to achieve some of his greatest results only because Riemann, Dedekind, Weierstrass, Frege, Abel and Galois had suggested new ways of abstract thinking about number continuum, functions and abstract groups.

EXERCISES

1. *Translate the given sentences into Russian identifying the form of the Mood of the predicate verb.*

1. Cantor wished people **understood** a paradoxical property of infinite sets, viz., an infinite set can be equivalent to one of its subsets. 2. It is important that every one **should realize** that one-to-one correspondence lies at the heart of Cantor's set theory. 3. Cantor insisted that the number properties of infinite sets **should be** strikingly different from those of finite sets. 4. Cantor suggested that an infinite set **should be conceived** and **studied** as a whole. 5. If Cantor **had not developed** his theory of infinite classes, Zeno's and Galileo's paradoxes **would not have been resolved.** 6. It is strange that the celebrated "continuum hypothesis" **should be** neither **proved** nor **disproved.** 7. If Cantor **had not derived** Theorem I on the power of the continuum, he **would not have deduced** from it that the continuum is uncountable. 8. Cantor insisted that the set of transcendental numbers **should have** the power of the continuum. 9. But for the paradoxes, Cantor's set theory **could have served** as a secure foundation for mathematics. 10. Most mathematicians continued to develop their theories as if the crisis in the foundations of mathematics **had not concerned** them.

II. *Translate the given sentences identifying the form of inversion.*

1. Not until paradoxes in Cantor's intuitive set theory were discovered, **were** the mathematicians **provoked** to start a thorough reconstruction of the entire foundations of mathematics. 2. So severe **were** the attacks on his work that Cantor began to doubt himself, became depressed and suffered mental breakdowns. 3. The programme of logistic school was not carried to sucessful completion, neither **was** the programme of formalistic school. 4. Not until the beginning of the current century **do** we **find** the origin of the new science — Mathematical Logic. 5. Only by making a "global analysis" of the whole real-number system, **did** Dedikind **manage** to develop his theory of "cuts". 6. Suppose you wish to count the number of elements in an infinite set, what **would** you **do?** 7. **Should** you ask me what Cantor's greatest innovation is, I would say — his proof that the set of points of a plane (or of space) has the same power as the set of points of a line. 8. The first crisis in mathematics occurred in number theory, so **did** the third. 9. The ultimate influence of the first crisis was beneficial (Eudoxus's theory of equal ratios), so **was** of the third (the origin of Mathematical Logic). 10. Only by investigating such basic issues as the nature of number **did** mathematicians succeed in developing modern number theory — the purest field of mathematics. 11. **Were I asked** now many fransfinite numbers exist, I would say that there exists an infinity of transfinite cardinal numbers, of which $\aleph_0$ is the smallest.

THE INTRODUCTORY TEXT

DEVELOPMENT OF MODERN MATHEMATICS

Research in the structural (topological) aspect of the real number system gave rise to one of the most fundamental concepts of modern mathematics — a concept which itself forms one of the characterizing features of modern mathematics, namely, the notion of "set". That the notion was about to come into full flower during the latter part of the nineteenth century may be seen in the works of Riemann, Dedekind, Weierstrass, and Cantor. Weierstrass gave an example of a function continuous over the real numbers but having no derivative at any point. Both Riemann and Cantor investigated properties of functions defined by Fourier-series expansions which exhibited such curious properties as to be given the epithet "pathological" (although modern set theory and topology have taught us that they are not deserving of such a label).

It became clear that the lack of a more penetrating description of the intuitively conceived structure of the real number system was a veritable scandal in mathematics. Only by making a more precise analysis of the structure **as a whole** — what might be termed a "global" analysis like that exemplified in Dedekind's theory of "cuts", as opposed to studying the properties of individual numbers — could a better approximation be achieved. And thus, out of sheer necessity, the theory of sets was born. As one looks back it seems amazing that so much was accomplished by the analysts of the XVII—XIX centuries without any firm foundation in a more clearly defined real number system.

The real power of the theory of sets, and the circumstances that gave it ultimately its key position in modern mathematics, may be found in the extension of **G Cantor's** researches into the real number continuum, which carried him on into an investigation of the nature of number. It is chiefly to elucidate the nature of number that the teacher uses the set concept in modern systems of teaching arithmetic. For by a consideration of sets and operations with them (union, intersection, etc.) one can arrive at a much better intuitive understanding of the nature of the natural numbers and operations with them. Even pioneering logicians such as Russell, who (vainly) tried to establish that mathematics is only an extension of logic, seized upon the set concept as the most suitable tool for the definition of the cardinal and ordinal numbers.

With the foundational works of **Dedekind, Weierstrass, Frege, Peano** and **Cantor** it seemed that the secure and definitive foundations of mathematics had been attained. This was not the case, however. All seemed right in this "best of all possible (mathematical) worlds". However, the feeling of confidence in the new foundations and mathematical rigour received a blow around the beginning of the twentienh century — a blow that gradually assumed the proportions of a crisis. Usually mathematics has benefited from its crises, a classical case is the crisis in Greek mathematics resulting from **the discovery of incommensurable line segments** and **the paradoxes of Zeno.**

Since the philosophy of the Pythagorean school was that whole numbers, or whole numbers in ratio, were the essence of all existing things (Numbers rule the Universe!) the discovery of incommensurables was regarded as a "logical scandal". The Pythagoreans tried to suppress the discovery but the truth cannot be suppressed. Zeno of Elea was famous for his paradoxes. A paradox is a statement which seems absurd but which is actually well founded. Zeno's paradoxes concern the structure of the continuum, infinite divisibility and the concept of motion. Zeno's paradoxes were not the only precedent in the history of scientific thought. Paradoxes — the genuine ones — arise whenever the conceptual apparatus of science is more or less radically revised. They are brought about by 1) the new conceptual apparatus that is not quite satisfactorily constituted; 2) by the use of old terminology not sufficiently adopted to the new concepts. Paradoxes — mathematicians claim — must be eliminated by any means: **the reconstruction of the new conceptual apparatus or by a revision of scientific terminology.** The mathematical concept of the infinite was the source of many paradoxes.

The prospect of an infinite process disturbed ancient mathematicians for here they were confronted with a crisis. They were unable to answer the subtle paradoxes Zeno of Elea proposed at about the same time that the devastating discovery of incommensurables was made. Aristotle and other Greek philosophers sought to answer the paradoxes of Zeno, but the replies were so inconvincing that mathematicians of the time concluded that it was best to avoid infinite processes altogether. An analogous "great crisis of foundations" (1900—1930) occurred with the revelation of contradictions in Cantor's intuitive set theory. In 1897 Cantor discovered a contradiction in his theory when he had failed to find an infinite set with cardinality between $\aleph_0$ and C, that is, an infinite set of points on a line segment that is not equivalent to the whole segment, and also not equivalent to the set of natural numbers. Cantor conjectured that no such set exists and the problem acquired the title "the continuum hypothesis". As it had happened before the "continuum hypothesis" revealed to the mathematicians unsuspected complexities in a seemingly rigorous Cantor's set theory, they began to doubt whether it could serve as a secure foundation for mathematics. The discovery of contradictions in set theory by Russell and Burali — Forti around 1900 precipitated more soul-searching than Zeno's paradoxes. The new foundations constructed by the nineteenth century mathematicians had developed a fissure that raised a question as to how rigorous the newly found rigour of the set theory really was.

As a response to "paradoxes" in Cantor's intuitive set theory, **Ernst Zermelo** founded in 1908 the **axiomatic set theory** with a notion of a set being regarded simply as an undefined object satisfying a given list of axioms. Further developments of the axiomatic set theory by the leading mathematicians and logicians resulted in the creation of the **abstract set theory** in which the symbols for "set", "union", "intersection" and so on may be rearranged only according to a given list of axioms and rules of inference. The abstract set theory did not solve the problem of sound foundations for mathematics. Although analysts, algebraists, and geometers continued to develop their theories, seemingly oblivious of the situation in the foundations, the crisis began to claim the attention of more and more mathematicians.

The philosophy propounded by **Kronecker** (1823—1891) was recalled. Kronecker had decried the "infinitistic" methods underlying the work of Weierstrass, Cantor, and the other creators of the new foundations. He maintained that "real" mathematics was only that which could be derived from the natural numbers ("intuitively given"), using only constructive methods. Nonconstructive existence proofs (for example, proving that every algebraic equation has a root by showing that assumption of nonexistence of such a root leads to contradiction) were to him worthless, and by his criteria only the rational numbers (or, more generally, the algebraic numbers), not the general real numbers were admissible. The controversy between Cantor and Kronecker was clouded with personal animosity, but at its roots were sharply divergent views on the philosophy of mathematics between **constructivists** (Kronecker) and **formalists** (Hilbert). So severe were Kronecker's attacks on his work that Cantor began to doubt himself, became depressed and suffered mental breakdown. After the contradictions in set theory were found, the Dutch mathematician **L. E. Brouwer** took up Kronecker's philosophy (c. 1908) and developed it further, gathering about him a small but an influential group of co-believers. However, it soon became evident that to adhere faithfully to this doctrine would result in having to abandon most of the new foundations.

Leaders among those who refused to accept such an extreme "cure" were **Russell, Whitehead,** and **Hilbert.** Taking their cue from the late nineteenth-century logicians Russell and Whitehead sought a secure foundation in the **tautologies of elementary logic**, together with **implication rules** that seemed safe from error. Although their work "Principia Mathematica" was never completed, it went far enough to show the necessity for bringing in axioms, concerning the infinite and the hierarchy of types. These axioms, if not of questionable validity at least, could not be considered purely "logical" in character.

Hilbert and his students did not begin actively carrying out their programme (called "formalism") until about the 1920's, and they were obviously influenced by the findings of Russell and Whitehead as well as the doctrines of Brouwer. They took the attitude that safety can be secured by acting as though mathematics were developed purely formally; the axioms and theorems were exhibited as pure formulas (in much the same way as in "Principia Mathematica") devoid of meaning for the purposes of the investigation, while the methods of deriving theorems (new formulas) were confined to such as are purely finitistic. If a formula having the form "Both F and $\sim$ not-F hold" (i. e., — $F \sim F$) is encountered, then one has contradiction; hence, the purpose was to show that such a formula could not be derived. That this programme, too, could not be carried to completion became evident in the 1930's, following **Gödel's famous theorem** of 1931.

It is characteristic of science that what may seem like failure often turns out to be the opposite. It is as important to show what cannot be done as to show what can be done. In the sense that their programmes could not be carried to successful completion, one may say that the "logistic" school (Russell) and the "formalist" school (Hilbert and his students) were failures. But from the standpoint of their influence on later development, they were quite successful, for they had given the major impulse for the creation of modern **Mathematical Logic**, now a recognized field of mathematics. The resulting impact on the modern point of view on the nature of modern mathematics was to be considerable. The crisis had not been resolved, but it had not stultified mathematics any more than had the earlier crises; and, as before, the ultimate influence was beneficial.

Reproduce the outline of the text. The following citations may direct you.

1. "Infinite is imperfect, unfinished and therefore unthinkable. It is formless and confused. It should be prohibited in mathematics" (**Aristotle**).
2. "Infinity and indivisibility are in their very nature incomprehensible to us" (**Galileo**).
3. "The notion of the number of whole numbers is self-contradictory and should be rejected" (**Leibnitz**).
4. "I protest against the use of infinite magnitude as something completed, which is never permissible in mathematics; one has in mind **limits** which certain ratios approach as closely as desirable while other ratios may increase indefinitely" (**Gauss**).
5. "God made the integers; all else is the work of man" (**Kronecker**).
6. "Transfinite numbers are as legitimate as rational numbers" (**Cantor**).
7. "Later generations will regard Cantor's work as a desease from which one has recovered" (**Poincare**).

8. "No one shall expel us from the paradise which Cantor has created for us" (**Hilbert**).

TEXT ONE

G. CANTOR AND THE ORIGINS OF TRANSFINITE SET THEORY

The nature of the infinite has always been a controversial topic. In Antiquity it appeared in the famous paradoxes of Zeno of Elea (450 B. C.) who argued with discomfiting clarity that motion is impossible because it requires that an object should pass through an infinite number of points in an infinite time. Zeno's paradoxes indicated that the number properties of infinite sets are strikingly different from those of finite sets. Mathematicians and philosophers from Greek times on devoted considerable thought to Zeno's questions without resolving his paradoxes and in the 17th century Galileo proposed another startling paradox of the infinite. He observed that the set of natural numbers could be placed in one-to-one correspondence with the set of squares of natural numbers. Thus, the sacred Euclidean notion that the whole is always greater than any of its parts was violated by infinite sets. Not until the 19th century did mathematicians realize that it is precisely this abnormal property that characterizes infinite sets.

In the 19th century mathematical infinity appeared only in its "potential" form. Mathematicians had customarily drawn a sharp distinction between **the infinite regarded as a complete quantity** and **the infinite regarded as a potential,** represented by an indefinite sum of series of numbers that tends toward some limit. Only the potential infinite were they willing to countenance. By speaking of limits it was possible to avoid paradoxes associated with actual infinities. There was no question of "infinitely great" or "infinitely small" and the symbol ∞ served only as a concise notation. Nevertheless, many great philosophers — Descartes, Spinoza, Leibnitz — discussed actual infinity. All the mathematical and philosophical doctrines associated with the acceptance or rejection of the actual infinity converge in the work of the German mathematician **Georg Cantor.** Despite the tentative endeavours of other mathematicians of the time, it was really **G. Cantor who founded the theory of the actual infinity.**

Cantor didn't set out with the aim of establishing a theory of infinite great magnitudes. In 1870 he started from a concrete mathematical problem in the theory of functions of a real variable. His problem involved distinguishing finitely or infinitely points of discontinuity and his first result on the theory of trigonometric series was the theorem: "If a function is continuous throughout an interval, its representation by a trigonometric series is unique". In 1872, when Cantor was 27, he published a paper that included a very general solution to his problem on number continuum together with the seeds of what was later to become the theory of transfinite sets. The main obstacles to a theory of real numbers are irrational numbers, such as $\sqrt{2}$ and π. Because the legitimacy of the rational numbers was not in question, Cantor followed an approach suggested by his former teacher K. Weierstrass: **any irrational number can be represented by an infinite sequence of rational numbers.** In this way all irrational numbers can be understood as geometric points on a real-number line just as rational numbers can. In spite of the advantages of Cantor's approach it troubled his contemporaries because it presumes the existence of sets of numbers having infinitely many elements, i. e., the concept of completed infinities; the idea rejected since the time of Aristotle, primarily because of the logical paradoxes they inevitably generate. Cantor upset the German mathematicians by showing that the infinite set of real numbers, represented by the continuum of points on a line is larger that the infinite set of all fractions. He also showed that the infinite quantities called by him transfinite numbers could be defined that describe such differences.

In a series of papers between 1872—1895 Cantor created the theory of transfinite numbers in much the way it exists today. In developing what he called the arithmetic of transfinite numbers Cantor gave mathematical content to the idea of the actual infinite and laid the groundwork for abstract set theory. Cantor made significant contributions to the foundations of the calculus and to the analysis of the continuum of real numbers. Cantor's most remarkable achievement was to show, in a mathematically rigorous way, that the concept of infinity is not an undifferentiated concept. Not all infinite sets are the same size; consequently, infinite sets can be compared with one another. For example, the set of points on a line and the set of all fractions are both infinite sets. Cantor was able to prove, in a well-defined sense, that the first set is greater in size than the second. In the 1890's Cantor succeeded in demonstrating that **there is a hierarchy of infinities, each one "larger" than the preceding one.** So shocking and counter-intuitive were Cantor's ideas to his contemporary mathematicians, that they attacked Cantor viciously. At first Cantor himself resisted the transfinite numbers because he believed that the idea of the actual infinite could not be consistently formulated and so had no place in rigorous mathematics. Nevertheless, slowly overcoming his own reluctance and the obstinate antagonism of fellow mathematicians, Cantor moved forward towards developing general concepts of actual infinity and transfinite numbers because, by his own account, he found that they were indispensable for mathematics. Because of his own early doubts he was able to anticipate opposition from diverse quarters, which he attempted to meet with philosophical arguments as well as mathematical ones. Moreover when he was called on to respond to his critics, he was able to muster his ideas with considerable force. One can't help appreciating the energy and single-mindedness with which he promoted his theory.

Cantor was not alone in studying the properties of the continuum in rigorous detail. In the same 1872 the German mathematician **R. Dedekind** also published an analysis of the number continuum that was based on infinite sets. "No matter how small the line segment, there are infinitely many rational points on it". Thus, the gist of Dedekind's remark was that in spite of the density of the rational points on a line segment there is still room to pack an infinite number of irrational points into the line. Cantor made this idea more precise. Dedekind's statement is consistent with a correct understanding of the continuum, but it conceals a serious weakness. If anyone had asked Dedekind how much richer the infinite set of points in the continuum was than the infinite set of rational numbers, he could not have replied. Cantor's major contribution to the question was published in 1874. Cantor borrowed Galileo's paradox and turned it into a means of comparing the size of infinite sets. He defined two sets as **equivalent** if a one-to-one correspondence can be established between the members of each set. For example, if a squad of soldiers, each carrying a gun, were to pass before us there would be one-to-one correspondence between soldiers and guns. Any set of numbers whose members can be matched one-for-one, or in effect counted by the set of positive whole numbers, Cantor called a denumerable (countable) set. **The set of real numbers, represented by the continuum of points on a line, is not denumerable.** If it were, the real numbers say between 0 and 1, could be paired one-for-one with the whole numbers.

Cantor said that two sets have the same cardinal number if and only if they can be placed in one-to-one correspondence. He proved that the set of all integers, the set of natural numbers, the set of rational numbers, and the set of all algebraic numbers have the same infinite (or transfinite) cardinal number, $\aleph_0$ (aleph null). Moreover, he was able to show that the set of real numbers has a cardinal number greater than $\aleph_0$ and that in fact there is **an infinity of transfinite cardinal numbers** of which $\aleph_0$ is the smallest. In the applications of set theory only two transfinite numbers $\aleph_0$ and C play an essential role.

In 1895, Cantor who dealt so successfully with the paradoxes of Zeno and Galileo, discovered a paradox in his own theory. He proved that there is a cardinal number greater than the largest possible cardinal number. In the well-ordered array of transfinite cardinals $\aleph_0, \aleph_1, \aleph_2, \ldots$ where does $C=2^{\aleph_0}$ fit it? It is at least $\aleph_1$ and Cantor conjectured that $C=\aleph_1$. This is Cantor's famous **Continuum Hypothesis.** Hilbert made it the first of his 23 unsolved problems. There was no progress made in solving this problem until 1938. In 1938 **Gödel** proved that the Continuum Hypothesis is consistent with the standard axioms of set theory.

There are three possibilities for a theorem: it may be provable, disprovable or undecidable. Gödel ruled out the "disprovable" possibility. In 1963 **Cohen** completed the task by ruling out the "provable" case. So it stands that the continuum problem is **undecidable** on the basis of the current axioms of set theory. Some day mathematicians may decide which of the $\aleph$-*s* is equal to C. Perhaps $\aleph_1$, $\aleph_2$ $\aleph_{17}$? At present, none of these is ruled out. Nevertheless, it is known, due to **Konig**, that C cannot be $\aleph_{\infty(\omega)}$. The theory of cardinal and ordinal numbers underwent a series of technical refinements under the leadership of **Russell, Zermelo,** and **Neumann.** However, Cantor's conception of transfinite number remains fundamental. In the course of years, when set theory displayed its usefulness in many fields of mathematics (topology, the theory of analytic functions, the theory of measure, the theory of functions of a real variable, the foundations of mathematics) it has become a tool of modern mathematics. Set theory is highly successful in its treatment of abstractions.

The knowledge of the fundamental features of set theory is of the greatest importance for an understanding of the foundations of logic and of mathematics, and indeed of the nature of mathematics as a whole. It is known that all of classical mathematics — indeed, that practically all of contemporary mathematics itself — can be developed within set theory. All of the basic concepts within classical mathematics can be defined in terms of set theoretical concepts; and all of the basic principles concerning these concepts can then be established within set theory with the help of these definitions. Many of the traditional concepts of mathematics, which prior to the rise of set theory were left at a rather vague and inexact level — e. g., the concepts of natural number, relation, function, finite and infinite — came to receive exact analysis within set theory as well as considerable generalization.

Quite clearly, the development of set theory has brought a good deal of conceptual clarity and precision to mathematical thinking. Set theory is especially important for understanding the twentieth century mathematics. The generality and abstractness and indeed the very rapid growth of the twentieth century mathematics have to be understood largely in terms of the extent to which this mathematics is permeated by the concepts and methods of set theory. Modern algebra and topology are especially important and obvious illustrations of this pervasive characteristic of modern mathematics.

It is an almost unique event in the history of science that a single person could found and develop a whole new branch of mathematics which has become a corner-stone in the foundations of many other branches of mathematics and the chief connecting link between mathematics and logic. In modern mathematics it has become increasingly rare to name a whole discipline after one man. Normally we speak of "algebra", "topology", "analysis", etc., without any reference to a particular mathematician. But just as we use phrases like "Euclidean Geometry" and "Newtonian Mechanics", so also do we use the phrase "Cantor's Set Theory". Cantor's set theory is included among the greatest revolutions in mathematics.

Read the text. Generalize its main ideas.

TEXT TWO

SET THEORY

Set theory is the mathematics of classes. Sets are classes. Basically "set" is simply a synonym of "class"; it has more currency than "class" in mathematical contexts. But this excess terminology is often used also to mark a technical distinction. There are advantages (and disadvantages) in holding with Cantor that not all classes are capable of being members of classes. There is in the notion of class no presumption that each class is specifiable; in fact, there is an implicit presumption to the contrary if we accept the classical body of set theory that comes down from Cantor. In theories that hold this the excess vocabulary has come in handy for marking the distinction: classes capable of being members are called 'sets"; the others have been called "proper classes". We can know this technical sense of "set" and still use the terms "set" and "class" almost interchangeably, for the distinction emerges only in the systems that admit proper classes and even in such systems the classes we have to do with tend to be sets rather than proper classes.

We have defined set theory as the mathematical theory of classes and described the notion of class yet with no inkling of what prompts set theory. This is best done rather by quoting the opening sentence of Zermelo's paper of 1908: "Set theory is that branch of mathematics whose task is to investigate mathematically the fundamental notions of "number", "order", and "function" taking them in their simple form, and to develop thereby the logical foundations of all of arithmetic and analysis". Because of paradoxes much of set theory has to be pursued more self-consciously than many other parts of mathematics. The natural attitude on the question what classes exist is that any open sentence (e. g., "$x \in a$") determines a class. Since this is discredited, as an open sentence can be true of some things and false of others and fail, after all, to determine any class at all (Russell's paradox), we have to be deliberate about the axioms of class existence and explicit about reasoning from them; intuition is not in general to be trusted here, the logical machinery is more in evidence in this part of mathematics than in most.

In this respect the literature on set theory divides conspicuosly into two parts. The part that concerns itself mainly with foundations of analysis gets on with the same measure of informality as other parts of mathematics. Here the sets concerned are primarily **sets of real numbers,** or **of points** and **sets of such sets** and so on. Here the paradoxes or antinomies do not threaten, for questions like "$x \in x$" do not come up. It is rather in what has been called **abstract set theory** as against point-set-theory, that the following topics are dealt with. First there are the general assumptions of the existence of classes and other general laws concerning them. Then, there is the derivation of a theory of relations from this basis and more particularly a theory of functions. Then, the integers are defined, and the ratios, and the real numbers and the arithmetical laws are derived that govern them. Finally, one gets on to infinite numbers: the theory of the relative sizes of infinite classes and the relative lengths of infinite orderings. These latter matters are the business of set theory as its most characteristic. They are the discovery or creation of Cantor and thus virtually coeval with set theory itself. It depends heavily on concepts and notations of modern mathematical logic the knowledge of which is so widespread among mathematicians and so necessary for those with an interest in the foundations of mathematics.

The following concepts are of fundamental importance to much of both modern pure and applied mathematics.

Sets. Cardinal Numbers, Finite and Transfinite

It is true that the cardinal number of a set may be the same as the cardinal number of proper subset or part. However, this cannot **happen** with finite sets, that is with sets that can be counted out at a steady rate of speed in a limited time. So far as finite quantities, or lengths, areas, angles and other magnitudes used in the elementary applications of mathematics are concerned, "the whole is greater than any of its (proper) parts" (Euclid) remains valid. In fact, it can be proved that a set is **infinite** if and only if it is equivalent to a proper part of itself.

Definition I. The cardinal number of an infinite set is called a **transfinite cardinal number** and denoted by $\aleph_0$ after *G*. Cantor who first studied transfinite numbers systematically. The cardinal number of the set of all points on the line-segment *AB* is denoted by *C*. It can be shown that $\aleph_0$ and *C* though they are both transfinite, are not the same numbers. In fact, there are many different transfinite numbers and an arithmetic of them can be worked out and studied systematically.

For example, we shall define the "sum" of two cardinal numbers, finite or transfinite, as follows:

Definition II. If the sets *X* and *Y* have no element in common, then by **the sum** $X+Y$ of the two cardinal numbers *X* and *Y* we shall mean the cardinal number of the set consisting of all the elements of the set *X* and all the elements of the set *Y* together.

Thus, the set *A* of all natural numbers has the cardinal number $\aleph_0$; the set *C* of all **even** natural numbers has same cardinal number $\aleph_0$ and the set *D* of all **odd** natural numbers also has the cardinal number $\aleph_0$. Hence, **the sum** of the cardinal number of C plus the cardinal number of D is the cardinal number of A or $\aleph_0+\aleph_0=\aleph_0$.

This is only one of the many surprising statements that arise in the arithmetic of transfinite numbers. Indeed, it can be demonstrated that $\aleph_0+3=\aleph_0$, $\aleph_0+8=\aleph_0$, $\aleph_0+n=\aleph_0$ by combining the elements of a set having the cardinal $\aleph_0$ with those of a set having the cardinal number 3, 8 or n and establishing a one-to-one correspondence with the natural numbers.

We can define that **the Cartesian product** $A \times B$ of two sets A and B is the class of ordered pairs (a, b), where a is any element of A and b is any element of B.

Definition III. The **product** of the cardinal number of a set A and the cardinal number of a set B is the cardinal number of the Cartesian product $A \times B$. Thus, we see that $\aleph_0 \cdot \aleph_0=\aleph_0$ $2 \cdot \aleph_0=\aleph_0$, $\aleph_0 \cdot 2=\aleph_0$, $3 \cdot \aleph_0=\aleph_0$.

Thus, the smallest transfinite cardinal is $\aleph_0$. A surprisingly large variety of sets in mathematics have this cardinality. There are $\aleph_0$ **prime numbers,** $\aleph_0$ **integers,** $\aleph_0$ **fractions,** $\aleph_0$ **even numbers,** $\aleph_0$ **odd numbers,** and, of course, $\aleph_0$ **natural numbers.** Any collection of objects that can be put into one-to-one correspondence with the natural numbers is referred to as countable or denumerable — in other words any set having the transfinite cardinal number $\aleph_0$ is a countable set. Although it is possible to study the concept of infinity with reference to **ordinal** as well as **cardinal** numbers we shall confine ourselves to **transfinite cardinal numbers** only.

Each finite cardinal n has an "immediate" successor $n+1$. This suggests the question of whether $\aleph_0$ also has this property. It may seem reasonable to suppose that there are also $\aleph_0$ **real numbers.** This conjecture is incorrect; there are actually **more** real numbers than there are natural numbers, integers or fractions. **The real numbers constitute an infinity greater than** $\aleph_0$. The transfinite cardinal number for them is designated differently by different writers either by C (the first letter of the word Continuum) or by $\aleph$, or by $\aleph_1$. Each symbol represents the set of real numbers, and the sets of points on a line, in a plane and in space; the total number of points is in each case the same. An interesting mathematical problem was posed by the question whether there is a transfinite cardinal number less than $\aleph_1$ but greater than $\aleph_0$ (Cantor's Continuum Hypothesis), which is still unsolved.

In addition there are transfinite cardinals representing infinites greater than $\aleph_0$ and $\aleph_1$. For example, there is the transfinite cardinal $\aleph_2$ representing the set of **all possible curves in a plane.** If we gave standard definition of what is meant by a transfinite cardinal number raised to a power which is also a transfinite cardinal number we could show that $\aleph_1=\aleph_0^{\aleph_0}$ and $\aleph_2=\aleph_1^{\aleph_1}$. It is possible to continue in this way and define $\aleph_3$ as $\aleph_2^{\aleph_2}$ and in fact, as Cantor claimed, there is an **infinity of transfinite cardinal numbers** but mathematicians have as yet been unable to conceive of a collection of objects having higher order of infinity. It seems that our imagination does not permit us to count beyond **three** when dealing with infinite sets.

Read the text, summarize its main ideas and give your own viewpoint concerning the origin and classification of set theory paradoxes.

TEXT THREE

SET THEORY PARADOXES

The relation between Cantor's set theory and mathematics never, in fact, ran smooth, due to the fact that Cantor's set theory deals with **"actual" or "completed" infinities.** At the beginning there was great resistance to this by the mathematical world, stemming in part from the famous dictum of Gauss (1831): "I protest against the use of an infinite magnitude as sómething completed, which is never permissilbe in mathematics". Gauss had in mind **infinite magnitudes** while Cantor's theory employs **infinite collections.** Just when Cantor's ideas were well on the way toward winning acceptance from most mathematicians, in the 1890's contradictions appeared in the upper reaches of his set theory. The contradictions or paradoxes focused attention on the foundations of set theory and of mathematics generally.

The seemingly simple and fundamental concept "set" is beset with difficulties. The free use of Cantor's intuitive notion of a set can lead to contradictions — this have become obvious since 1890. Puzzles and paradoxes have become so much to the force that one might regard the theory of infinite quantities as a mathematical divertisement. This is far from being the correct evaluation. Despite the paradoxes to which Cantor's work led and which still remain to be cleared up in a thoroughly satisfactory manner, many mathematicians have come to see that he made the only real progress man is capable of making in science.

By the end of the nineteenth century Cantor had developed set theory to a point where it constituted a special branch of mathematics and during the 1890's his set theory, which had at first met with little favour, came to enjoy considerable popularity. But at the same time there were signs that not all was well. In 1895 Cantor himself discovered an antimony, or contradiction within the theory of self-ordered sets. In 1897 **C. Burali-Forti** rediscovered the same contradiction, which later came to bear his name. Neither Cantor nor Burali-Forti proposed a way of avoiding the contradiction, however. Things became much more serious when **B. Russell** in 1902 discovered his famous paradox, which lay not at some fairly remote corner of set theory, as did the Burali-Forti paradox, but at the very foundations of set theory. Set theory was once again suspect, and even was an object of ridicule. As Poincaré who had always been very sceptical of set theory, delightfully put it, "set theory was no longer barren, for it now had given a birth to a contradiction!"

In 1926 F. P. Ramsey (1909—1930) proposed a distinction of the paradoxes known at that time into two types: **logical** or **mathematical paradoxes,** and **semantical paradoxes.** Ramsey argued that paradoxes of the latter sort by virtue of making reference to language (meaning, truth, definability) cannot be stated within mathematics, in which there is no reference to such matters, and thus **there is no need to consider them at all in attempting to devise ways of avoding paradox within mathematics.** This reasoning is not as conclusive as Ramsey apparently thought, but his classification is helpful and has been widely used.

The most famous of the logical, or mathematical paradoxes is **Russell's paradox.** This paradox proceeds as follows. First we define a class K, say, as the class of all those classes that are not elements of themselves. The class of dogs, for example, is not itself a dog, and this is not an element of itself. By the definition of K, then, the class of dogs is an element of K. And so are most, if not all, of the classes that first come to mind. Now we ask whether K itself is an element of K. We see immediately that K **is** an element of K if and only if it **is not** an element of K. It follows by the propositional logic — since all formulas $(A \equiv \sim A)$ $(A \wedge \sim A)$ are tautologies — that the class K both **is** and **is not** an element of itself. But this is a contradiction, or a paradox.

As a well-known example of the **semantical** paradoxes, we have **the paradox of the liar.** This paradox, in one form or another, goes back to ancient times. In one of its forms it proceeds as follows. Consider a man who says, "I am lying" and then says nothing further. If this man is telling the truth, then (as he says) he is lying; if, however, he is lying, then he is telling the truth in saying so. It follows by the propositional calculus of logic that he is both lying and telling the truth, which is a contradiction. This paradox is a **semantical** paradox because it makes reference to certain uttered words, which express a lie. True, this paradox does indeed at first seem frivolous and unworthy of serious consideration. Yet, it is as genuine a paradox as any other, and must be taken seriously if any paradoxes are taken seriously at all.

In all the above paradoxes a contradiction makes it appearance and it is not immediately apparent just **what** to do in order to avoid contradiction. This was accomplished by Axiomatics. One of the very earliest axiomatic approaches to set theory (preceeded only by the work of Frege and Russell) is the axiomatic set theory of **Zermelo.** Zermelo was, of course, in 1908 well aware of the numerous paradoxes which had been brought to light by that time and his axiom system was especially designed so as to prevent these paradoxes from appearing within his theory. Zermelo's set theory avoids the two logical paradoxes we have just presented. As for the semantical paradoxes, we have remarked that it is not completely convincing to argue that since these paradoxes make reference to semantical considerations, while no such reference appears within mathematics, it follows that these paradoxes cannot appear within mathematics, in particular within set theory.

All that follows is that they can make no **direct** appearance within mathematics (here excluding metamathematics). This fact, however, does not itself exclude the possibility that they make some **indirect** appearance there. For there may be some isomorphism, not at first evident, between semantic concepts and mathematical concepts which would permit a translation of these paradoxes into paradoxical statements within mathematics. Until such a possibility is ruled out, we cannot conclude then, that the semantical paradoxes are of no concern to the mathematician in general, or to the set theoretician in particular.

Read and translate the text. Agree or disagree. Give your own resolution or (dis) proof.

TEXT FOUR

RUSSELL'S PARADOX RESOLVED

Dr. Anthony Standen *New Scientist, 24 August, 1972*

An American chemist who describes himself as "a very old-fashioned philosopher" here re-examines this famous paradox connected with the theory of sets. He shows that the full description of a set calls for two distinct statements: if these are mutually contradictory — and apparently they are in the case of Russell's paradox — the set cannot be said to exist, and there can be no paradox.

When the German mathematician **Frege** had been working on set theory for over 20 years, and was about to bring out his second volume on the subject, he received a letter from **Bertrand Russell,** drawing his attention to "the set of all sets that are not members of themselves". This seems to lead to the conclusion that "if it is a member of itself, then it is not a member of itself, but on the other hand, if it is not a member of itself then it has to be a member of itself", which (as Euclid would say) is absurd. This, apparently, drove Frege to the greatest distraction, and has been a very serious concern of mathematical logicians ever since. Attempts have been made to get round it by a variety of expedients, such as laying it down that "No totality can contain members which are defined **only** by means of that totality and are therefore dependent on that totality". But each pronouncements have an empirical **ad hoc** character, and do not arise naturally from the fundamental principles of all reasoning.

It is the purpose of this paper to show that Russell's alleged paradox is no paradox at all; that far from being a contradiction within the framework of the old-fashioned logic, it holds in reality no contradiction, and its apparent difficulty is most easily resolved. The solution of the paradox is most easily seen (but does not depend on) application of the old, **old** logic; not John Stuart Mill; nothing so skitishly modern as Descartes even; but Aristotle and the scholactics.

Scholastic philosophy distinguished between **true beings,** and **beings of reason,** and I shall now revive this distinction. True beings are any things that unquestionably exist; such as you, or I, or the Empire State Building, or this house. Beings of reason, or **entia rationis,** are things that can be thought of although they do not fully have being; they do not really and truly exist. Examples are: two (or any other numeral); the square root of 20; a line (having no breadth or thickness); beauty; intelligence; or ever so many more things that can easily be thought of. All mathematicals are beings of reason; also all abstractions; anything less than an actually existent something. Among true beings the law of contradiction holds: A thing cannot both **be** and **not be** in the same respect at the same time. This impossibility of contradiction is a fundamental condition imposed by being.

Beings of reason are also subject to the law of contradiction. We cannot think of a line, which has no thickness or breadth, and at the same time has some thickness or breadth; it must be either one thing or another. If a certain object of thought is a triangle, then it has three sides; if anyone says "two-sided triangle" he is uttering an impossibility. We **cannot** think of a thing that is both two-sided and three-sided. Beings of reason do not, in the truest sense of the word, exist. There is no such things as "two" or "three" or "four", etc., lying around. They "exist", if that is the word, only in our minds. But the fact that they are subject to the law of contradiction gives them that much resemblance to the condition of true being, and it is for this reason that they are called "beings of reason".

Mathematicians speak of "existence theorems"; they will discuss whether rational roots to such and such an equation "exist". They are referring not to true being, but to that shadowy resemblance of being that is possessed by being of reason. Within this realm in order to show "existence", it is only necessary to show freedom from contradiction. As soon as that is done, the being-of-reason is shown to "exist"; the mathematician has his "existence theorem" or knows that it is possible to find rational roots to his equation.

It is otherwise with true beings. They must in any case be free from contradiction, but in order to show that they exist, we must prove that they really do exist, we must find them. If anyone were to say "winged horse", or "five-legged elephant" these expressions are valid, taken as referring to beings of reason, for they contain no **logical** contradiction. They are, however, of a very high degree of physiological improbability, and we can take it as certain that there are no such things in true existence.

Having shown that the distinction between true being and being of reason has some power of clarifying trains of thought, we can apply it to the theory of sets. A set, of course, is a being a reason; it does not fully exist. Since the **only** limitation on beings of reason is the law of contradiction, I can think of anything whatsoever as a set, **providing only** that it does not involve contradiction.

Sets whose members (**all** of them) are true beings cannot be members of themselves. For sets that contain beings of reason, or contain exclusively beings of reason, there is this possibility. It is quite easy to construct sets that are members of themselves, and also sets that are not members of themselves. And of course it is perfectly true to say that **every** set is either a member of itself or not a member of itself.

Let us now consider that alleged "set of all sets that are not members of themselves" and face the apparent paradox that "if it is a member of itself, then it is not a member of itself, but on the other hand if it is not a member of itself, then it qualifies to be a member of itself". It is to be noted that this exercise in mental gymnastics concerns beings-of-reason through and through. A variant form of the "paradox" is often put out, as a popularisation, or even as party game, in which it is made much more easy to grasp by using true beings. One form of this goes as follows:

"In a certain village there is a barber, who shaves all those and only those who do not shave themselves". Who shaves the barber? Or does he go unshaven? If he does, then he does not shave himself, and we said, didn't we, that he shaves **all** those who do not shave themselves? Intriguing paradox! Now a barber is an actually existing something, and not a being of reason at all. He cannot be made to pass out of existence by any chop logic. And so, once this apparent paradox has been expressed in terms of true beings, its solution can be found fairly easily. There might be two villages. In one of them people would say "The barber here shaves **everyone** who doesn't shave himself. And when you come to think of it, he also shaves one person who does shave himself, since he is clean shaven, and shaves himself". In the other village "The barber here shaves **only** people who do not shave themselves. If there is anyone who ever shaves himself, even once in a while, our barber does not touch him. Come to think of it, he doesn't shave quite **all** of the non-self-shavers, because he wears a beard, and he does **not** shave himself". This barber also would continue in existence.

And so the cause of the apparent paradox is easily seen; you cannot have a barber who shaves **all** the non-self-shavers, and at the same time shaves **only** non-self-shavers, because he either shaves himself or he does not. "**All** those and **only** those..."; this condition is an impossibility. It appears, then, that the description of a set is not, as it verbally appears to be, a simple matter of saying "all this", "all that", or "all the other thing". A set is defined by **two** statements, the "**all**" statement and the "**only**" statement. **And in this realization lies the solution to Russell's paradox.** For if the definition of a set requires two separate statements, then there exists the possibility that, in some form of words that appears to define a set, the two statements could be contradictory. This is exactly what has happened with Russell's so-called set.

"Set of all sets that are not members of themselves". We must examine the "all" statement and the "only" statement. If this set really contains all sets that are not members of themselves, then we can easily prove that it does not consist only of such sets, but goes on to contain one that is a member of itself, namely itself (for if it were not a member of itself, then it would have to contain itself). But if the set contains **only** sets that are not members of themselves, then it is easy to see that it does not contain all such sets, but at the most all except one of them, the exception being itself (for otherwise a contradiction arises at once).

Russell's "set", then, is not a set at all. It is not even a being of reason, for it inherently involves contradiction. It is like the expressions "two-sided triangle" or "square circle". There are two related sets; let us call them *A* and *B*. *A* is a member of itself, and it contains all, but does not consist only of, sets that are not members of themselves. *B* is not a member of itself, and though all its members are non-self-members it does not contain all of such sets. One of these is missing from its membership, namely itself. All sets, then, without exception, are either members of themselves or not members of themselves. There is no need to bring in any **ad hoc** special qualifications. There is no need for example, to exclude the set of all sets. This set (naturally a member of itself) involves no contradiction, and there is nothing wrong with it as a being of reason. **But Russell's alleged "set" is not a set at all.**

We have been led to this conclusion by bearing in mind a distinction that was made in scholastic philosophy. But the solution to Russell paradox does not **depend** on any particular philosophical viewpoint. The apparent paradox came in from not realizing that the expression "set of..." means **two** conditions, not just one. Once it is realized that "set of" means "all **and** only" then, in any kind of logic, old fachioned, new, or Far Out Modern, it can be seen that Russell's "set" is nothing more than a contradiction in terms.

When Frege received that communication from Betrand Russell in 1902 he is reported to have replied "die Arithmetik its ins Schwanken geraten" — arithmetic totters. We can now see (with the wisdom of hindsight) that he should have replied "I need pay no more attention to your so-called "set" than I would to 'square triangle' or 'five-sided parallelogram'". And this would have saved his anguish, and many years of subsequent botheration.

At least so it seems to this very old-fashioned philosopher.

Read the text. Reproduce it, expressing its main ideas, the reasoning and the proof in more popular terms. Give your appraisal of the text.

TEXT FIVE

RUSSELL'S PARADOX IN PERSPECTIVE

Dana Scott — *New Scientist,*
Professor of Mathematical Logic at Oxford — *12 October, 1972*

Logicianns and others have criticized the article on Russell's paradox of **Dr. Anthony Standen.** Here a professor of logic **Dana Scott** explains how and why Russell devised his paradox and why set theory is not so simple.

Dr. Anthony Standen claimed to have resolved the well-known puzzle about the set of all non-self-membered sets. Unfortunately, though much of what he says is correct, he has failed to understand the point of the paradox, and the principle of set existence which he himself puts forward also leads directly to contradiction. The matter can be described in simple enough terms; and the record must be set straight since despite what Dr. Standen says, Frege and Russell knew perfectly well what they were doing — along with a large number of mathematicians, logicians and philosophers who have thought about the foundations of mathematics over the last 70-odd years.

First, we need a quick review about definitions of sets. If a set A can be uniquely characterized at all, its definition can be put in the following form: $A=\{x: \ldots x \ldots\}$.

In this equation the brackets {and} denote "set abstraction". We read the equation as: "A is the set of all (not only!) those x such that the property $\ldots x \ldots$ holds". In other words, to say that $x \in A$ (i. e., x is a member of A) it is necessary and sufficient that $\ldots x \ldots$ holds. Dr. Standen makes the point that such a definition is two-sided (he says **all and only;** logicians might emphasize **if and only if**), but this is not at all the central issue. We might only mention here that we are using as a basis **assumption** that sets (whatever they are) are uniquely determined by their elements. Otherwise said, if A and B are sets such that for all x we have $x \in A$ if and only if $x \in B$ then (by assumption) $A=B$. This assumption is part of the generally accepted concept of sets (as beings of reason), and it is responsible for the possibility of the above normal form for definitions.

Well then, what is the question? It is just this: which properties $\ldots x \ldots$ can be allowed in defining sets? It is a question of **existence.** If a (definable) set is already known to exist, then we can force its definition into the standard form. But which forms can be used **creatively** to assure set existence? Dr. Stranden believes that the answer is easy, but he is wrong.

Frege seemed to think, in effect, that **arbitrary** properties determine sets. (The set is sometimes called the extension of the property.) Russell showed that Frege was wrong by using his now classical example: $R=\{x: x \notin x\}$. Frege's general principle would allow this set to exist. But as Russell argued, such a set R cannot exist, because the mere assumption of its existence leads to a contradiction. Dr. Standen gives the argument about R quite correctly while getting the significance of the example wrong. Russell knew that R cannot exist — and Frege agreed with him — the real point is: what sets **do** exist? The "paradox" lies in the fact that Frege's original assumption about **all** properties determining sets leads to contradiction. It had all seemed straightforward at first, but the situation proved to be rather more complex.

The obvious first move to make is to put some restriction on the allowable properties. Russell put forward his "theory of types" for this purpose. He proposed that we regard all beings (real or imaginary) as beings divided into types. The objects of a given type would form a set, say T. Now T consists of **all** things of that type, but we generally wish to select out"subsets". Russell would allow then such definitions as: $A=\{x: x \in T \text{ and } \ldots x \ldots\}$ (i. e., x is a member of T, having the property x) where now the variable x has been restricted to a predetermined type. Furthermore, Russell would allow **all** subsets of T to form **a** new type, say T', where we could write $T'=\{X: X \subseteq T\}$ with $\subseteq$ standing for the subset relation between sets. Of course types do not emerge out of air: we need to **assume** the existence of certain basic types as well as allow for the passage from T to T' (and possibly other constructions).

Now Russell's theory of types is a perfectly reasonable suggestion. In the original form in which he cast it there were certain technical difficulties and some clumsy features. These were eliminated in the Zermelo theory of sets which was developed with additional improvements by such men as Skolem, Fraenkel, Bernays, von Neumann, Tarski, Gõdel, and (rather differently) by Guine.

Dr. Standen suggested the following criterion of set existence: "a set, of course, is a being of reason, it does not fully exist. Since the **only** limitation on beings of reason is the law of contradiction, I can think of anything whatsoever as a set, **providing only** that it does not involve contradiction". But Dr. Standen seems to overlook the fact that **two** statements, logically consistent by themselves, may be incosistent when taken **together.** Sets do not exist in isolation: the assumption that one set exists may very well preclude the existence of another — though each at first sight may seem equally desirable.

Just to dot the "i" and cross the "t" consider that Dr. Standen allows us the universal set $U=\{x:x=x\}$ that is, the set of **all** things. Is this reasonable? He says, "there is no need, for example to exclude the set of all sets. This set (naturally a member of itself) involves no contradiction, and there is nothing wrong with it as a being of reason". Given a set T, what would Dr. Standen say to the set $A=\{x:x\in T$ and $\dots x\dots\}$? The work of Russell and Zermelo shows (in a variety of ways) why this principle of set existence is self-consistent. And this is so for arbitrary properties $\dots x\dots$ and arbitrarily given predetermined sets T.

These two self-consistent definitions are, however, incompatible. If we were to allow the substitution of Dr. Standen's set U for T and $x\notin x$ for "$\dots x\dots$", then A above reduces simply to Russell's "set", because the clause "$x\in U$" is logically redundant. We thus reach the same old paradox. It is not just a question of the difficulty of applying Dr. Standen's criterion of self-consistency, it is more a problem of the **scope** of the application.

In considering theories of set existence, one must have regard for the whole universe (domain) of sets and the general laws assumed. What has come out of the insights of Russell and his followers is that **there are many theories of sets, and their choice is not just a matter of logic.** The "anguish" of Frege is something we must live with but it is an anguish that can be studied scientifically.

ASSIGNMENT

There is circularity of self-reference in Russell's paradox. Nevertheless, because of its set-theoretical form it played a large role in the historical development of set theory. Despite the vast literature devoted to Russell's paradox and the variety of means to resolve it, there is at present **no one** explanation or resolution which is universally accepted. Why? Your viewpoint. The suggestion of your own for resolving the paradox.

VOCABULARY EXERCISES

I. *Give the Russian equivalents of:*

a) **set** *n*: an outset, a sunset, a tea-set, a shaving-set, a dinner-set, a TV-set, a set of writer's works.

b) **set** *adj.*: a set function, a set look, a set smile, in set terms (phrases), at a set time.

c) **to set** *v.*: 1. We set $a=b$. 2. A transformation **sets up** a correspondence between numbers and points. 3. Look, the moon **is setting.** 4. It will be cooler after the sun **has set.** 5. His star **is set.** 6. She **sets** the table for five people. 7. The young plants **should be set out** at intervals of six inches. 8. Who **sets** this rumour **about,** I wonder. 9. **Let's set aside** all formality and get down right to the point. 10. The train stopped at the station to **set down** three passengers. 11. Rules **have been set down** and must be obeyed. 12. The rainy season **has set in.**

II. *Give the English equivalents of the Russian phrases choosing one of the words given below.*

much, a lot of, a set, a good deal of, plenty of, a great number of, a pack of, an aggregate of.

Множество
(система)
(совокупность)
(набор)

1) точек
2) хлопот
3) друзей
4) действительных чисел
5) денег
6) времени
7) упражнений
8) огорчений
9) свойств
10) групп
11) переменных
12) подмножеств
13) замечаний
14) упорядоченных пар
15) функций
16) уравнений
17) многоугольников
18) индексов
19) значений
20) алгоритмических языков
21) квадратов натуральных чисел
22) пар целых чисел
23) ошибок
24) символов
25) болельщиков
26) трудностей

III. *Distinguish the meanings of the given verbs and the related nouns with the examples of your own.*

to contradict — to say the opposite — to disagree — to disbelieve — to deny.

Contradiction

In philosophy as in science to **contradict (contradiction) is a powerful stimulus to thought.** Contradiction is the stuff of which dialectics is made. Dialectics is the stuff on which philosophy feeds. Paradoxes have played a dramatic part in intellectual history often foreshadowing revolutionary developments in mathematics and logic. Whenever in any discipline we discover a problem, that cannot be solved within the conceptual framework that supposedly should apply, we experience an intellectual shock. The shock may compel us to discard the old framework and adopt a new one. It is to this process of intellectual **disagreement** and molting that we owe the birth of many of the major ideas in science. The paradox of incommensurables led to the concept of the continuum; Zeno's paradox of Achilles and the tortoise gave birth to the idea of convergent infinite series. Antinomies (internal contradictions in mathematical logic) eventually blossomed into Gödel's theorem. The paradoxical result of the Mishelson-Morley experiment on the speed of light set the stage for the theory of relativity. The discovery of the wave-particle duality of light forced a reexamination of deterministic causality, the very foundation of scientific philosophy and led to quantum mechanics.

Faced with a contradiction that seems fundamentally irreconciable one can ignore it or **deny** it, worship it or try to remove it. Zeno's Achilles-tortoise paradox evokes the first of these reactions in most people; "common sense" and observation **say the opposite:** the faster runner always overtakes a slower one, there is no contradiction. G. Hegel constructed his system of "logic" on **denial** of the "law of noncontradiction", which states that **a proposition cannot be both true and false.** The dialectical materialists, who adopted Hegel's philosophy but turned it upside down, declared that contradiction is inherent in the "laws of motion and matter", or the very nature of reality. No system of formal logic, however, can be built on contradiction as a fundamental principle. **The law of noncontradiction is an indispensable foundation for any rigorously constructed deductive system and indeed no such system has ever denied this law.** Since contradiction cannot be tolerated in formal logic, any paradox that arises must be honestly faced and if possible resolved and removed. The resolution of paradoxes has been an immensely profitable exercise in the history of mathematics.

Paradoxes arise, it seems, **from failure to pursue a generalization far enough.** Incommensurables appeared paradoxical because the Greeks could not pursue the generalization of number as far as continuum. The famous unsolved problems of Antiquity could not be solved as long as the definitions of geometrical constructions were restricted only to straightedge-and-compass constructions. The generalizations of modern mathematics have got rid of most of the old paradoxes. Toward the end of the 19th century it seemed that mathematics was on the road to complete emancipation from the illusions of "common sense" and achievement of an ideal science. Many mathematicians believed that with further formalization of its foundations and structure mathematics would become a perfectly self-contained, internally consistent system constructed of purely logical concepts and free of all paradoxes. That was a wain hope. In the 20th century it has become clear that mathematical logic itself cannot be free from paradoxes.

In 1930's **Kurt Gödel** finally issued a verdict that has not so far been challenged: We must resign ourselves to living with incomplete mathematical systems in which "undecidable propositions" will forever be cropping up. This follows from Gödel's proof that **no mathematical system that includes the principle of transfinite induction can be both complete and free of contradictions.** Gödel's theorem asserts that there are undecidable propositions that cannot be proved to be true or false by any chain of deductive logic starting with the postulates of the system. If his theorem had been proved in an earlier age, it would have been greeted with **disagreement** and **disbelief** as the most startling of paradoxes, because of the strongly held notion (implied by the "**law of the excluded middle" which states that either a proposition or its negation must be true**) that a well-formed proposition must be either true or false and that the failure to prove its truth or falsity only meant that the problem had not yet been solved. Today, however, mathematicians take the implications of Gödel's theorem in stride, since it is now recognized that the "law of the excluded middle" (the assumption of a two-valued logic) is merely a postulate, not an immutable law. One might say that the growing maturity of a mathematical theory often proceeds by the step-by-step solution of paradoxes involving at each stage the shedding of intuitive notions and the acquisition of an enriched, more generalized framework of thought. Paradoxes and other unsovable problems can sometimes be solved by broadening the logical framework in which they are presented.

IV. *Give one Russian equivalent of the following groups of words.*

a) Discussion — dispute — argument — debate — controversy / refutation — counter argument — disproof / confidence — truth — faith / negation — denial / research — survey — inquiry / support — help — backing / instant — the shortest space of time.

b) To oppose — to bring arguments against — to deny — to controvert / to refute — to disprove — to prove wrong — to prove mistaken / to do without — to dispense with / to disturb — to bother — to trouble — to annoy — to be anxious — to worry — to plague / to clarify — to clear up — to make clear — to elucidate / to appear — to emerge — to crop up / to be confident — to be sure — to be certain.

c) Empty — null — devoid — nought — vacuous / confidential — private — secret / instantaneous — happening at an instant.

LAB. PRACTICE

Grammar Rules Patterns

The Subjunctive Mood

I. *Turn the facts into unreal, improbable, hypothetical statements.*

Model. should would could might ought to	} know ...	Cantor says, "A set is any collection of definite well-distinguished objects of our intuition or of our thought conceived as a whole (=as one entity or totality) (should). We **should (would)** say (=we'd say) it is hardly a definition at all. It **might be** rather an intuitive demonstration of the set concept.

1. We **say** that Cantor's concept of a set is too wide and it may **lead** to contradictions and inconsistencies. (would, might) 2. We **don't claim** that the concept of a set is easy to define precisely. (should) 3. One **recognizes** that set is both an everyday word and a mathematical term. (could) 4. It **will be** difficult to say what a rigorous definition of the mathematical term "set" is. (would) 5. Mathematicians **can't hold** that everybody knows the meaning of mathematical term "set". (could) 6. Mathematicians themselves **admit** that the term "set" involves difficulties. (would) 7. Specifying a set the mathematician **has to answer** the question: "What are its members?" (would) 8. A set **may be defined** by membership property alone. (might) 9. The mathematician **says** that the relation of being a member of is a basic one. (ought to) 10. Some specification of the sets in the axiomatic set theory **is** obviously necessary. (would)

II. *Change the form of the infinitive to refer the statements to the Past.*

Model.

should
would
could
might
ought to

+ have known

Mathematicians **would prefer** to study a number system as a whole (The late XIX c. mathematicias.)
The late XIX c. mathematicians **would have preferred** to study not individual numbers but a number system as a whole.

1. Mathematicians **would reveal** unknown properties of individual numbers. (Pythagoreans) 2. The precise analysis of the structure of the real number system as a whole **might be termed** a "Global" analysis. (Dedekind's) 3. By performing a global analysis a rigorous and consistent foundation for the system concerned **could be achieved.** (in the XIX c.) 4. Dedekind's theory of "cuts" **could be** an example of the outcome of such analysis. 5. Cantor's set theory **might serve** as a secure foundation of mathematics. (but for the paradoxes). 6. The crisis **would occur,** (then due to the reappraisal of concepts and mathematical rigour) 7. The crisis **might be settled** easily. (but for the disagreements among schools of thought) 8. The crisis **could result** in the creation of a new philosophy. (at that time) 9. We **should say** that mathematics extended its number systems. (with the transfinite numbers introduced by Cantor) 10. Mathematicians **would maintain** that not philosophical controvercies but rigorous mathematical theories constitute secure foundations. (Gauss)

Clauses

The Object Clause

a) *Express a wish about each of the following statements.*

Model.

to be {be / (was) **were**

can→could

Unfortunately, I **am not** a mathematicians.
I wish...
I wish I **were** (was) a mathematician.
I wish I **knew** mathematics.

1. Cantor **is not** our contemporary. We wish...
2. Cantor **can't see** that his set theory holds key position in modern mathematics. We wish...
3. The infinity **does not exist** in reality. We wish...
4. Infinite sets **are not easy** to define rigorously. We wish...
5. Continuum **can't be constructed** from its parts. We wish...
6. The calculus of transfinite numbers **is not readily mastered.** We wish...

b) *Refer the wish to the Past.*

Model. We wish Cantor's contemporaries **didn't attack** his ideas so viciously.
We wish(ed) Cantor's contemporaries **hadn't attacked** his ideas so viciously. Cantor wished his contemporaries...

1. We wish Cantor's set theory **were recognized** from the outset. We wished...
2. We wish Cantor's colleagues **appreciated** his way of reasoning. We wished...
3. We wish they **understood** that one-to-one correspondence was Cantor's fundamental idea. We wished...

4. We wish they **could grasp** the meaning of the concept of the power of an infinite set. We wished...

5. We wish they **didn't reject** the idea of equivalence of infinite sets. We wished...

6. We wish they **didn't doubt** Cantor's diagonal procedure. We wished...

The Comparison Clause

a) *Transform the sentences adding the phrase "mathematicians operate (use, employ)" according to the model.*

Model. **as if** (as though)	Numbers are ideas (They **do not exist** in reality). **Mathematicians operate** with numbers **as if they existed** (could, might exist) in reality.

1. Infinite quantities are widely used in mathematics. (They **have** physical counterparts) 2. Cantor's transfinite numbers are meaningful. (They **don't exist** in reality) 3. $\aleph_0$ is the symbol representing the number of positive integers. (It **doesn't tell** the number of objects in that collection) 4. C is informative. (The Number Continuum doesn't exist in reality) 5. The empty set is important. (It **doesn't exist** in reality) 6. The arithmetical expressions such as $\aleph_0+3$; $\aleph_0+\aleph_0$; $\aleph_0\cdot\aleph_0$ make sense. (Cardinal numbers don't exist in reality)

b) *Refer the action in the clause to the Past.*

Model. Mathematicians deal with infinity as it **were** an actuality.
Cantor dealt with infinity as if it **had been** an actuality.

1. Non-mathematicians accept natural numbers as though they **were** intuitively **given.** (Kronecker) 2. These mathematicians present mathematics as if it **were developed** purely formally. (Hilbert and his followers) 3. Some logicians treat mathematics as though it **were** only part of logic. (Russell and Whitehead) 4. Mathematicians develop their theories as if they **had** rigorous foundations. (Gauss) 5. Some mathematicians behave as if the crisis in the foundations **didn't concern** them. (The early XX c. mathematicians) 6. Axiomatic set theory regards a set as if it **were** an undefined term satisfying a list of axioms. (Zermelo)

The "If-" clauses

a) *Combine the two sentences by "if"-clauses I, II, III and make the statements unreal.*

Model. The term **is defined** precisely. It **doesn't lead** to contradictions.
I If the term **is defined** precisely, it **doesn't (will not, cannot) lead** to contradictions.
II If the term **were defined** precisely, it **would not lead** to contradictions.
III If the term **had been defined** precisely, it **would not have led** to contradictions.

1. The concept of a number **is elucidated.** Rigorous theories of number **are developed.** 2. There **exists** a one-to-one mapping of two sets. The sets **are** of equal power. 3. The set **is** infinite. Transfinite cardinal number **assigns** its power. 4. There **exists** a membership relation for the set. The Axiom of Choice **establishes** its well-ordering. 5. The concept of

cartesian product **is defined.** It **helps** to classify a function in a general way.

b) *Refer the actions to the Past.*

Model. If set theory **were developed** axiomatically, the contradictions **would not be revealed.** (Cantor)
If set theory **had been developed** axiomatically by Cantor, the contradictions **would not have been revealed.**

1. If the incommensurables **were not discovered,** the first crisis **would not occur.** (by Pythagoreans) 2. If paradoxes **were not** so subtle and colourful, mathematicians **would not** pay attention to them. (Zeno's) 3. If this paradox **were** easy to resolve, it **would not be** the goal of both specialists and laymen. (Russell's) 4. If Gödel's incompleteness theorem **were not proved,** rigorous and consistent philosophy of mathematics **would be created.** (in the XX c.) 5. If axiomatic set theories **were not developed,** a new science Axiomatics **would not originate.** (early in the XX c.) 6. If mathematics **had** no "subject matter", it could be reduced to logic. (by Russell and Whitehead)

c) *Combine the two statements by if-clauses and make them unreal.*

Model. Set theory **is** fundamental in mathematics.
Mathematicians **employ** it in many branches.
If set theory **were not** fundamental in mathematics, mathematicians **would** (could, might, ought to) **not** employ it.

1. Set theory **displays** its usefulness in many fields of mathematics. Modern mathematicians wholly **accept** it. 2. Modern mathematics **is characterized** by a higher level of abstraction. It becomes incomprehensible for a nonspecialist. 3. Mathematicians **attacked** Cantor's set theory viciously. Cantor **suffered** mental breakdowns. 4. Transfinite numbers **were introduced** by Cantor. The number system **was extended.** 5. Mathematical Logic **was developed.** More rigorous foundations of mathematics **were laid down.** 6. Abstract set theories **were created.** Greater mathematical rigour **was achieved.**

The Suppositional Mood

Replace the infinitives in brackets by the Suppositional Mood form.

The Subject Clause (should) + do

Model.	is	necessary	
	was	important	that the students of mathematics
It	will be	reasonable	(to know) set theory theorems
	has been	demanded	

It is necessary that the students of mathematics (should) know set theory theorems.

1. It is advisable that every one (to get familiar) with set theory symbols and their meaning. 2. It is desirable that we (to make) a list of Cantor's innovations. 3. It is clear that the Continuum Hypothesis (not to be rejected). 4. It is improbable that the Axiom of Choice (not to be appreciated) by a specialist. 5. It is demanded that a mathematical theory (to have) a rigorous foundation.

The Object Clause

Model. We { suggest / demand / propose / insist / claim, etc. } that every theorem (to be derived) deductively. **We insist that every theorem (should) be derived deductively.**

1. Cantor recommended that an infinite set (to be conceived) as a whole. 2. He suggested that infinite sets (to be matched one-to-one). 3. Cantor held that there (to be) an infinity of transfinite cardinal numbers. 4. Gödel's incompleteness theorem claims that no finitely axiomatizable system (to be) complete. 5. Philosophers insist that there (to be) no "real" progress in foundational questions due to Gödel's theorem.

The Clause of Purpose lest+should do

Model. a) Each set theory symbol is precisely defined (not to be mixed up) with its counterpart in speech.
Each set theory symbol is precisely defined **lest it should be mixed up** with its counterpart in speech.

1. Cantor designated the power of the Continuum by the letter C (not to be confused) with $\aleph_0$ infinte sets power. 2. In 1908 Zermelo created a formal system of axioms for set theory (not to lead to contradictions). 3. Zermelo's axiomatization of set theory was in the spirit of Hilbert's formalism (not to crop up paradoxes). 4. Weierstrass eliminated infinite quantities from the calculus (not to lead to inconsistencies). 5. The practical mathematician generally dismisses philosophical controversies (not to be deflected) from his own work.

b) **so that..., that..., in order that...** { can, may (could, might) do }

Model. Cantor gives a lot of illustrations **so that** people (to understand) that set theory refers to "real" mathematical objects.
Cantor gives a lot of illustrations so that people **can (could) understand** that set theory refers to "real" mathematical objects.

1. One must study Cantor's way of proving theorems in order that one (to see) good and beautiful mathematics. 2. Cantor extended the number concept so that he (to justify) the introduction of transfinite numbers. 3. Axiomatic set theories originated in order that paradoxes (to be escaped). 4. Gödel's theorem is the greatest barrier in order that a mathematician (to create) the consistent foundation for his theory. 5. Russell and Whitehead developed the abstract set theory (not to be plagued) by paradoxes.

Inversion

Make statements more emphatic.

I. **If-clauses.**

1. If set theory **were** not so complex, it would have been accepted from the outset.

2. If Cantor **had had** co-believers he wouldn't have been plagued so much by his colleagues' attacks.

3. If Cantor **could have resolved** to Continuum Hypothesis, there would have been no stimulus for further research in set theory.

4. If an infinite set **existed** in reality, mathematicians would have little problem with the infinite.

Should an infinite set **exist** in reality...

5. If you **asked** we wherein Cantor's greatness lies, I should say in the originality of his basic ideas. **Should you ask me...**

II. **Only, never, little, hardly ... when, no sooner ... than, not only ... ... but also.**

1. Mathematicians never display agreement in viewpoints during the transition period in the philosophy of mathematics.

Never do mathematicians **display...**

2. Cantor created his theory of sets only in 1895.

Only in 1895 **did Cantor create** his theory of sets.

3. Mathematicians couldn't resolve Zeno's paradoxes until Cantor created his set theory.

Not until Cantor created his set theory, **could mathematicians resolve** Zeno's paradoxes.

4. The controversy concerning Cantor's set theory started hardly Burali-Forti published his paradox.

Hardly did Burali-Forti publish his paradox when...

5. Mathematicians began to worry over the foundations of mathematics **as soon as** Russell's paradox became known.

No sooner did Russell's paradox become known, than...

6. The empty set had been used before Cantor by Boole in his algebra of classes, but it only later found its place in set theory through Zermelo's work.

...only later through Zermelo's work **did** the empty set **find** its place in set theory.

III. **so ..., neither ..., nor ..., no more ...**

1. The rational numbers are enumerable. The same holds for the algebraic numbers.

The rational numbers **are** enumerable, **so are** the algebraic numbers.

2. There are $\aleph_0$ natural numbers, and **there are** $\aleph_0$ prime numbers.

There are $\aleph_0$ natural numbers **so are there** $\aleph_0$ prime numbers.

3. Every cardinal number n has a successor $n+1$, $\aleph_0$ has this property too ($\aleph_0$, $\aleph_1$, $\aleph_2$...).

Every cardinal number n has a successor $n+q$, **so has** $\aleph_0$ ($\aleph_0$, $\aleph_1$, $\aleph_2$, ...).

4. Cantor did not prove the Continuum Hypothesis. His followers failed to do it either.

Cantor did not prove the C. H., **neither (nor) did** his followers.

5. Gödel ruled out the "disprovable" possibility in 1938. Cohen ruled out the "provable" case for the C. H. in 1963.

Gödel ruled out the "disprovable" possibility for the C. H. in 1938, **so did Cohen** for provable case in 1963.

6. The Formalists did not settle the fundamental questions in the foundations of mathematics. The Logicists did not cope with it either.

The Formalists didn't settle the fundamental questions in the foundations of mathematics, **neither (no more) did** the Logicists.

7. The first crisis occurred in number theory. The third crisis happened in the same field of mathematics.

The first crisis occurred in number theory, **so did** the third.

8. Kronecker did not accept Cantor's work, Poincaré rejected it either.
Kronecker did not accept Cantor's work, **neither did** Poincaré.

9. Frege and Weierstrass sought to build new foundations of mathematics. Cantor contributed a lot to the elucidating fundamental questions in foundations.
Frege and Weierstrass sought to build new foundations of mathematics, **so did** Cantor.

10. The creators of axiomatic set theory did not construct secure foundations for mathematics (due to Gödel's theorem).
This theorem destroyed the hope of the abstract set theory creators in this respect.
The axiomatic set theory creators didn't construct secure foundations for mathematics, **neither did** the abstract set theory creators.

CONVERSATIONAL EXERCISES

I. *Confirm the following statements, giving possible justifications.*

1. Mathematics and Logic, historically speaking, have been entirely distinct studies. 2. Mathematics was connected with science, Logic with the Greeks. 3. Classical mathematics used Logic to generate theorems. 4. Zeno's paradoxes showed that classical mathematics was not perfectly clear. 5. Logic is the youth of mathematics, and mathematics is the manhood of Logic. 6. The modern very close relationship of mathematics and Logic has become obvious. 7. Nowadays mathematics has become more logical, Logic — more mathematical. 8. There is no point at which a sharp line can be drawn between them. In fact the two are one. 9. The new foundations of mathematics were based solely on pure logic. 10. The builders of modern foundations used deductions that were purely logical. 11. Mathematics, however, can't be grounded exclucively on Logic. Logic alone is not sufficient. 12. Mathematics does have extralogical "subject matter". 13. The mathematical concept of the infinite has yielded the origin of a new discipline — Mathematical Logic. 14. Mathematical Logic is now a recognized and powerful field of mathematics. 15. The symbolic notation of Mathematical Logic is an indespensable tool of set theory.

II. *Agree or disagree.*

1. The concept of the infinite has nothing to do with the controversy between the schools of philosophy of mathematics. 2. The language of the main schools in the philosophy of mathematics diverged. 3. All concepts should be derived from experience **(Classical expericists).** 4. Infinite quantities must be eliminated from mathematics altogether **(Intuitionists).** 5. Intuitionist mathematics does not belong to science as it consists in mental constructions. 6. Intuitionism is the only possible way to construct mathematics. 7. A mathematical entity exists if its non-existence is impossible **(Existentialists).** 8. The universal validity of this thesis should be denied. 9. All true mathematical statements are true by convention **(Conventionalists).** 10. The self-evidence of the existence of infinite quantities must be proved by "finitistic" means **(Formalists).** 11. Mathematics is essentially the study of formal systems. 12. Formalists are all primarily mathematicians rather than philosophers. 13. What is true for the intuitionist is also true for the formalist. 14. Mathematics does not have any "subject matter" as it deals with pure relations among abstract concepts **(Logicists).** 15. Mathematics must be reduced to Logic. 16. To say that mathematics is Logic is me-

rely to replace one undefined term by another. 17. The notion of infinity is entirely foreign to pure Logic. 18. Gödel's incompleteness theorem represents the greatest obstacle to a satisfactory philosophy of mathematics.

III. *Add some more information, details, facts and reproduce the topic "The foundations of mathematics". The following statements may prove helpful.*

1. Mathematics needs rigorous logical foundations. 2. It is unambiguous mathematical questions not matters of taste, that are being investigated in the foundations of mathematics. 3. Dedekind, Frege and Cantor are the most celebrated mathematicians for their work in the foundations of mathematics. 4. The problem of the nature of mathematical entities has been the central question in the researches of the foundations of mathematics. 5. One of the most important questions for the foundations of mathematics is that of the relation between mathematics and logic. 6. The discovery of paradoxes in set theory threatened the very foundations of mathematics and logic. 7. Most mathematicians, however, tend to view with profound indifference the problems of rigorous foundations. 8. The fact that the foundations of some mathematical concept or theory are not secure do not prevent mathematicians from using them. 9. Despite the crisis in logic and theory of sets, there was a feeling of confidence that the new foundations for analysis, geometry and abstract algebra could be safely used to build new theories. 10. New mathematical sciences were developed (e. g., Mathematical Logic, the set-theoretic topology, etc.), applications of the newer theories were made. 11. Mathematical theories originally invented purely for mathematical purposes have ultimately found application in other sciences. 12. The problem of the rigorous logical foundations of mathematics has not yet been completely solved. 13. Of the various attempts made by mathematicians, logicians and philosophers to solve the problem none has resolved every difficulty. 14. Nevertheless, modern mathematics is independent of the physical world for its theoretical justification. 15. It had laid down its own foundations.

IV. *Read the following statements, add some more details (information) available and relate Cantor's life story.*

1. G. Cantor was born in 1845 in St. Petersburg, Russia. 2. However, Cantor should properly be ranked among the German mathematicians. 3. Cantor spent the greater part of his life in German universities. 4. The three great mathematicians at Berlin — Kummer, Weierstrass and Kronecker — stimulated Cantor's interest in number theory. 5. Cantor's work until 1870's gave no hint of the great innovator which he was to become. 6. Function theory, group theory, trigonometric series were Cantor's main concern. 7. Dedekind's abstract logical way of thinking greatly influenced Cantor. 8. His work with trigonometric series led him to the theory of point sets and to the transfinite ordinals. 9. As Cantor's work on transfinite numbers advanced, Kronecker's opposition grew. 10. In the period of 1879—1884 Cantor published articles with his complete theory of sets. 11. There was tremendous opposition to Cantor's work by many prominent mathematicians of that time. 12. Hilbert and Minkovsky were the first in Germany to recognize the originality and significance of Cantor's logic, reasoning and his theorems. 13. It was not until 1897 at the first International Congress of mathematicians, held in Zurich, that Cantor was generally recognized. 14. Other mathe-

maticians came to employ Cantor's ideas in their work. 15. The early twentieth century saw the spread of Cantor's set-theoretic manner in mathematics. 16. Cantor's set theory has infiltrated into and largely transformed most branches of mathematics. 17. In 1915 Cantor's seventieth birthday was celebrated as an event of international importance.

V. *Explain in your own words what is meant by the following assertions.*

1. There is a remarkable permanency in the concern with the infinite in mathematics. 2. The infinite does not exist in nature; it is nowhere to be found in reality. 3. The meaning of the infinite has never been completely clarified. 4. The role played by infinite quantities in mathematics has been enormous. 5. The mathematics of the infinite was the central obstacle that had stopped Pythagoras and halted his successors for over two thousand years. 6. Eudoxes discovered a way round the obstacle or perhaps through the thickest part of it. 7. The builders of the modern continuum followed Eudoxes's path and sought to give it a firmer foundation. 8. References to the infinite are meaningless (Hilbert). 9. Infinite quantities should be eliminated from the calculus (Gauss, Cauchy, Weierstrass). 10. Mathematics must not base assertions concerning the existence of infinite structures on physical considerations. 11. Both Frege and Dedekind used the concept of the "actual infinite" to provide the foundation of arithmetic independent of intuition and experiment. 12. But it was G. Cantor who systematically developed "actual infinite" in his set theory. 13. No physical structure (not even an infinite one) can serve as a standard model for set theory. 14. Cantor's infinities are abstract, entirely divorced from the physical world. 15. The reaction to Cantor's actual infinite took, in fact, a very dramatic form. 16. The third "crisis" was precipitated by too bold a use of infinite classes. 17. Nevertheless, Cantor's belief in the actual existence of the infinities of set theory still predominates. 18. Infinite sets are accepted in modern mathematics with no reservation.

VI. a) *Characterize the given below definitions as (in)correct, (in)adequate, (in)accurate, (non)rigorous, (non)scientific, ect. and choose or give your own one that may be used as a general definition. Some examples of infinite sets should be also supplied.*

1. A set is **infinite (transfinite)** if it does not contain an object which is greater than or equal to each of the others. 2. A set is **infinite** if it contains either no smallest or no largest element. 3. A set is **infinite** if and only if its elements can be put into one-to-one correspondence with those of a proper subset. 4. A set is **infinite** if and only if it is equivalent to a proper part of itself. 5. A set is **infinite** if and only if there exist a proper subset equivalent to it.

b) *What is meant by the following assertions.*

1. The non-enumerable totality of the real numbers is called the "Continuum" by the mathematicians. 2. The prime characteristic of the structure of the contimuum is that the infinite decimal fractions cannot be "separated" from each other. 3. Not only do the real numbers lie "dense" — as the rational numbers do — but there are absolutely no gaps between them; they lie continuously. 4. The Continuum cannot be constructed from its parts. 5. The symbol ∞ for "infinity" must not by any means be confused with a symbol for a number.

VII. *Choose the "correct" definition of function. Justify your choice.*

Many texts were examined for definitions of "function", the given below are those that are most frequently used.

Definition I — A function is a set of ordered pairs whose first elements are all different.

Definition II — When the value of one variable depends on another, the first is a function of the second.

Definition III — If to each permissible value of x there corresponds one or more values of y, then y is a function of x.

Definition IV — If y is a function of x, then it is equal to an algebraic expression in x.

VIII. *Repeat the given statements and expand them with examples, illustrations, reasons and evidence of your own.*

1. In the first period of its existence set theory was practically and exclusively the creation of G. Cantor. 2. Set theory caused many mathematicians to regard it with a certain degree of distrust and reluctance. 3. Cantor's doctrine was attacked on all sides. 4. So violent was this reaction that the most fruitful concepts and deductive methods of set theory were threatened. 5. In the course of years, however, set theory displayed its usefulness in many fields of mathematics. 6. Further development of set theory went mostly in an abstract way. 7. Axiomatic and abstract set theories were little concerned with other branches of mathematics. 8. The object of set theory is to investigate the properties of sets from the most general viewpoint. 9. The ideas and methods of set theory penetrate current mathematics. 10. In set theory one can define many important mathematical concepts, e.g., of a function, sequence, the power of the set, etc. 11. Concepts such as the **union** and **intersection of sets, countability, closed sets, one-to-one correspondence** are now classical. 12. The concept of a number has also come to be better analyzed in terms of set theory. 13. Operations on sets are analogues to arithmetic operations, e.g., sets have the properties of commutativity, associativity and distributivity. 14. These properties do not depend on whether the set consists of numbers, points or other mathematical entities. 15. The symbols of set theory are both simple and efficient, though they are not mere shorthand symbols. 16. Each symbol differs from its counterpart in speech in virtue of the fact that it **is precisely defined.** 17. Set theory has provided a language and symbolism to synthesize mathematics both the old and the new. 18. In every domain of mathematics we have to deal with sets. 19. Thus, set theory is the fundamental mathematical theory.

IX. *Reproduce the text using the statements given after it.*

The Axiom of Choice

Systems of infinitely many simultaneous correspondences are a matter of course in current mathematics, or may even be considered as characteristic of mathematics. Within the framework of the logical and mathematical procedures which were usual and recognized up to the end of the 19th century, one is not permitted to take infinitely many steps in choosing arbitrary elements which are not determined by a definite law. **This exclusion of an infinity of choices** was based by some critics on the argument that any logical procedure must be brought to an end within a finite length of time which would apparently be impossible in this case. Yet, this argument is hardly tenable, for the process of thinking should be regarded as instantaneous and not as taking a definite length of time. A deeper analysis shows that procedures involving infinitely many arbitrary steps have been avoided in the past not for the reason mentioned above, but because these procedures were considerel to be meaningless, not merely nonconstructive.

Imagine a denumerable set whose elements are **pairs of shoes**: a first, second, ..., n-th, ... pair for every positive integer n. Is the set of all these **pairs** equivalent to the set of all **shoes** contained in the pairs? The answer is, of course, in the affirmative. We may assign to the first pair the left shoe of the first pair; to the second pair the right shoe of the first pair; to the third pair the left shoe of the second pair, and so on. Then, the left shoe of the n-th pair corresponds to the $(2n\text{-}1)$st pair, the right shoe of the n-th pair to the $(2n)$th pair. Evidently this rule yields a mapping of the set of all pairs onto the set of all shoes, hence the sets are equivalent; there are $\aleph_0$ pairs and $\aleph_0$ shoes. However, the situation changes completely if we consider **infinitely many pairs of stockings** instead of pairs of shoes. The difference lies in the fact that manufactures produce identical stockings for both feet. Certainly, we may start by assigning to the first pair an **arbitrary** stocking of this pair and to the second pair the other stocking of the first pair, etc. Yet, now we can continue this procedure only finitely many times, unless we are prepared to admit an infinity of arbitrary choices of stockings out of the pairs of stockings. So long as we exclude this, in accordance with mathematical tradition throughout history, we cannot determine whether the set of all stockings has the same cardinal $\aleph_0$ as the set of all pairs.

Of course, the significance of this example is merely expositional, not scientific. Nevertheless, this is an example in which an important problem is unsolvable without the use of **an infinity of choices**. Thus there emerges a new principle of logic and mathematics, which was discovered only at the beginning of the twentieth century, and which is indispensable for proving many important statements in various branches of modern mathematics. This principle is called the **Axiom of Choice** (after Zermelo, who first introduced it explicitly in 1904), or the **multiplicative principle** (after Russell, who found a better formulation in 1906).

Axiom of Choice: If S is a disjoint set of nonempty sets, i.e., such that any two members of S have no common element, then **there exists** at least one set C which contains a single element out of each element of S.

Any such set C is called **a choice set** of S. By means of this formulation we have eliminated the function concept and arbitrary "choices" and the problem has been reduced to the existence of sets. The axioms of set theory alone should determine what sets do actually exist. As shown by Zermelo (1908), the general axiom of choice can be derived from the above axiom by constructive procedure which are generally recognized in logic and mathematics. The history of the axiom of choice is interesting and may in some respects be compared with the most famous axiom in mathematics, Euclid's parallel axiom. In the 1880' and in 1890's Cantor had already in the proof of well-ordering theorem used an argument which is logically equivalent to the axiom of choice; yet he had done so implicitly and was not conscious of using a new principle.

Between 1904—1910 papers rejecting the axiom of choice were published in many leading mathematical journals. Poincare (1910) remarked: "This dispute about Zermelo's ingenious axiom is rather strange; one side rejects the axiom of choice but accepts its proof, the other accepts the axiom but not the proof of the theorem". Poincare himself belonged to the 'other", who were a small minority. Many mathematicians still take a negative attitude towards the axiom of choice. Many objections against the axiom of choice are based on misunderstanding and are therefore void, in as much as they ignore the **purely existential nature** of its statement. Because of this existentiality it is quite natural and to be expected that one cannot deduce from the well-ordering theorem **where** in the series of cardinals the cardinal $\aleph$ of the continuum is to be found. This question constitutes the **continuum hypothesis**; to solve it, we need a constructive formation of the corresponding choice set. As a matter of fact, except for a partial (though profound) result of Gödel in 1938—1940, the attempts made by outstanding mathematicians since 1880's to solve this problem remained unsuccessful until 1963. It is psychologically understandable that the mistrust toward the axiom of choice has deepened, because it is of no help in the solution of the continuum problem. In 1963 P. J. Cohen showed in an ingenious proof that the problem is **unsolvable**; that is to say, various positions in the series of alephs for the cardinal of the continuum are compatible with the axioms of set theory.

Yet, only those mathematicians and philosophers who in principle acknowledge only constructive, not existential, procedures are entitled to reject the axiom of choice for such reasons; among these, in particular, are intuitionistic and neo-intuitionistic schools. However, insofar as they keep to their principles, they restrict the methods of mathematics to such an extent that outside of arithmetic only narrow fields can be investigated. Actually, psychological rather than logical reasons played a leading part in the rejection of the axiom of choice. Prominent among them was the aversion to the well-ordering theorem, which was considered too strong a statement that yielded too few "practical" results. On the other hand, most critics had to admit that if the axiom of choice were accepted they could discover no shortcoming in Zermelo's proof. Hence the only way out is the rejection of the axiom of choice.

It is a strange phenomenon, which rarely occurs in the exact sciences, that discussions become stagnant over a period of many decades. Yet, this is what happened to the axiom of choice, and hardly any fundamentally new ideas have emerged in the discussions. On the other hand, the fact that the axiom of choice is compatible with the other principles of set theory was proved by Gödel in several profound papers since 1938. The same papers also deal with the continuum problem. Comparing Gödel's result with the situation described, we see that the main objective was either a proof that the **negation** of the choice axiom is also compatible, or a proof that the negation is contradictory. The situation is somewhat similar to that which had existed in geometry with respect to the axiom of parallels. The anologue of absolute geometry in the present case is that part of set theory which can be treated without reference to the axiom of choice.

Finally, we should outline the significance and the applications of the axiom of choice in various branches of mathematics. The axiom is used throughout analysis especially in the theory of real functions as well as in set theory and in wide domains of topology. As far as set theory is concerned, the **arithmetic** of transfinite cardinals, ordinals, and order types is essentially based upon the choice axiom. Its indispensability in abstract algebra is obvious. It should be stressed again that the axiom of choice is needed for a complete analysis of the concepts of finite set and finite number (cardinal, ordinal). Only by using the axiom of choice can we prove that any set or cardinal is either finite or infinite. Hence any cardinal either equals $\aleph_0$ or is greater than $\aleph_0$. Thus although the axiom of choice has entered mathematics only recently, it has proved indispensable for the structure and development of mathematics.

Statements

1. The Axiom of Choice (A. Ch.) asserts: Given any collection of disjoint non-empty sets there exists a set having exactly one element from each of the given sets. 2. The first explicit statement of the axiom was given by Zermelo in 1904. 3. Cantor's implicit use of the axiom is obvious in the proof that every transfinite set contains a countably infinite subset. 4. Neither the A. Ch. or its negation can be deduced from the other axioms of set theory. 5. If we assume the A. Ch. then there exists a relation for any set which establishes its well-ordering. 6. The A. Ch. is generally accepted as one of the basic principles of mathematical reasoning. 7. The A. Ch. is safe as its use cannot introduce any contradiction. 8. The A. Ch. is indeed a fundamental principle of inference in mathematics. 9. There are many logically equivalent mathematical assertions of the A. Ch. 10. The A. Ch. cannot be dispenced with in modern mathematics.

X. *Give a proof or explain in your own words the meaning of Cantor's set theory a) theorems; b) basic concepts.*

a) 1. A countable union of countable sets is countable (denumerable). 2. The set of integers is countable. 3. The set of algebraic numbers is countable. 4. The set of rational numbers if countable. 5. The set of real numbers if not countable. 6. For any set A, there does not exist a function mapping A onto its power set $P(A)$. (Russell's paradox). 7. Let A and B be sets such that there exists a one-to-one map of A and B and one-to-one map of B into A. Then there exists a one-to-one correspondence between A and B. 8. Let "f" be a one-to-one map of a set A onto itself. Let C be a subset of A containing $f(A)$. Then there exists a one-to-one correspondence between A and C. 9. Any set can be well ordered. Any set of cardinal numbers is well-ordered. The relation $\leqq$ on cardinal numbers is a partial ordering. 10. For any sets A and B there exists a one-to-one function of A into B or one-to-one function of B into A (or both). 11. Every finite set is such that is equivalent to none of its subsets. 12. Every transfinite set has subsets which are equivalent to it. 13. For every transfinite cardinal number there exists a next one greater. There exists an infinity of transfinite cardinal numbers, e. g., $\aleph_0$, $\aleph_1$, $\aleph_2$ (representing the set of all possibe curves in a plane). 14. There exists a transfinite cardinal number less than $\aleph_1$ but greater than $\aleph_0$. (The Continuum Hypothesis).

b) 1. Cardinal numbers characterize **the power** of infinite sets. 2. One can classify infinite sets with respect to their power. 3. To sets having the same power as the set of all natural numbers (countaly infinite sets) we assign the cardinal number $\aleph_0$. 4. To the set of all real numbers we assign the number C (the power of the continuum). 5. The concept of "cartesian" product allows us to define the concept of a **function** (or a mapping) in a general way. 6. If there exists a one-to-one mapping of the set X onto the set Y then these sets are of **equal** power. 7. The equality of powers is the generalization of the idea of equal number of elements. 8. The significance of this generalization depends on the fact that it can be applied to both finite and infinity sets. 9. In the applications of set theory only two of transfinite cardinal numbers $\aleph_0$ and C

play an essential role. 10. It is possible to assign a meaning to expressions such as $\aleph_0+3$; $\aleph_0+\aleph_0$; $2\aleph_0$; $\aleph_0\cdot\aleph_0$ and even $2^{\aleph}$ if all these numbers are to be interpreted as cardinal numbers. 11. It is possible to define $\aleph_3$ as $\aleph_2^{\aleph_2}$ but mathematicians have as yet been unable to conceive of a collection of objects having such a high order of infinity. 12. It seems that human imagination does not permit one to count beyond **three** when dealing with infinite sets.

XI. *Agree with the following negative statements.*

1. Cantor's viewpoint didn't triumph for a long time. 2. No other mathematical concepts have had as great an impact on modern mathematics as has the set notion. 3. Set theory is not finite and exhausted. 4. Cantor posed the Continuum Hypothesis (C. H.) but failed to solve it. 5. No mathematician can claim that the C. H. could be disposed of by denying its existence. 6. The truth or falsity of the C. H. cannot be determined within the set theory. 7. There is no verification or refutation procedure for the C. H. 8. Mathematicians cannot deny that the C. H. is distinguished from other hypotheses by its uniqueness, independence and inner beauty. 9. The C. H. is not so far solved though it is the first on the list of the famous Hilbert's unsolved problems. 10. No new methods have been proposed for its solution. 11. No new results have been obtained for the time being. 12. The C. H. is not an easy nut to crack, neither was Euclids parallel postulate. 13. Cantor did not state the Axiom of Choice explicitly. 15. The Axiom of Choice does not introduce any contradictions. 15. Neither the Axiom of Choice or its negation can be deduced from the other axioms of set theory. 16. The Axiom of Choice cannot be disposed with in current mathematics.

XII. *Disagree with the following negative statements.*

1. Set theory is not an independent branch of mathematics. 2. The creation of set theory is not Cantor's alone. 3. Cantor didn't found any new science. 4. Cantor didn't deal with Number Continuum at all. 5. Cantor didn't advance his research to infinite sets. 6. Cantor didn't extend the number system. 7. Set theory is nothing more than a hypothetical theory; the mathematicians find it amusing to study. 8. Set theory is no more than luxury in mathematics. 9. Set theory is not akin to logic — an objective constituent of mathematical reasoning itself. 10. Totalities of infinitely many members cannot be made accessible for mathematical treatment. 11. Set theory fails to demonstrate how mathematicians can calculate with infinite totalities. 12. One cannot use the term "Continuum" for the non-enumerable totality of the real numbers. 13. Cantor's set theory had no influence on mathematics. 14. Much of classical mathematics has never been reinterpreted in set-theoretic terms. 15. Cantor's contribution has never been recognized and appreciated. 16. Cantor's set theory is of no use nowadays.

XIII. *Supply the given below statements with a reason, proof, illustration, justification, applications, etc.*

1. The lack of rigorous foundations of set theory led to the contradictions revealed in the theory. A contradiction arises from a contradictory definition. 2. As a result of employing imprecise and non-rigorous definitions contradictions or paradoxes began to crop up in set theory. 3. In developing set theory mathematicians paid too little attention to the validity of their deductive methods. 4. The paradoxes though at first scattered became progressively more acute and serious. 5. Although parado-

xes are entirely remote from ordinary mathematical reasoning, they did point out the need for care in determining which properties describe sets. 6. More than ten paradoxes were discovered; Russells' paradox in particular, had a catastrophic effect. 7. Russell's paradox is the prototype of a whole family of paradoxes. 8. It became obvious that one should find a way of avoiding paradoxes without betraying mathematics. 9. Mathematicians came to realize that the way could be attained if the nature of infinite were fully elucidated. 10. Too many different remidies and cures were offered to resolve paradoxes. 11. To avoid the paradoxes Russell introduced the theory of types and the axiom of infinity. 12. Russell's type theory cuts out the paradox but at a very high price in complexity. The theory is arbitrary and artificial. 13. Russell's paradox is turned into a proof that a certain object is not a member of a class (set). 14. Zermelo's axiomatic set theory claims to have removed those contradictions. **15.** Zermelo banned universal class and allowed the formation only of subclasses with a given property. 16. Paradoxes were the indicators of the crisis which has still not been resolved to the satisfaction to all concerned. 17. Mathematicians have learnt to take advantage of paradoxes and most profound results in modern Logic have arisen from the analysis of paradoxes.

XIV. *Is the paradox of Tristram Shandy (Laurence Sterne A. M. The Life and Opinions of Tristam Shandy, gentleman. Б. В. Л. Т. 61) logical or semantical? Prove it.*

Tristram Shandy was hopelessly perplexed. He had begun to write his autobiography and found that he could record only half a day's experiences in one day of writing. Consequently, even if he **were to start** writing at birth and even if he **were to live** forever, he **could not record** his whole life, for at any time only half of his life **could be recorded.** And yet if **he did live** on indefinitely he **ought to be able** to record his whole life, for the experiences of his first ten years **would be recorded** by the end of his twentieth year; and so on. Thereby every year of his life **would be reached** at some time. Hence depending on which way he reasoned, he **could** or **could not complete** his autobiography. The longer Tristram puzzled over this paradox, the more confused he became and the farther he was from a decision. Tristram's inability to resolve the paradox was really to be expected, for his problem involved an infinitude of time and **could be resolved** by use of the theory of infinite classes.

Were we seriously **concerned**, we **could use** Cantor's equation $\aleph_0 = 2\aleph_0$ to solve the dilemma of Tristram Shandy. Tristram was puzzled because he could record only half a day's experiences in one day, so that even if he **were to live** an infinite number of years he apparently **could record** only half of his life. On the other hand, it was equally clear to him that if he **were to live** forever, every year of his life **would be recorded** at some time. The mathematical theory of infinite quantities supports the latter argumest. If he **were to live** $2\aleph_0$ years, he **could record** $\aleph_0$ years of his life. But to live $2\aleph_0$ years is to live $\aleph_0$ years, and so Tristram **could favour** posterity with his completed autobiography.

Equations involving $\aleph_0$ seem incorrect to us because we are accustomed to thinking in terms of what holds for finite numbers. Yet there is nothing illogial here. Properties that hold for finite numbers need not hold for transfinite number, nor **does the reverse** need to be the case. Only through Cantor's modern theory of sets **can** many trouble-making problems **be solved**, new insights **be gained**, "known mathematics" **be made** more rigorous and precise and analyses of mathematical situations

become clarified and often simplified. In fact, the theory of ses has reached the heart of the philosophical foundations of mathematics.

XV. *Give your own suggestions or a way of resolving the "Barber Paradox". Most people hearing the Barber Paradox for the first time soon come up with the following suggestions.*

1. The barber is a woman or a young boy (the barber, in fact, is a man who pays daily tribute to the blade). 2. The barber is not himself one of the villagers (but he is). 3. The totality we are talking about does not include the barber (but it does). 4. There is no barber in the village. Everyone shaves himself. 5. If the assumption that there is a barber leads to a contradiction, then this assumption is false; there is no barber. 6. The barber paradox is an unsuccessful attempt to express Russell's paradox in a popular form. 7. The paradox is nothing but a proof that there is no barber in the village. 8. The barber paradox does not describe a real situation; hence it is not a paradox at all.

XVI. *Compose questions and a) answer them in writing at home according to the model; b) practise them orally in class.*

Model.	If you asked Should you ask	me	что такое «множество»	I	should would could might ought to	say that...
	If I were asked Were I asked If I were to be asked Were I to be asked					

Should you ask me what a "set" is, I'd say that unless otherwise specified "set" is an undefined concept in modern mathematics.

1. ... что породило понятие «множество» в математике...
2. ... что такое конечное (бесконечное, счетное, несчетное, пустое) множество...
3. ... как возникла теория множеств...
4. ... кто создатель теории множеств...
5. ... как обогатилась античная математика в результате первого кризиса...
6. ... почему математики заговорили о новом кризисе в математике в конце XIX в....
7. ... каковы характерные черты этого кризиса...
8. ... почему современники Кантора не считали теорию множеств надежным основанием для математики...
9. ... почему развитие математики пошло по линии дальнейшей формализации и более высоких уровней абстракции...
10. ... многих ли математиков беспокоил новый кризис в основаниях математики...
11. ... какие школы математиков пытались создать новые основания математики...
12. ... разрешен ли кризис в настоящее время...
13. ... каковы основные нововведения Кантора...
14. ... каковы основные открытия Кантора...
15. ... почему теория множеств занимает ключевое положение в современной математике...
16. ... разрешены ли парадоксы теории множеств...
17. ... как обогатилась современная математика в результате кризиса начала XX в.

18. ... какие наиболее фундаментальные понятия современной математики...

19. ... как оценить число элементов в бесконечном множестве, не пересчитывая их...

20. ... сколько четных натуральных чисел...

XVII' *Say it in English.*

Общепризнанно, что теория множеств создана Георгом Кантором. Действительно, с 1872 по 1897 г. Кантор опубликовал ряд работ, в которых были систематически изложены основные результаты теории множеств, включая теорию точечных множеств и теорию трансфинитных чисел (кардинальных и порядковых). Именно Кантору мы обязаны основными понятиями этих теорий и введением в математику рассуждений нового типа, которые он применил для доказательства основных теорем теории множеств. Кантор доказал (1874) неэквивалентность множеств рациональных и действительных чисел; ввел (1878) общее понятие мощности множества и разработал основы отображения и сравнения множеств, высказал (1883) континуум-гипотезу. Развитие теории множеств протекало в обстановке острой критики и борьбы. Особенно острыми были выступления берлинского профессора Л. Кронекера, основные работы которого относились к алгебре и теории групп. В вопросах оснований математики Кронекер был приверженцем арифметизации математики, т. е. он стремился свести все трудности, связанные с обоснованием любой области математики, к натуральному ряду. Около 1900 г. было обнаружено, что рассуждения, казавшиеся сходными с рассуждениями Кантора, приводят к противоречиям. В 1897 г. Бурали-Форти опубликовал парадокс наибольшего порядкового числа, да и сам Кантор обнаружил парадокс относительно существования множества всех множеств. В 1903 г. Б. Рассел опубликовал еще более поразительный парадокс о множестве всех множеств, не содержащих самих себя в качестве своего элемента. Некоторые математики заключили, что при рассмотрении множеств нельзя просто полагаться на интуицию, хотя множества являются фундаментальными открытиями для математики и человеческого мышления. Другие математики отвергали всю теорию множеств, считая ее ошибочной и несостоятельной. Вопросы обоснования теории множеств, исследование и уточнение понятий, которые лежат в основе теории множеств, а также выделение тех рассуждений, которые приводят к противоречиям; исследования пределов применения теории множеств привели в XX в. к созданию специальной науки — математической логики, составляющей важную часть оснований современной математики. В эпоху, когда развивалась теория множеств, большой интерес вызывали аксиоматические системы геометрии и арифметики (теории чисел). Казалось вполне естественным попытаться построить системы аксиом и для теории множеств. В 1908 г. были опубликованы две системы, которые построили независимо друг от друга, Цермело и Рассел. Построенная Кантором общая теория мощностей множеств, отображений, операций над множествами, свойств упорядоченных множеств составила основное содержание всех последующих аксиоматических теорий множеств. Цермело первым попытался явно и строго формализовать методы, которые Кантор применил неявно. Цермело избегает противоречий, налагая некоторые ограничения на объем совокупностей, которые можно рассматривать как множества. Абстрактная теория типов Рассела также дает средство избегать парадоксы, но оказывается, что существенная часть математики не может быть изложена в рамках теории типов, так как нельзя быть полностью уверенным в непротиворечивости теории типов. (Теорема Геделя). Были созданы и многие другие теории множеств (Неймана, Бернайса, Куайна, Френкеля, Сколема и др.) с иными и более современными системами аксиом, которые являются объектом теоретических исследований в наши дни. Теорию множеств Кантора сейчас называют «наивной теорией множеств», так как ее идеи и методы не были формализованы и аксиоматизированы. Однако в алгебре, топологии, функциональном анализе, теории меры, теории аналитических функций и т. д. именно «наивная» теория множеств Кантора используется главным образом, теория оказавшая такое огромное воздействие на развитие современной математики и исследования оснований истории и философии математики. Дальнейшее углубление исследований по основаниям математики сосредоточивается на преодолении логических трудностей, возникших в общей теории множеств средствами математической логики.

COMPOSITION

"If one wants to use a short slogan which hits at the very centre of modern mathematics, then one may well say that **it is the science of the infinite**" (Hilbert).

Look through all the texts of the lesson again, collect the information and write two-pages-long composition on the topic: "The Mathematics of the Infinite".

COMPREHENSION EXERCISES

Questions

1. Set theory is no doubt familiar to every student of modern mathematics, isn't it? 2. What is Cantor's definition of a set? Why does it look reasonable at first, but show its contradictory character afterwards? 3. A set is determined by whether or not a thing is an element of the set. Is the **membership property** the only way to determine a set? 4. How are sets commonly designated in current mathematics? 5. Is everything for us a set? Is every set a set of sets? 6. Does there exist a transfinite cardinal number for each infinite set? 7. Can one show or prove that every transfinite cardinal is an aleph? 8. Transfinite numbers are new names for incommensurables, aren't they? 9. Can the word "number" at all justified for $\aleph_0$? 10. Can one show that some or all of $\aleph_0$, $\aleph_1$, $\aleph_2$, ... are not vacuous? 11. Why does there exist only one null or empty set? 12. Natural numbers are of fundamental importance of mathematics, aren't they? Why? 13. Can infinity be constructed? Can the infinite be "non-enumerable"? 14. Why did Cantor's viewpoint triumph completely after all? 15. In what fields of mathematics did the crises (I, II, III) occur? 16. What comprises the philosophy of mathematics? 17. What are philosophical problems that a philosopher of mathematics ought to be interested in? 18. What is meant by the "foundations of mathematics"? 19. What mathematicians contributed to the creation of secure and rigorous foundations of mathematics? 20. What has Mathematical Logic done for the philosophy and foundations of mathematics? 21. What is the meaning of Gödel's incompleteness theorem and its importance for the philosophy of mathematics? 22. Did axiomatic and abstract set theories remove contradictions in Cantor's set theory? 23. Did abstract set theories resolve set theory paradoxes? 24. Why are mathematicians so anxious to find a way out of the paradox? 25. What are the possible ways of escaping paradoxes? 26. What is your favourite paradox? Why? 27. Why is the ma-

thematician not satisfied until he clearly exhibits the system (set) of postulates (axioms) for the system concerned?

Problems

1. Show that the set of all even whole numbers is infinite. Show how to pair this set, one-to-one, with the set of whole numbers. 2. Explain why the set of all people over five feet tall is not an infinite set. 3. Show that there are infinitely many positive integers whose last digit is a 5 by establishing a one-to-one correspondence with a proper subset. 4. Prove that there are $\aleph_0$ positive multiples of 11. 5. Show that $\aleph_0+12=\aleph_0$. 6. Prove that $\aleph_0 \cdot 2=\aleph_0$ by combining the elements of $\aleph_0$ sets containing 2 numbers each. 7. Establish a one-to-one correspondence to show that there are as many points on a line segment of length 1 as there are on a line segment of length 5. 8. Establish a one-to-one correspondence to show that there are as many points on the circumference of a circle as there are poins on a line extending indefinitely in both directions. 9. Consider a set R consisting of all the whole numbers and a set T consisting of all the natural numbers

$$R=\{0, 1, 2, 3 \ldots\}$$

$$T=\{1, 2, 3, 4 \ldots\}$$

Is T a proper subset of R? Can you give a rule for one-to-one matching between R and T (note that both sets are endless?

Discussion

1. Zeno sought to defend mathematics against common sense but he made no defence of mathematics. He simply denies that mathematics has to make common sense. Are today's mathematicians on Zeno's side? Explain.

2. However much mathematicians recoiled from infinite quantities, by the middle of the nineteenth century mathematics could no longer dispense with the concept. Why?

3. The mode of thought which serves for finite collections is no reliable guide to an understanding of infinite collections. It was the failure of mathematicians before Cantor's time to understand that they must abandon some habitual way of thinking about quantity that kept them from developing the subject of transfinite numbers. Explain the meaning of the phrase "habitual way of thinking about quantity".

4. There are no logical difficulties in Cantor's concept of transfinite numbers. Is this really the case?

5. Cantor's father had urged him to study engineering, a more profitable pursuit than teaching mathematics. Cantor started to follow this down-to-earth career and ended by contributing to the most abstract regions of mathematics. How did he manage to do it?

6. We can say that Cantor's work got the reception that **innovation and originality** usually encounter: neglect, ridicule and even abuse. Agree or disagree.

7. Toward the end of Cantor's life (he died in 1918), the uncommon sense of his logic finally gained some recognition from a few colleagues. Why?

8. Today Cantor's work is so widely and completely accepted that many profound mathematicians are quite willing to devote themselves

to the solution of further problems with transfinite numbers. Confirm or deny.

9. Set theory should be included among the great scientific revolutions which has transformed the mathematical outlook since the end of the XIX c. Agree or disagree.

10. In spite of old and recent attacks on set theory, the majority of today's mathematicians agree with Hilbert's dictum: "No one shall expell us from the paradise which Cantor has created for us". Why?

11. "For set theory there is in principle no barrier between the finite and the infinite, indeed here the infinite is the simpler one of the two" (Weyl). Agree or disagree.

12. The notion of a set has always been implicit in mathematics (Boole, Frege, Weierstrass), however, it was G. Cantor who first explicitly introduced and developed it. Why did Cantor come to consider sets? What resulted from his researches? Explain.

13. There exist questions to which the intuitive idea of a set does not give a unique answer. What are those questions?

14. The developments concerning Cantor's Continuum Hypothesis and the Axiom of Choice are often explained by analogy with non-Euclidean Geometry. Why?

15. Attitude toward the Axiom of Choice ran through various degrees of skepticism, from total rejection to complete acceptance. In topology the axiom is used inhesitatingly, as little of the subject can apparently be derived without its use. Why do mathematicians use the Axiom of Choice in Analysis, measure theory, etc.?

16. The independence of the Continuum Hypothesis and the Axiom of Choice is particularly important for Axiomatics. Why?

17. Among the ordered sets, the **linearly ordered sets** are very important. Why? Is the set of all integers well-ordered? Give some exampes of well-ordered sets.

18. In his method of proof Zeno anticipated **Cantor's diagonal principle**. Explain what is meant by this phrase.

19. Discuss the state of alarm which reigned in the world of mathematicians and logicians during the first decade of the XX c. in spite of many brilliant achievements.

20. Mathematicians are no longer very much impressed by the paradoxes. Why? Are the paradoxes no longer menacing?

21. Classify the famous paradoxes and present some ways of escaping or resolving them.

22. Paradoxes of set theory have been the cause of important study and intensive research in the foundations of mathematics. Why?

23. Different schools of thought in the philosophy of mathematics. Their achievements, contributions and failures.

24. The schools of thought displayed diverse viewpoints on possible solutions of the set theory paradoxes and on the questions of what constitutes admissible and justifiable methods in mathematics. What school do you favour more? Why?

25. There are justifiable and unjustifiable methods in mathematics and acceptable results are those that are obtainable by justifiable methods. Give some illustrations of methods, proofs, practices, techniques, etc., that are legitimate and therefore justifiably used in current mathematics.

26. The greatest disagreement among mathematicians concerns the logicists' claim that mathematics can be reduced to Logic. The reasons, to your mind?

Number Continuum

An infinite set is defined to be one that can be put into one-to-one correspondence with a part of itself whereas a finite set cannot be. Thus the set of positive integers is infinite because there is a one-to-one correspondence between the whole class and the even numbers which are only a part of that class. Can every infinite collection be put into one-to-one correspondence with the positive integers? By no means. The set of all numbers between 0 and 1, a collection that includes whole numbers, fractions, and irrationals, cannot be put into one-to-one correspondence with the positive integers. Hence the two collections cannot be equal in number. The number of numbers between 0 and 1 is represented by the transfinite number *C*. Accordingly, any collections of objects in one-to-one correspondence with all the numbers between 0 and 1 must also contain *C* objects.

An example of a set of *C* objects is furnished by the **points on a line segment.** Consider a line and a fixed point 0 on that line. Let us attach to each point on the line the number that expresses the distance of that point from 0, with the added condition that distances to the right of 0 are to be positive and those to the left, negative. There is, then, a one-to-one correspondence between the numbers from 0 to 1 and the points on the line to which the numbers are attached. This implies that the number of these points is *C*. Stated arithmetically, the set of positive real numbers is in one-to-one correspondence with the real numbers between 0 and 1 and hence the number of positive real numbers is *C*. The number of points on a line segment and the number of points on an entire half — line are **the same** despite the fact that one is infinite in length and the other is just one unit long. Actually a line segment could have been two units long or any other finite length and our result would have been the same. Hence the number of points on any line segment is always *C*. This conclusion, like others, seems to violate our intuition. What right have we, however, to expect more points on the larger of two line segments? What precise knowledge about points and lines supports such an expectation? Euclidean geometry does require that any line segment contain an infinite number of points since any line segment, however small, can be bisected; but **this geometry says nothing about the number of points on a segment.** Cantor's theory does, and it informs us that any two line segments, regardless of their lengths, possess **the same** number of points. This conclusion is not only logically sound but it also permits us to dispose of some perplexing questions about the nature of space, time and motion that had bothered philosophers for over two thousand years.

Our intuitions of space and time suggest that any length and any interval of time, no matter how small, may be further subdivided. The mathematical formulation of these concepts takes into account this property. For example, any line segment may be bisected by a precise Euclidean construction. The mathematical line contains additional properties. Any length consists of points, each of which has no length; moreover, these points are related to each other as are the numbers of the number system. Now between any two numbers there is an infinite number of other numbers; for example, between 1 and 2 there are 1 1/2, 1 1/4, 1 1/8 and so on. Hence, between any two points on a line there is an infinite number of other points. Similarly, the mathematical concept of time regards time as consisting of instants each with no duration, which follow each other as do the numbers of the number system. Thus twelve o'clock is an instant and there is an instant corresponding to any number of seconds after twelve o'clock that we can name. It is true, then, for instants as for points on a line that **there is an infinite number of instants between any two.**

Zeno's Paradoxes

There are difficulties in mathematical concepts of length and time which were first pointed out by the Greek philosopher Zeno, but which can now be resolved by use of Cantor's theory of infinite classes. Let us consider a formulation by Betrand Russell of Zeno's Achilles and tortoise paradox. Achilles and the tortoise run a race in which the slow tortoise is allowed to start from a position that is ahead of Achilles' starting point. It is agreed that the race is to end when Achilles overtakes the tortoise. At each instant during the race Achilles and the toirtoise are at some point of their paths and neither is twice at the same point. Then, since they run for the same number of points, the tortoise runs through as many distinct points as does Achilles. On the other hand, if Achilles is to catch up with the tortoise, he must run through more points than the tortoise does since he has to travel a greater distance. Hence, Achilles can never overtake the tortoise.

Part of this argument is sound. We must agree that from the start of the race to the end the tortoise passes through as many points as Achilles does, because at each instant of time during which they run each occupies exactly one position. Hence there is a one-to-one correspondence between the infinite set of points run through by the tortoise and the infinite set of points run trough by Achilles. The assertion that because he must travel a greater distance to win the race Achilles will have to pass through more points than the tortoise is not correct, however, because, as we know, the number of points on the line segment Achilles must traverse to win the race is **the same** as the number of points on the line segment the tortoise traverses. We must notice, however, that the number of points on a line segment has nothing to do with its length. It is Cantor's theory of infinite classes that solves the problem and saves our mathematical theory of space and time. For centuries mathematicians misunderstood the paradox. They thought it merely showed its poser Zeno was ignorant that infinite series may have **a finite sum.** To suppose that Zeno did not recognize it is absurd. The point of the paradox could not be appreciated until mathematics passed through the third crisis. Does it make sense to speak of completing an infinite series of operations? Cantor holds that it does make sense to talk of testing an infinity of cases. The paradox is not that Achilles doesn't catch the tortoise, but that he does!

In his fight against the infinite divisibility of space and time Zeno proposed other paradoxes that can be answered satisfactorily only in terms of the modern mathematical conceptions of space and time and the theory of infinite classes. Consider an arrow in its flight. At any instant it is in a definite position. **At the very next instant,** says Zeno, it is in another position. When does the arrow go from one position to the other? How does the arrow manage to get to a new position by the very next instant? The answer is that there is no next instant, whereas the argument assumes that there is. Instants follow each other as do numbers of the number system, and just as there is not next larger number after 2 and 2 1/2 there is no next instant after a given one. **Between any two instants an infinite number of others intervene.**

But this explanation merely exchanges one difficulty for another. Before an arrow can get from one position to any nearby position it must pass through an infinite number of intermediate positions, one position corresponding to each of the infinite intermediate instants. How does it ever manage to get to that nearby position if it has to pass through an infinite number of intermediate ones? This too is no difficulty. To traverse one unit of length an object must pass through an infinite number of positions but the time required to do this may be no more than one second, for even one second contains an infinite number of instants. There is, however, a greater difficulty about the motion of the arrow. At each instant of its flight the tip of the arrow occupies a definite position.At that instant the arrow cannot move, for an instant has no duration. Hence at each instant the arrow is at rest. Since this is true at each instant, the moving arrow is always at rest. This paradox is almost startling; it appears to defy logic itself. The modern theory of infinite sets makes possible an equally startling solution. **Motion is a series of rests.** Motion in nothing more than a correspondence between positions and instants of time, the positions and the instants each forming an infinite set. At each instant of the interval during which an object is in "motion" it occupies a definite position and may be said to be at rest.

Does this mathematical concept of motion satisfy our conception of the physical phenomenon of motion? Does not our intuition suggest that motion is something more than an object being in different positions at different instants of time? Here again our intuition cannot be trusted too much. A "motion" picture is no more than a series of stills flashed on the screen at the rate of sixteen per second. That is, it consists of motionless pictures presented to the eye at a rate rapid enough to give the illusion of motion. This motion, then is no more than a series of rests. The mathematical theory of motion should be more satisfying to our intuition for it allows for an infinite number of "rests" in any interval of time. Since this concept of motion also resolves paradoxes it should be thoroughly acceptable. The basic concept in the study of infinite quantities is that of a collection, a class, or a set of objects, as for example, a set of points on a line, and a set of instants in time. Unfortunately, this seemingly simple and fundamental concept is beset with difficulties, that revealed themselves in Zeno's paradoxes.

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